SFUSD Math Core Curriculum

Grade 4

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Unit 4.3: Whole Number Division

Big Idea

Division and multiplication are inverse operations and often either of them can be used to solve problems involving division situations.

Unit Objectives

- Students represent division using concrete and visual models as well as equations..
- Students connect the area model to a partial quotient algorithm to solve division problems.
- Students interpret the remainder in division word problems.

Unit Description

This unit covers how to divide whole numbers of up to 4 digits by 1 digit. They work with division as the inverse operation to multiplication. They work with a wide variety of contexts that lend themselves to division. They develop their understanding of division as rectangles of given areas (dividend) with one known length (divisor) and one unknown length (quotient). They use rectangles to develop an understanding of partial quotients, first with a diagram and then with a partial quotient algorithm. Students learn how to interpret the remainder based on the context of the division problem

CCSS-M Content Standards

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular

arrays, and/or area models.
For additional resources to support students' fluency with division facts, see the folder 4.3 Additional Division Fluency Resources.

Progression of Mathematical Ideas

Prior Supporting Mathematics	Current Essential Mathematics	Future Mathematics
In Grade 2, students practiced skip counting by 5s, 10s, and 100s. They "divided" shapes into two, three, and four equal parts. In Grade 3, students learned concepts of multiplication, including arrays. They learned to interpret whole number quotients of whole numbers, such as 56 ÷ 8, as equal shares. They used multiplication and division within 100 to solve word problems involving equal groups, arrays, and measurement quantities. Finally, they determined the unknown whole number in a multiplication or division equation such as 8 x ? = 48. In Unit 4.2, students learned to multiply multi-digit numbers using area models and partial products.They learned to think of multiplication and division in terms of comparing one whole number to another whole numbers, such as 25 is five times as many as 5.	In this unit, students find whole number quotients with up to four-digit dividends and one-digit divisors using strategies based on place value. They interpret the remainder in the context of the problem. **Note:** The standard long division algorithm is not a Grade 4 or Grade 5 standard. Rather, students learn to divide using place value understanding.	In Grade 5, students will fluently multiply numbers using the standard algorithm. They will find whole number quotients of up to four-digit dividends and two-digit divisors using strategies based on place value or properties of operations. Students will extend their understanding of division to include division of decimals to the hundredths place using concrete models or properties of operations. They will divide whole numbers by unit fractions and unit fractions by whole numbers. In Grade 6, students will fluently interpret and compute quotients of fractions. They will fluently divide multi-digit numbers using the standard algorithm.

Unit Design

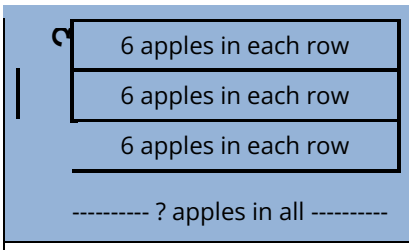
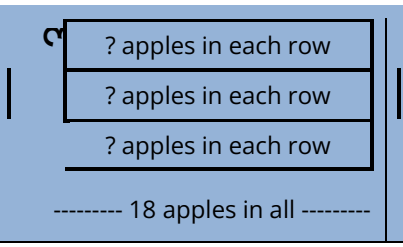
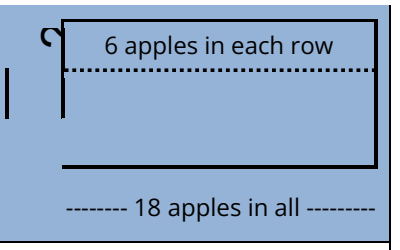
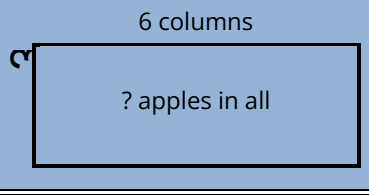
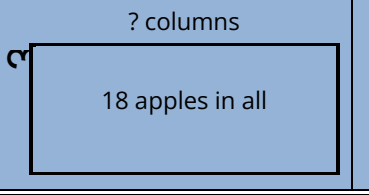
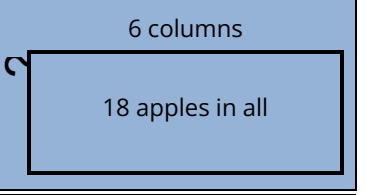
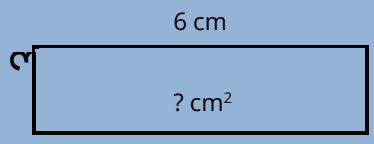
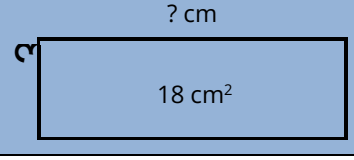
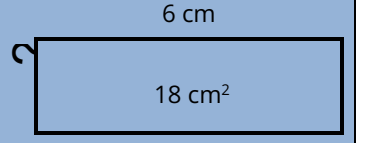
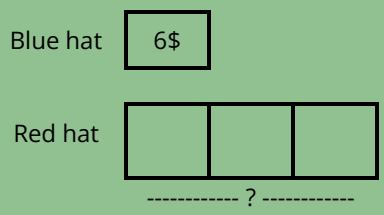
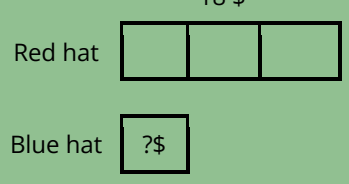
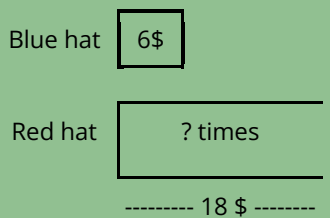
Entry Task: *What do you already know?*
Apprentice Task: *What sense are you making of what you are learning?*
Expert Task: *How can you apply what you have learned so far to a new situation?*
Milestone Task: *Did you learn what was expected of you from this unit?*

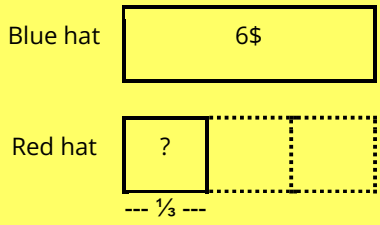
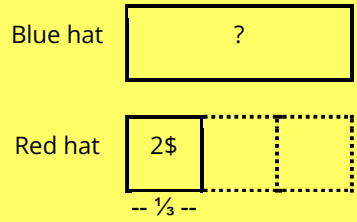
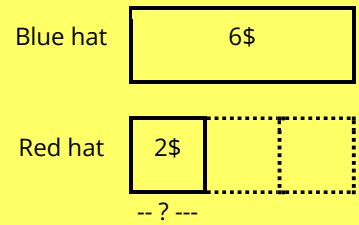
Multiplication and Division Situations in the CCSS-M, with Tape Diagrams

Blue problem types are introduced in Grade 3; green are introduced in Grade 4; yellow are introduced in Grade 5

In this unit, students mostly encounter the Equal Group type of division situation, though they also work with arrays in the *Division Array* game which is repeated throughout the unit, and they work on problems involving multiplicative comparison in Lesson Series 1 and the Apprentice Task.

