What you'll Learn About

- How use slopes to create a solution to a differential equation

Given the function $y = \frac{1}{2}x^2$. Find what $\frac{dy}{dx} = x$

At each grid point representing integers, calculate the value of the derivative and draw a short line segment with that slope.

$x = 0 \quad \frac{dy}{dx} = 0$

$x = 1 \quad \frac{dy}{dx} = 1$

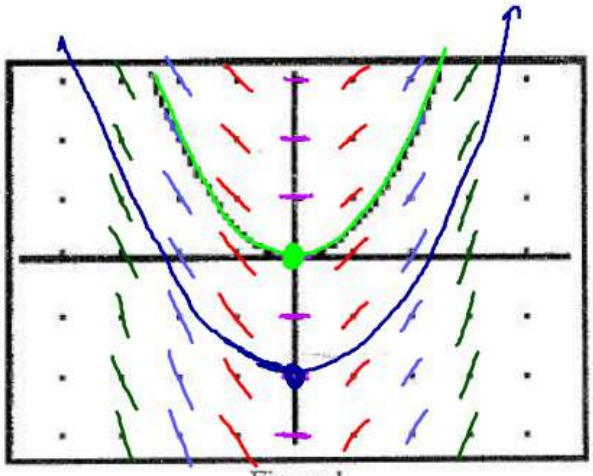


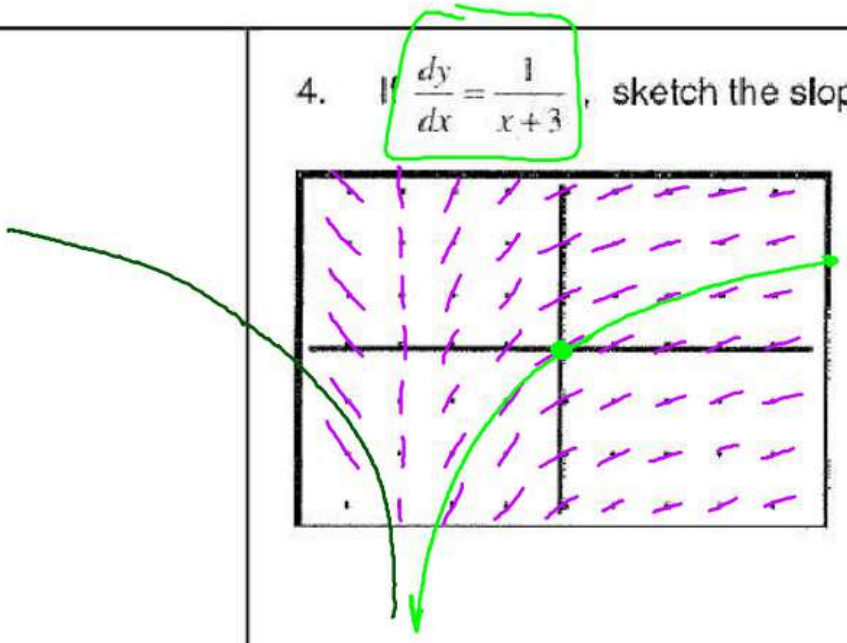
Figure 1

A) What family of functions seems to match all the slope fields?

Parabolas

B) What is an initial condition of the function graphed?

4. If $\frac{dy}{dx} = \frac{1}{x+3}$, sketch the slope field



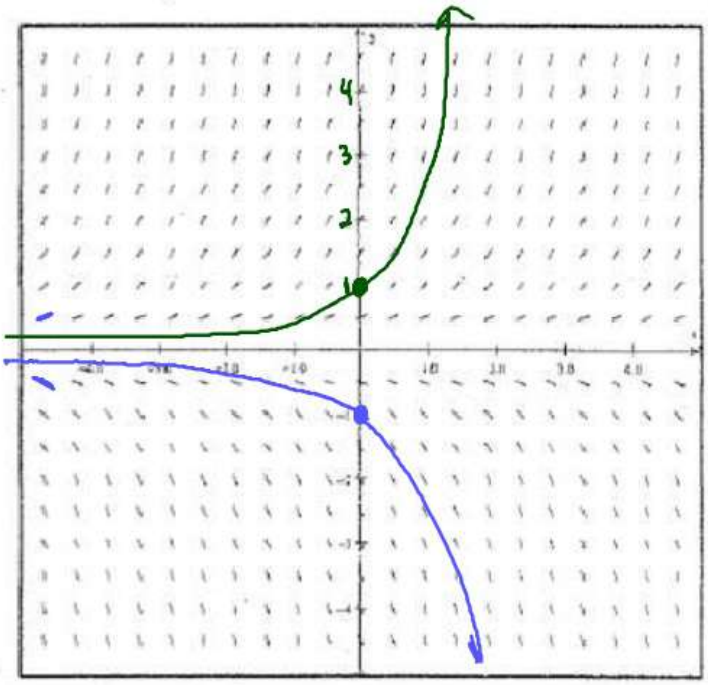
$x = 0 \quad \frac{dy}{dx} = \frac{1}{3}$

