Table OV-2. Standards for Mathematical Practice (MP)

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meanings of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **Students build proofs by induction and proofs by contradiction. CA.3.1** (for higher mathematics only).

Table OV-2 *(continued)*

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a Web site, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP.6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

MP.7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square, and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Table OV-2 *(continued)*
MP.8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Table OV-3 summarizes the eight MP standards and provides examples of questions that teachers might use to support mathematical thinking and student engagement (as called for in the MP standards).

Table OV-3

Summary of the Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
MP.1 Make sense of problems and persevere in solving them. • Mathematically proficient students interpret and make meaning of the problem to find a starting point. • Analyze what is given in order to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Monitor their own progress and change the approach if necessary. • See relationships between various representa-tions. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Continually ask themselves, "Does this make sense?" • Can understand various approaches to solutions.	• How would you describe the problems in your own words? • How would you describe what you are trying to find? • What do you notice about _________? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. What might you change? • Talk me through the steps you have used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you [organize, represent, show, etc.] _________?

Table OV-3 *(continued)*

Summary of the Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
MP.2 Reason abstractly and quantitatively. • Mathematically proficient students make sense of quantities, and the relationships between quantities, in problem situations. • Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and flexibly use operations and their properties. • Create a logical representation of the problem. • Attend to the meaning of quantities, not just how to compute them.	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is _________ related to _________? • What is the relationship between _________ and _________? • What does _________ mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide that you needed to use _________ in this task? • Could we have used another operation or property to solve this task? Why or why not?
MP.3 Construct viable arguments and critique the reasoning of others. • Mathematically proficient students analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others, and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve or revise the argument. • Compare two arguments and determine if the logic is correct or flawed.	• What mathematical evidence would support your solution? • How can we be sure that _________? How could you prove that _________? • Will it still work if _________? • What were you considering when _________? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? (What was unknown?) • Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? • What is the same and what is different about _________? • How could you demonstrate a counter-example? • I think it might be clearer if you said _________. Is that what you meant? • Is your method like Shawna's method? If not, how is your method different?

Table OV-3 *(continued)*	
Summary of the Standards for Mathematical Practice	**Questions to Develop Mathematical Thinking**
MP.4 Model with mathematics. • Mathematically proficient students understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the mathematics they know to solve everyday problems. • Simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram, and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving or revising the model. • Ask themselves, "How can I represent this mathematically?"	• What math drawing or diagram could you make and label to represent the problem? • What are some ways to represent the quantities? • What is an equation or expression that matches the [diagram, number line, chart, table, etc.]? • Where did you see one of the quantities in the task in your equation or expression? • How would it help to create a [diagram, graph, table, etc.]? • What are some ways to visually represent __________? • What formula might apply in this situation?
MP.5 Use appropriate tools strategically. • Mathematically proficient students use available tools including visual models, recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	• What mathematical tools could we use to visualize and represent the situation? • What information do you have? • What do you know that is not stated in the problem? • What approach would you consider trying first? • What estimate did you make for the solution? • In this situation, would it be helpful to use a [graph, number line, ruler, diagram, calculator, manipulatives, etc.]? • Why was it helpful to use __________? • What can using a __________ show us that __________ may not? • In what situations might it be more informative or helpful to use __________?

Table OV-3 *(continued)*	
Summary of the Standards for Mathematical Practice	**Questions to Develop Mathematical Thinking**
MP.6 Attend to precision. • Mathematically proficient students communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand the meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • What would be a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language, definitions, properties (and so forth) can you use to explain ________? • Can you say it in a different way? • Can you say it in your own words? And now say it in mathematical words. • How could you test your solution to see if it answers the problem?
MP.7 Look for and make use of structure. • Mathematically proficient students look for the overall structures and patterns in mathematics and think about how to describe these in words, mathematical symbols, or visual models. • See complicated things as single objects or as being composed of several objects. Compose and decompose conceptually. • Apply general mathematical patterns, rules, or procedures to specific situations.	• What observations can you make about ________? • What do you notice when ________? • What parts of the problem might you [eliminate, simplify, etc.]? • What patterns do you find in ________? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to ________? • In what ways does this problem connect to other mathematical concepts?

Table OV-3 *(continued)*	
Summary of the Standards for Mathematical Practice	**Questions to Develop Mathematical Thinking**
MP.8 Look for and express regularity in repeated reasoning. • Mathematically proficient students see repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details in the problem-solving steps. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results.	• Explain how this strategy works in other situations. • Is this always true, sometimes true, or never true? • How would we prove that ________? • What do you notice about ________? • What is happening in this situation? • What would happen if ________? • Is there a mathematical rule for ________? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice? • How is this situation like and different from other situations using this operation?

Adapted from Kansas Association of Teachers of Mathematics 2012, 3rd Grade Flipbook.

Ideally, several MP standards will be evident in each lesson as they interact and overlap with each other. The MP standards are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (**MP.1**), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender and reinforce a "growth mindset."[5] In Dweck's estimation, "[g]rowth-minded teachers tell students the truth [about being able to close the learning gap between them and their peers] and then give them the tools to close the gap" (Dweck 2006).

Structuring the MP standards can help educators recognize opportunities for students to engage with mathematics in grade-appropriate ways. In figure OV-1, the eight MP standards are grouped into four categories. These four pairs of standards can also be given names, beginning with the rectangle on the far left and then moving from the bottom to the top with the other three rectangles. These names can become a sentence teachers might ask at the end of every day—for example, "Did I *Make Sense of Math* and *Math Structure* by using *Math Drawings* to support *Math Reasoning*?" This approach can help teachers to continually incorporate the core of the MP standards into classroom practices.

5. According to Dweck, a person with a growth mindset believes that intelligence is something that can be nurtured and gained. When people with this type of mindset do not meet the expected level of performance on a test or an assignment or have difficulty understanding a concept, they work hard at it, believing that if they just try hard enough, they will achieve the desired outcome.