	3 x 6 = ☐	**3 x ☐ = 18 and 18 ÷ 3 = ☐**	**☐ x 6 = 18 and 18 ÷ 6 = ☐**
Equal Groups of Objects	**Unknown Product** There are 3 bags with 6 apples in each bag. How many apples are there in all? 3 bags 6 apples in each bag | | | ---------- ? apples in all ---------- *Measurement example:* You need 3 lengths of string, each 6 inches long. How much string will you need altogether? 3 pieces of string each piece is 6 inches | | | ---------- ? total inches ----------	**Group Size Unknown (Partitive)** If 18 apples are shared equally into 3 bags, then how many apples will be in each bag? ---------- 18 apples ---------- ? apples in each bag | | | 3 bags *Measurement example:* You have 18 inches of string, which you cut into 3 equal pieces. How long will each piece of string be? ------ 18 inches of string ------- each piece is ? inches | | | 3 equal pieces	**Number of Groups Unknown (Quotitive)** If 18 apples are to be packed 6 to a bag, then how many bags are needed? ---------- 18 apples ---------- 6 apples in each bag | ⋮ | ? bags *Measurement example:* You have 18 inches of string, which you cut into pieces that are 6 inches long. How many pieces of string will you have? ------ 18 inches of string ------ each piece is 6 inches | ⋮ | ? pieces
Arrays[2] and Area[3]	***Equal groups language***		
	Unknown Product There are 3 rows of apples with 6 apples in each row. How many apples are there?	**Unknown Factor** If 18 apples are arranged into 3 equal rows, how many apples will be in each row?	**Unknown Factor** If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?

			
	Row and column language		
	Unknown Product The apples in the grocery window are in 3 rows and 6 columns. How many apples are there? 	**Unknown Factor** If 18 apples are arranged into an array with 3 rows, how many columns of apples are there? 	**Unknown Factor** If 18 apples are arranged into an array with 6 columns, how many rows are there? 
	3 x 6 = ☐	**3 x ☐ = 18 and 18 ÷ 3 = ☐**	**☐ x 6 = 18 and 18 ÷ 6 = ☐**
Arrays[2] and Area[3]	***Area***		
	Unknown Product What is the area of a 3 cm by 6 cm rectangle? 	**Unknown Factor** A rectangle has an area 18 square centimeters. One side is 3 cm long. How long is a side next to it? 	**Unknown Factor** A rectangle has an area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? 
Compare	***A > 1***		
	Larger Unknown A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?  *Measurement example:* A rubber band is 6 cm long. How long will the rubber band be when	**Smaller Unknown** A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?  *Measurement example:* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How	**Multiplier Unknown** A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?  *Measurement example:*

	it is stretched to be 3 times as long?	long was the rubber band at first?	A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
	A < 1		
	Smaller Unknown **⅓ x 6 = ☐** A blue hat costs $6. A red hat costs 1/3 as much as the blue hat. How much does the red hat cost? 	**Larger Unknown** **⅓ x ☐ = 2** A red hat costs $2 and that is 1/3 of the cost of a blue hat. How much does a blue hat cost? 	**Multiplier Unknown** **☐ x 6 = 2** A red hat costs $2 and a blue hat costs $6. What fraction of the cost of the blue hat is the cost of the red hat? 

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Making Sense of Remainders

An important new topic in this unit is the interpretation in remainders. In Grade 3, students saw that division problems either resulted in even groups or did not. In 4th grade, students think more deeply about what happens when the groups are not even. We call what is left over the *remainder*. What one does with the whole part and the remainder depends on the situation. In Lesson Series 3, students learn that the possibilities for the interpretation rests on 4 questions:

- Is the unknown the number of groups or the number in each group? (in other words, is the situation partitive or quotitive?)
- Do we need to include everything?
- What should we do with the groups?
- What should we do with the remainder?

This chart summarizes the ways students can interpret groups and remainders

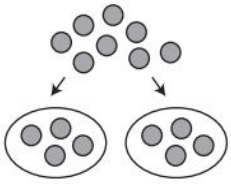
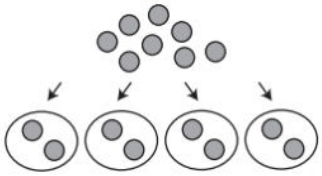
Include the remainder		**Leave the remainder out**	
PARTITIVE (number in each group unknown)	**QUOTITIVE** (number of groups unknown)	**PARTITIVE** (number in each group unknown)	**QUOTITIVE** (number of groups unknown)

Use the remainder to make uneven groups*	Use the remainder to make an extra group*	Ignore the remainder	Ignore the remainder
Use only the remainder	Use only the remainder	Use only the remainder	Use only the remainder

* In grade 5, students will consider whether to divide the remainder into fractional parts.

About Partitive and Quotitive situations

The distinction between Partitive and Quotitive situations is summarized in this diagram.

Sharing Interpretation: 8 ÷ 2 means 8 divided or shared evenly, as between 2 people	**Grouping Interpretation:** 8 ÷ 2 means determine how many groups of 2 can be made with 8
	

For more, please Read "Two Ways of Thinking about Division" by Donna Curry available via TERC and see the video in the Math Teaching Toolkit: http://www.sfusdmath.org/videos.html

About notation for remainders

Some of us were taught to write (7 ÷ 2) as 7 ÷ 2 = 3 R1. The problem is that "3 R1" is ambiguous and may cause confusion. Bill McCallum, one of the authors of the Common Core Standards, wrote this on his blog:

> *7 ÷ 3 = 2 1/3 is really a Grade 5 understanding, although 7/3 = 2 1/3 is fine for Grade 4. Bear in mind also that writing something like 7 ÷ 3 = 2 R 1 is simply wrong ... the thing on the right is not a number, and this usage leads to strange things like 7 ÷ 3 = 9 ÷ 4 (because they are both "equal" to 2 R 1). Correct usage would either use words like "3 goes into 7 twice with a remainder of 1" or, if you want to write a correct equation, you would write 7 = 2 × 3 + 1.*

For this reason, we will either write out the solution: *Pablo can make 2 groups of 3 and will have 1 person left over* or write an equation using 7 = (2 x 3) + 1 notation.