$$\frac{dy}{dx} = \frac{1}{x+3}$$

$$\int dy = \int \frac{1}{x+3}\, dx$$

$$y = \ln|x+3| + C$$

Given the slope field shown below answer the following questions.



a) Sketch a path of the unique solution that passes through (0, 1).

b) Sketch a path of the unique solution that passes through (0, -1).

c) What familiar functions do these resemble? exponential

d) Given $\frac{dy}{dx} = y$, verify your guess analytically.

$$\frac{dy}{y} = \frac{y\,dx}{y}$$

$$\int \frac{1}{y}\,dy = \int 1\,dx$$

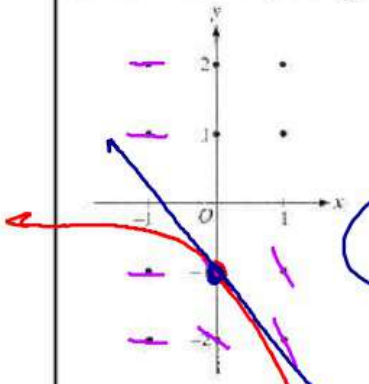
$$\ln|y| = x + C$$

$$y = e^{x+C} \qquad y = Ae^{x}$$

(-1, 0) $m = \frac{0}{0}$

5. Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$

a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated and for $-1 < x < 1$, sketch the solution curve passing through the point (0, -1).



$x+1=0$ $y=0$ (x-axi

$x=-1$

(0,-2) $m = -\frac{1}{2}$

(1,-1) $m = \frac{2}{-1} = -2$

(0,-1)

(1,-2) $m = \frac{2}{-2} = -1$

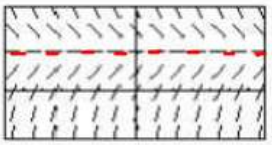
b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$

$y = -1 - 1x$

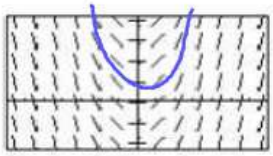
c) Find the particular solution y = f(x) to the given differential equation with the initial condition f(0) = -2.

Match the following differential equation with the correct slope field

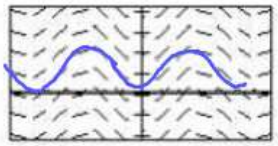
(A)



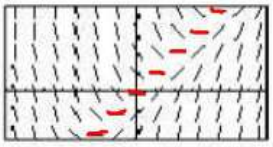
(B)



(C)



(D)



7. $\frac{dy}{dx} = \sin x$

C

$y = -\cos x + C$

8. $\frac{dy}{dx} = x - y$

D

$0 = x - y$

$x = y$

9. $\frac{dy}{dx} = 2 - y$

$y = 2$

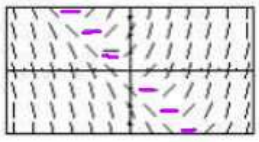
A

10. $\frac{dy}{dx} = x$

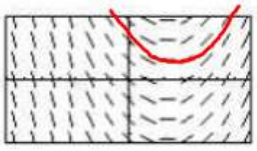
$y = \frac{1}{2}x^2$

B

(A)



(B)



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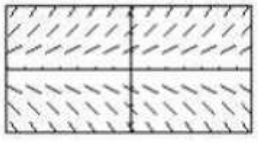
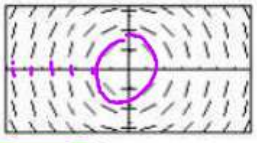


(D)



11. $\frac{dy}{dx} = 0.5x - 1$

B

12. $\frac{dy}{dx} = 0.5y$

C

13. $\frac{dy}{dx} = -\frac{x}{y}$

UNO $y = 0$ (x-axis)

D

14. $\frac{dy}{dx} = x + y$

A