Unit Overview (19 days)

	Days	**Description**	**Core Math**
Entry Task	1	**Amusement Park Rides**: Students determine divisibility of 40 into 6, 5, 4, and 2 groups and how to show this with diagrams.	Division can be thought of as equal grouping. Some numbers cannot be divided equally into whole number quotients by given numbers.
Lesson Series 1	5	Students discern when a situation calls for multiplication, division, or either. They explore connections between a variety of strategies to represent	Multiplication and division are inverse operations. The factors of a number are also divisors or quotients of that number. So, when we consider a

		division, such as tape diagrams, arrays, number lines, and area models.	number being divided by another, we can use multiplication or knowledge about factors.
Apprentice Task	1	**The Baker**: Students use division of two-digit by one-digit numbers in context. In the process, students connect their previous understanding of multiplication arrays and factors to division.	Multiplication and division are inverse operations. Factors of a given number are also divisors or quotients of that number. So, when we consider a number being divided by another, we can use multiplication or knowledge about factors.
Lesson Series 2	5	Students use the area model from multiplication and apply it to division as a missing side length. This leads to an exploration of area models as tools to represent partial quotients, and finally to a more generalizable method for dividing using partial quotients.	Any dividend can be broken up into smaller parts, which are easier to divide. This division relies on understanding of partial products as well as the relationship between the dimensions of a rectangle and its area.
Expert Task	1	**School Report Cards**: Students divide three-digit numbers by one- and two-digit numbers and apply these quotients to find the cost of paper for school report cards.	The quotient tells us the quantity in each group when there is equal sharing.
Lesson Series 3	5	Students focus on the meaning of the remainder in division contexts. They learn that the remainder of a whole number division problem can be interpreted in a variety of ways that are dependent on context. The last day of this lesson series is a re-engagement day with division concepts.	Not every division problem has a whole number quotient. In cases when there is a remainder, we have to make decisions that make sense with the situation.
Milestone Task	1	**Field Trip**: Students apply concepts of division and/or multiplication as well as the interpretation of remainders to a real-world context of students and adults on school buses.	Division helps us solve real-world problems in which we want to create equal groups. When there is a remainder, we have to interpret it based on the context of the problem.

Resources

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Student Pages **Blackline Masters** **Materials**

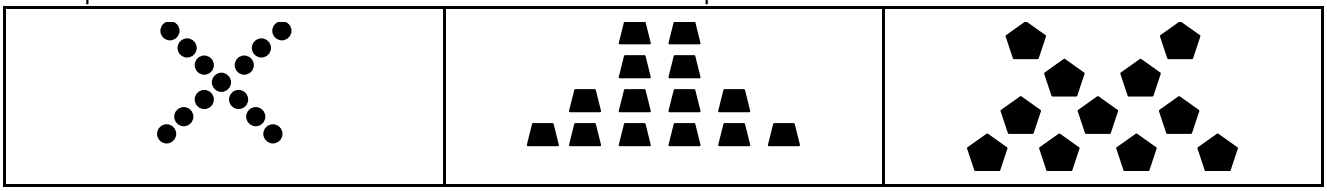
Entry Task S = Spanish C = Chinese	Unit 4.3 Family Letter S C Amusement Park Rides S C		Counters Poster paper Optional video: tinyurl.com/ogtxuj7
Lesson Series 1	Division Array Game S C Multiplication or Division? S C Cecilia's Plums Part 1 S C Cecilia's Plums Part 3 S C	Multiplication or Division? BLM S C Division Array Number Cards BLM Cecilia's Plums Part 2 BLM S C Cecilia's Plums Solutions Teacher	Dice Counters Envelopes Document camera
Apprentice Task		The Baker BLM S C	Poster paper Scissors and tape Markers
Lesson Series 2	Do I Multiply or Do I Divide? S C Partial Quotient and Division Rectangles S C Practice Partial Quotient and Division Rectangles S C Angel Island Camping Trip S C	Division Array Game BLM S C	Dice Number cards (from LS1) Envelopes Poster paper Markers Sticky notes (optional) video: Partial Quotients: http://goo.gl/75G69Y
Expert Task		School Report Cards BLM S C	Poster paper Markers Optional: Scissors, Tape or glue, Sticky notes
Lesson Series 3	Division with Remainders S C Interpret the Remainder S C Bay Area Basketball League S C	Rocking Remainders BLM S C Re-engagement Division Crossword BLM S C Re-engagement Division Rectangles BLM S C Re-engagement Make 24 BLM S C Re-engagement Sharing Tips BLM S C	Division with Remainders Presentation S C 1 die for every 2 students Calculators (optional) Poster paper Markers Scissors Sticky notes (optional) Tape or glue iPad and/or computers (optional)
Milestone Task		Field Trip BLM S C Constructed Response S C	

Math Talks Bank

How could I use this strategy during this unit?
Below are the Math Talks suggested for this unit. These Math Talks are also listed within each lesson. See the Math Teaching Toolkit at www.sfusdmath.org/math-talks for ideas about deepening your Math Talks. Math Talk Visuals are here.

Expression Talk

The Expression Talks in this unit continue with the same process as in Unit 4.1.



Number Talk with Multiplication and Division

The Number Talks with Multiplication / Division in this unit continue with the same process as in Unit 4.2.

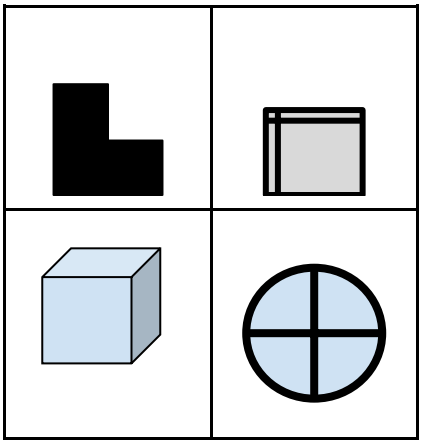
25 x 7	72 ÷7	99 x 6

Which One Does Not Belong?

The WODB in this unit continue with the same process as in Unit 4.2.

red	**six**
240	**10 + 5**

63	**500**
150 x 3	**10 ÷2**



Number Strings

The Number Strings in this unit continue with the same process as in Unit 4.2.

5 x 5 5 x 10 5 x 2	3 x 10 3 x 20 3 x 30	4 x 10 4 x 5 4 x 2	5 x 5 5 x 10 5 x 2	6 x 10 6 x 5 6 x 6

85 ÷ 5	3 x 1 96 ÷ 3	3 x 1 48 ÷ 4	*85 ÷ 5*	6 x 2 *96 ÷ 6*

Entry Task

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Core Math	Division can be thought of as equal grouping. Some numbers cannot be divided equally into whole number quotients by given numbers.
Description	**Amusement Park Rides** Students determine divisibility of 40 into 6, 5, 4, and 2 groups and how to show this with diagrams.
CCSS-M Standard(s)	**Number and Operations in Base Ten[2]** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [2.] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
Resources and Setup	• Amusement Park Rides S C • Counters • Poster paper, 1 per pair (8.5 x 14 is a good size) • Short roller coaster video (optional): http://tinyurl.com/ogtxuj7 • Amusement Park Rides Suggested Answer Guide Teacher • Optional: Divide and Ride / Read Aloud (Math Stories) video
Homework	Entry Task HW S C

Lesson Plan	
LAUNCH **1** (10 min)	Tell students that they will be helping a group of 40 friends decide what ride to go on at an amusement park. You may want to show a short video of a roller coaster to get them interested. http://tinyurl.com/ogtxuj7 Tell students that they will be working with partners on this problem. They will make a small poster to show their work. Remind them of the **Rule of Four** as a way to think about different representations of their thinking. For more information on the **Rule of Four** see Digital Math Teaching Toolkit http://www.sfusdmath.org/rule-of-four.html Pass out counters so that partners may use them to model the division problems. Pass

	out poster paper about 15 minutes into the task so that students may first concentrate on the math before showing their work on a poster.
EXPLORE **2** (40 min)	Circulate as students work with partners and ask them to justify their reasoning. **→Key Math to Observe** • How are students diagramming the division into equal parts? • How do students talk about 40 divided by 6, including the remainder? • How do students think about the amusement park rides with 39 instead of 40 people?
SUMMARIZE **3** (10 min)	**→ Core Math to Emphasize** • 40 can be divided equally among 5, 4, 2, but not 6. • 39 cannot be divided equally among any of the given numbers. • In this problem, division is about the number of equal groups that can be made (quotitive division). Based on your observations, choose 2–3 partners to share their thinking about the problem. Instead of debriefing the whole task, choose one or two questions that bring up the most interesting discussion points. More information about Quotitive Division is in Lesson Series 1 Day 4.

Notes	Universal Support
• Students may not know how to represent this problem in a variety of ways. • Students may not know what to do when one number does not evenly divide into another one.	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students. **Considerations for emerging bilingual students:** • Provide verbal instructions if needed. • Provide sentence frames as needed. • The video of the amusement park ride will offer context.

Extensions	Have students determine the number of students that can fill up cars for all of the rides.

Lesson Series 1 **Overview**

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Description

Students connect the concept of division to the concept of multiplicative comparison through the use of visual models such as tape diagrams and area models. They learn the partial quotient algorithm and connect it to an area model with a missing side length (missing factor). They apply these ideas to solving problems with single-digit divisors and up to four-digit dividends.

Standards

Number and Operations in Base Ten[2]

Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

	Day 1	Day 2	Day 3
Core Math	Multiplication and division are inverse operations. Multiplication situations involve iterating equal groups into wholes while division situations involve taking wholes and partitioning them into equal groups.	Array models previously used in multiplication also work for division.The array model can be used to answer two questions: *How many groups? How many in each group?*	Division can be represented with equal groups, number lines, arrays, and area models with missing factors.
Description	Students analyze three problems and decide how to use either multiplication or division to solve them.	Students play a game called *Division Rectangles* to reinforce the connection between division and the array model.	This is Part 1 of a 3-day arc of lessons in which students re-engage with the various visual models for division that they learned in 3rd grade and describe how they are connected.
Resources and Setup	Multiplication or Division? S C Multiplication or Division? BLM S C Document camera	Division Array Game S C Division Array Number Cards BLM Envelopes Counters Dice 	Cecilia's Plums Part 1 S C Cecilia's Plums Possible Solutions Teacher

Homework	Day 1 HW S C	Day 2 HW S C	Day 3 HW S C

	Day 4	Day 5
Core Math	Number lines, tape diagrams, arrays, and area models are all connected to each other and can be used to solve division problems.	Number lines, tape diagrams, arrays, and area models are all connected to each other and can be used to solve division problems. When the dividend is a large number, it is useful to break it down to its place value components.
Description	This is Part 2 of a 3-day arc. Students return to the previous lesson's problem and analyze select visual models to see how they are connected to each other.	This is Part 3 of a 3-day arc. Students use what they know about representing division of two digits to explore area models for larger numbers.
Resources and Setup	Cecilia's Plums Part 2 BLM S C	Cecilia's Plums Part 3 S C
Homework	Day 4 HW S C	Day 5 HW S C

Lesson Series 1 – **Day 1**

Core Math	Multiplication and division are inverse operations. Multiplication situations involve iterating equal groups into wholes while division situations involve taking wholes and partitioning them into equal groups.
Description	Students analyze three problems and decide how to use either multiplication or division to solve them.
CCSS-M Standard(s)	**Number and Operations in Base Ten[2]** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [2.] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
Resources and Setup	Multiplication or Division? S C Multiplication or Division? BLM S C Document camera
Homework	Day 1 HW S C

Math Talk

Number Strings

Write one problem at a time and ask students how they solved for the answer for each one. Move through the multiplication problems quickly. When you write the division problem, ask students how they could use the multiplication problems to help them find the quotient.

5 x 5
5 x 10
5 x 2
85 ÷ 5

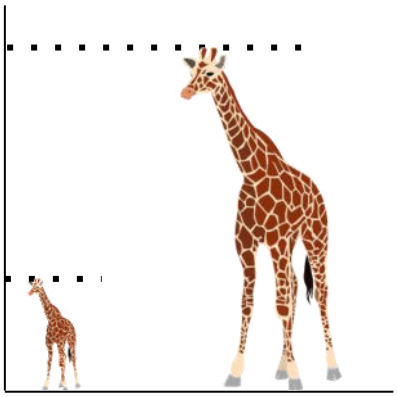
Lesson Plan	
LAUNCH	Tell students that today they will be extending their work with multiplication to think

1

(10 min)

about division as well.

Project the first problem from the Multiplication or Division? BLM. Have students talk with a partner to think of ways they could figure out the height of the mother giraffe. They can record their thinking on the Multiplication or Division? student page. After students have had some time for partner talk, ask for volunteers to share how they figured out the mother giraffe's height. Record their thinking.



Possible strategies

Estimate by iterating lengths

180 + 180 + 180

3 x 180

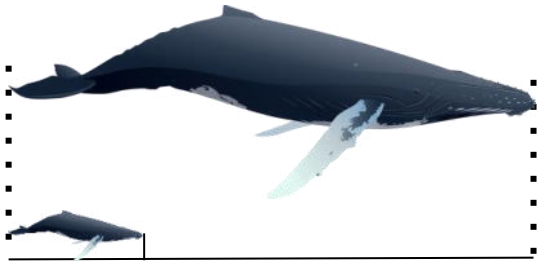
EXPLORE

2

(35 min)

Tell students that they will be working with their same partners on two more questions. They may be able to use multiplication, but they also may be able to use division.

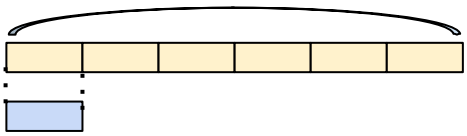
As students work, circulate and take notes on specific student examples you want to discuss that highlight the ways multiplication and division are similar as well as different.



Possible strategies

- Estimate by iterating lengths
- 4 + 4 + 4 ... until 16 is reached
- 4 x 4 = 16 (multiplication fact)
- 4 x __ = 16 (missing factor multiplication)
- 16 ÷ 4 = 4 (division)

This problem can be solved with division strategies as well as with multiplication strategies.



Possible strategies

- Estimate then test by iterating
- 6 x __ = 72 (missing factor multiplication)
- 72 ÷ 6 = 12 (division)

While this problem can be represented with multiplication or division, it is best solved using division strategies.

→Key Math to Observe

- Do students represent their problems with any of the suggested strategies?
- Do students connect ideas, such as multiplication and repeated addition, or the inverse nature of division and multiplication?

SUMMARIZE

3

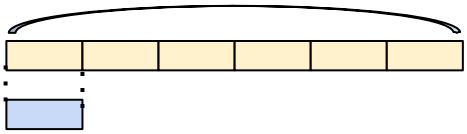
(15 min)

→ Core Math to Emphasize

- Multiplication situations involve iterating equal groups into wholes, while division situations involve taking wholes and partitioning them into equal groups.

Select student work that illustrates the connections between the operations. In particular, look for examples of repeated addition and connect those to multiplication. Also look for work that will support a conversation about the inverse relationship between multiplication and division.

The last problem will most clearly lead to discussing ways to represent and solve division problems. While it is possible to use a guess-and-check strategy to solve this problem, it is much more direct and efficient to use division.



6 x __ = 72 (missing factor multiplication)

6 x 10 = 60; 6 x 11 = 66; 6 x 12 = 72

72 ÷ 6 = 12 (division)
60 ÷ 6 = 10 and 12 ÷ 6 = 2
So 72 ÷ 6 = 10 + 2 = 12

Use this problem and this diagram to discuss how this represents division because we know the whole (72 kg) and we know how it is partitioned (6 equal parts), and what we want to find out is how many in each partition.

	 **Notebook Prompt (5 minutes)** One thing I learned today about solving division problems with drawings and numbers is _____.

Notes	Universal Support
• Students may struggle to attach division meanings to models they previously used for multiplication. • The tape diagram in problem 3 may not make sense to students: they may confuse what is the whole and what are the parts. • Students may not remember or understand division as a missing factor multiplication problem.	**Considerations for students with learning differences:** • Reread the problem from the previous lesson. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** • Discuss the meaning of the contexts and use visuals to reinforce this.

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Lesson Series 1 – **Day 2**

Core Math	Array models previously used in multiplication also work for division. The array model can be used to answer two questions: *How many groups? How many in each group?*
Description	Students play a game called *Division Arrays* to reinforce the connection between division and the array model.
CCSS-M Standard(s)	**Number and Operations in Base Ten[2]** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [2.] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Resources and Setup	• Division Array Game S C
	• For each pair of students:
	  ○ Division Array Number Cards BLM (Copy on tag board and cut out ahead of time.)
	  ○ Envelopes for the number cards (Save these for Lesson Series 2 and 3.)
	  ○ 1 six-sided die
	  ○ 25 counters (cm cubes or something similar)

Homework	Day 2 HW S C

Math Talk

Number Strings

Write one problem at a time and ask students how they solved for the answer for each one. Move through the multiplication problems quickly. When you write the division problem, ask students how they could use the multiplication problems to help them find the quotient.

3 x 10
3 x 20
3 x 30
3 x 1
96 ÷ 3

Lesson Plan	
LAUNCH	This game uses arrays to show the relationship between multiplication and division.

1

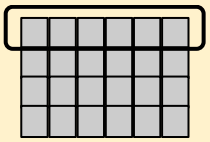
(10 min)

Tell your students they will be learning a new game that they will be playing with a partner several times during this unit to help them think about and model division.

The best way to introduce this game is to play it with a volunteer in front of the class. Students draw a number card for the number of counters. They roll a die for the number of rows. Students lay out the array. Their score is the number of counters in the first row. If all the counters were used in the rectangular array, then the score is triple the number of counters in a row.

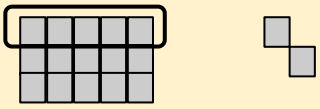
Division Array Game

Round	Rows	Counters per row	Total number of counters	Leftover Counters	Score
Example	4	6	24	0	18
Example	3	5	17	2	5
				Total Score	



4 x 6 array

Score:
Each row has 6 counters.
There are none left over, so the score is triple that = 18



3 x 5 array

Score:
Each row has 5 counters.
There are 2 left over, so the score 5

EXPLORE

Circulate to see how students are interpreting the rules. Notice where students struggle and what they find easy about this game.

2

(35 min)

→Key Math to Observe

- Do students form accurate arrays?
- Do students know the multiplication facts to calculate how many points they get?
- Are students connecting arrays to previous work on multiplication and division?
- Do students understand that this game is based on pure probability and that there is no real strategy to win?

SUMMARIZE

→ Core Math to Emphasize

3 (10 min)	• This game reinforces the relationship between multiplication and division. • Not every number is divisible by another number A debrief of this game might include a discussion on probability. It also can include a discussion of the usefulness of knowing the multiplication tables.  **Notebook Prompt (5 minutes)** One thing I learned about division from this game was _____.

Notes	Universal Support
• Students may struggle with setting up accurate arrays based on the dice. • Students may try to learn the rules of the game without really understanding the underlying concept of arrays in division. • Students may not immediately understand and/or accept that multiplication and division are inverse operations and can often be represented by the same models.	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. • Have students form teams of two, so that pairs are playing against each other. Partners can work together to set up the arrays and determine how many points they get. **Considerations for emerging bilingual students:** • Sentence frames: ○ I rolled _____. ○ I made an array that is __ by __. ○ I have _____ left over.

Additional Notes	Keep all materials organized so that the game may be used again easily in Lesson Series 2 and 3 as a re-engagement activity. This interactive Activity - **Seeing Factors** - Is a virtual version of this game. It shows whole number divisors of a number as rectangles with and without remainders.

Lesson Series 1 – **Day 3**

Core Math	Division can be represented with equal groups, number lines, arrays, and area models with missing factors.
Description	This is Part 1 of a 3 day arc of lessons in which students re-engage with the various visual models for division that they learned in 3rd grade and describe how they are connected.
CCSS-M Standard(s)	**Number and Operations in Base Ten[2]** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
Resources	Cecilia's Plums Part 1 S C Cecilia's Plums Possible Solutions Teacher
Homework	Day 3 HW S C

Math Talk

Number Strings

Write one problem at a time and ask students how they solved for the answer for each one. Move through the multiplication problems quickly. When you write the division problem, ask students how they could use the multiplication problems to help them find the quotient.

4 x 10
4 x 5
4 x 2
3 x 1
48 ÷ 4

Lesson Plan	
LAUNCH **1** (10 min)	Tell students that today they will be working with a division problem and showing different ways to solve it. Read the problem aloud. Ask students what a plum is and why Cecilia would be wanting to bag them into groups of 5.

Cecilia needs to pack **45** plums in bags. If she can put **5** plums in one bag, how many bags does Cecilia need?

Have students think of at least two different ways to solve the problem. Ask them to go to their desks and work independently for about 5 minutes to show different ways to solve the problem. Tell them that when you say it is time, they will be sharing their solutions with each other. If someone has a way to solve the problem that is different from their way, students will copy this way onto their paper.

EXPLORE

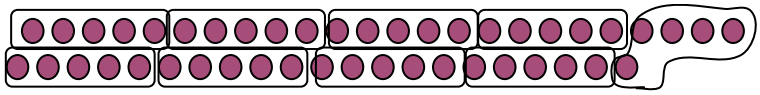
2

(40 min)

Have students work independently for about 15 minutes as they represent their work. Circulate and notice how students are making sense of the problem and representing their work in diagrams and equations.

Select student work that illustrates some or all of the following strategies. If you do not find examples similar to those below, look for a close approximation or for other, similar connections between drawing representations.

Tell the students that you will be using their work as examples for the class and that you will ask them to help explain them.

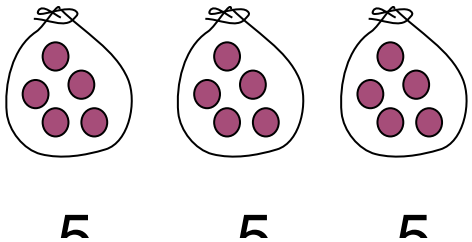
1. Draw all and circle groups of 5	
2. Draw groups of five until there are 45 total	

Call the class together after 15 minutes to discuss the visual models you have **selected** at this point. If possible, put a model of **#1 Draw All** as your first example and ask the student to explain his or her thinking. Ask the class if they understand this strategy and whether they have further questions.

Continue with an example of **#2 Draw Groups of Five** and have the student explain his or her thinking. Ask the class if they understand the strategy and whether they have further questions.

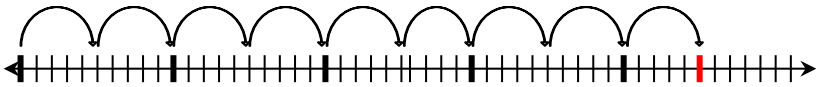
Ask the class to discuss with partners the following questions: How are these two

strategies similar? (*Both strategies involve counting of drawings; the first draws all the plums, the second draws each bag of plums and continues until there are 45. Both show actual representations of plums grouped in bags*.)

3. Skip Counting	

Ask the class if they understand the strategy and whether they have further questions. Ask the class to discuss with partners how the skip counting strategy is related to the previous two strategies.

Continue with an example of **#4 Number Line** and have the student explain his or her thinking. If you do not have a student sample of number line, you can use the #3 Skip Counting to elicit the skip counting strategy through questions such as *Can this be represented on a number line? If so, how?*

4. Use a number line	

For the last 10 minutes of this Explore part of the lesson, have students work with partners to represent their work in each of the four ways discussed.

If students have another way of solving this problem, allow them to work on it for a while, but hold off on discussing it until a later lesson.

Circulate as students are doing this work and **select** new student work that can help illustrate the connection between the drawings and the repeated addition and/or number line.

→Key Math to Observe

- Do students interpret this problem as a division problem and represent their work with equal groups?
- Are students able to describe how these visual models are similar and connected to each other?

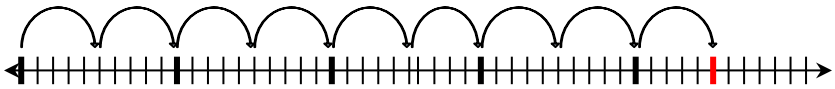
SUMMARIZE	Use student work to **connect** the repeated addition to the number line.

3

(10 min)

Sequence the work, if possible, to draw out the concept of multiplication as an efficient way of adding the same number many times.

Use questions to help students think about how division is connected to multiplication (groups of groups). This will be a crucial understanding throughout this unit on division.

Skip counting	**5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 45** **9 groups of 5** **9 x 5**
Use a number line	 **9 jumps of 5** **9 x 5**

→ Core Math to Emphasize

- Division is the act of separating groups of things into equal groups.
- Division can be modeled in a variety of ways, either starting from the whole group and separating, or by creating equal groups until the whole is achieved.
- Drawings help for thinking about division, and these drawings can be translated to numbers and operations, such as repeated addition and multiplication.

Notebook Prompt (5 minutes)

One thing I learned today about solving division problems with drawings and numbers is _____ .

Notes	Universal Support
• Students may not remember how to represent a division problem. • Students may not accurately count out 45 plums and so their division work will be inaccurate. • Students may not know how to connect repeated addition to jumps on a number line.	**Considerations for students with learning differences:** • Provide real objects to count and separate into groups. **Considerations for emerging bilingual students:** • Make sure students understand the context of the problem. Use realia, such as a real bag and a picture of a plum to make sense of the context. • Partner EL students with bilingual students who can help interpret the language of the word problems.

• Students may not immediately understand how to connect all four representations.	

Lesson Series 1 – **Day 4**

Core Math	Number lines, tape diagrams, arrays, and area models are all connected to each other and can be used to solve division problems.
Description	This is Part 2 of a 3-day arc. Students return to the previous lesson's problem and analyze select visual models to see how they are connected to each other.
CCSS-M Standard(s)	**Number and Operations in Base Ten[2]** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
Resources	Cecilia's Plums Part 2 BLM S C
Homework	Day 4 HW S C

Math Talk

Number Strings

Write one problem at a time and ask students how they solved for the answer for each one. Move through the multiplication problems quickly. When you write the division problem, ask students how they could use the multiplication problems to help them find the quotient.

5 x 5
5 x 10
5 x 2

85 ÷ 5

Lesson Plan	
LAUNCH **1** (10 min)	Review the problem from the previous lesson: Cecilia needs to pack **45** plums in bags. If she can put **5** plums in one bag, how many bags does Cecilia need? Show the summary from the previous lesson. 3. Skip counting: **5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 45** **9 groups of 5**

	9 x 5
4. Use number line	 9 jumps of 5 9 x 5

Ask students to think about how this problem can be represented and solved using other models: arrays, rectangles, and tape diagrams.

Tell students that they will be working on the Cecilia's Plums problem from the previous lesson and making connections between how they solved it to arrays, rectangles, and tape diagrams.

Pass out Cecilia's Plums Part 2 S C and tell students to cut out the sections, paste them on a poster, and describe how each representation is similar to or different than the others.

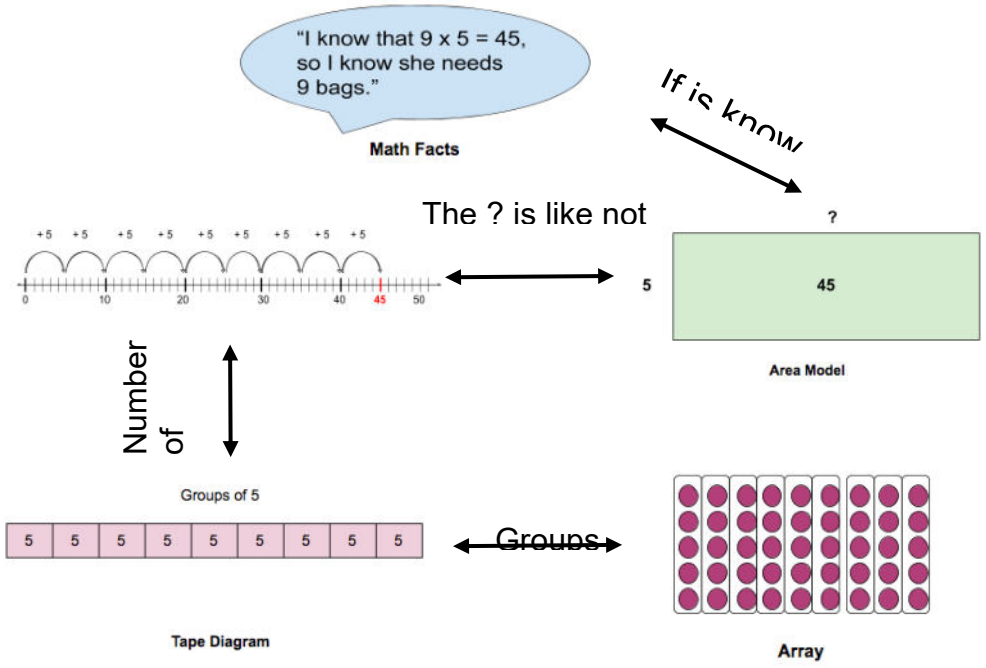
EXPLORE

2

(35 min)

Circulate as students work with partners. Observe how students might be using the previous lesson's work to think about how the representations are related.

Some sample connections made might be:



→Key Math to Observe

- Are students making explicit connections between the array and the area model

(rectangle)?
- Do students understand where the unknown value is and what it tells us?

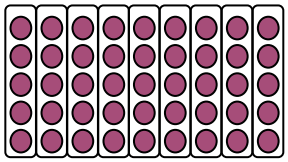
SUMMARIZE

3

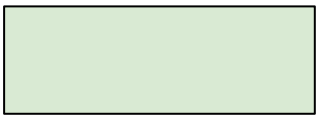
(15 min)

Based on your observations, have students share some of the connections they made between the different representations. The point is not that each representation is equally important but, rather, that there are many ways to show the same idea.

End the lesson with an emphasis on arrays and area models. Discuss how these representations are related to each other. For example, with arrays we can continue to draw groups of 5 until we have reached 45, then we can count how many groups we have. The unknown value in this case is the number of groups. In the area model, the unknown value is one of the side lengths, which is similar to the number of groups.

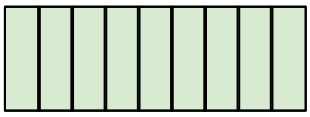


Array



Area Model

In fact, we can use what we see in the array model and show it in the area model:



Area Model

By representing it this way, we can see that the unknown value "?" is really 9. We can use this way of representing division to solve problems with much larger numbers.

→ Core Math to Emphasize

- Array models and area models can be used to represent division problems. Instead of finding the total number of items (arrays) or the area (rectangles), we are solving for the number of groups (arrays) or a missing dimension (rectangles).

Notebook Prompt (5 minutes)

An array is similar to a rectangle because they both _____.

Notes	Universal Support
• Students may not see how an array or a rectangle can represent both a multiplication as well as a division problem. • Students may not accurately count out 45 plums and so their	**Considerations for students with learning differences:** • Reread the problem from the previous lesson. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it.

division work will be inaccurate. • Students may feel more comfortable with "count all" strategies versus counting groups.	**Considerations for emerging bilingual students:** • This is the second day of the same problem. The context should be clear. Clearly explain what an array is and what an "unknown value" is. • Sentence frame: *An array is similar to a rectangle because they both _____.*

Lesson Series 1 – Day 5

Core Math	Number lines, tape diagrams, arrays, and area models are all connected to each other and can be used to solve division problems. When the dividend is a large number, it is useful to break it down to its place value components.
Description	This is Part 3 of a 3-day arc. Students use what they know about representing division of two digits to explore area models for larger numbers.
CCSS-M Standard(s)	**Number and Operations in Base Ten[2]** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
Resources	Cecilia's Plums Part 3 S C Optional: Base 10 Blocks
Homework	Day 5 HW S C

Math Talk

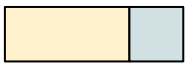
Number Strings

Write one problem at a time and ask students how they solved for the answer for each one. Move through the multiplication problems quickly. When you write the division problem, ask students how they could use the multiplication problems to help them find the quotient.

6 x 10
6 x 5
6 x 6
6 x 2

96 ÷ 6

Lesson Plan	

LAUNCH **1** (10 min)	Tell students that they will be working with a different version of Cecilia's Plums, because now she has a whole lot more of them. Cecilia needs to pack **145** plums in bags. If she can put **5** plums in one bag, how many bags does Cecilia need? Ask students what operation they would use to solve this problem and why? (They would likely choose multiplication or division based on the previous two days' work, but this problem can also be solved using repeated addition or repeated subtraction. If a student says either of those, discuss the pros and cons of using repeated addition/subtraction instead of discarding it as wrong.) Tell students that they will be working on this problem alone for about 10 minutes. Suggest they use strategies from the previous lessons, or any new strategies they want to use because the numbers have changed. Offer students Base-10 Blocks.
EXPLORE **2** (35 min)	Circulate as students work on this problem. **→Key Math to Observe** • Which strategies from the previous lessons are students using? • Are students trying out new strategies based on the size of the numbers? • Do students use knowledge about decomposition of numbers and/or multiplication to solve this? For example, do they notice that 145 is 100 + 45, so that they can use the solution from the previous lesson's problem to help them solve this one?
SUMMARIZE **3** (15 min)	**→ Core Math to Emphasize** • Area models can be used to represent division problems. • When the numbers get large, it is often easier to break them up into smaller parts, solve them, then put them back together. Use student work to make **connections** between the area models (rectangles) from the previous problem (45 divided by 5) and today's problem (145 divided by 5). Base-10 Blocks can help students see this connection. For example, they may use what they know from the previous Cecilia's Plums problem to set this one up as such:  From this point, students need to think about 5 x _?_ = 100. They may know the answer to be 20, or they may need to continue to break down 100 into smaller parts, such as 50 and 50, and find the missing lengths, then add them up. 

	 **Notebook Prompt (5 minutes)** One thing I learned today about solving division problems with drawings and larger numbers is _____.

Notes	Universal Support
• Students might find some of the word problems difficult to understand. • Students may struggle to justify their classification of some word problems, particularly when they involve fractions or decimals.	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** • The language from previous parts of the problem is basically the same. The numbers changed. Clarify any language students may still be confused or unsure about.

Apprentice Task

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Core Math	Multiplication and division are inverse operations. Factors of a given number are also divisors or quotients of that number. So, when we consider a number being divided by another, we can use multiplication or knowledge about factors.
Description	Students use division of two-digit by one-digit numbers in context. In the process, students connect their previous understanding of multiplication arrays and factors to division.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Resources and Setup	• The Baker BLM S C (two per group) • Poster paper • Markers • Scissors and tape or glue (optional, to cut out questions and attach them to posters) • The Baker Suggested Answer Guide Teacher
Homework	Apprentice HW S C

Lesson Plan	
LAUNCH **1** (10 min)	Ask students about their experience going to a bakery. What are some of their favorite baked goods? Clarify vocabulary: bread loaves, muffins, donuts, and cookies. This task is designed to be done in groups. Either use pre-existing groups you have set up in math class or assign these groups before the lesson. **Noticing equitable participation:** Launch this task using the **Multiple Abilities** Strategy. Be thoughtful about creating groups. Create random groups or design groups to support different learners in your class. If you create random groups, be explicit about why you are doing this. For example, tell students that by creating random groups, you are showing confidence in their ability to work with anyone in the classroom. If you set up groups based on perceived strengths and support needs, be careful to give every student some specific roles and responsibilities to avoid accidental tutoring situations or dominance by some students over others. Share a list of strengths you will be looking for students to bring to their work on this task. Your list should include a variety of student mathematical behaviors as well as content-specific strengths. Tell students: *These are some of the things that will help you with this task. Each of you can do some of these things, although probably none of you can do all of these things, really well. By working in a group you can combine strengths to learn from this task.* **Some Suggested Strengths for Multiple Abilities Lists** • Understand the written task and explain it to your partner. • Ask questions of others and explain your thinking. • Organize how to pack things into boxes. • Understand numerical information in a table. • Multiply multi-digit numbers by other numbers. • Add multiple numbers together. • Explain how your partner and you solved the task. **Note:** For more information on how to use Multiple Abilities Lists to launch a task, see Digital Math Teaching Toolkit http://www.sfusdmath.org/multiple-abilities-strategy.html Tell students that you will give each group of two students one task sheet, scissors, glue or tape, and one large piece of paper for them to make a poster of their work. Tell them that they will be creating a group poster and that their work should be organized and clear.

EXPLORE **2** (40 min)	Circulate as groups work. Periodically announce time in the class period, to help students pace themselves. Try to answer group questions instead of individual questions. **→Key Math to Observe** • Can students draw visual models for the division problems in Question 1? • Are students able to determine "twice as many" and "half as much"? • Do students understand that they are not necessarily obliged to use all four box sizes?
SUMMARIZE **3** (10 min)	**→ Core Math to Emphasize** • Division represents equal shares. • Not all numbers can be divided equally by a given number. • Twice as many is multiplicative comparison (x 2). Half as much is also multiplicative comparison, but with fractions. Based on your observations, you might choose to do a **Gallery Walk** to share work. If there are specific issues with visually representing the division problems, you might **select** two or three posters and have groups **connect** their work with one of the questions.  **Notebook Prompt (5 minutes)** One thing I learned today about solving division problems with drawings and larger numbers is _____.

Notes	Universal Support
• Students may be unclear about how to find different combinations of baked goods to meet the requirements of the last question. • Students may assume they need to use division, and subsequently not think they can use knowledge of factors for parts of this task. **Vocabulary:** baker, bread loaves, muffins, different-sized boxes, baked goods	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication chart for students who need it. **Considerations for emerging bilingual students:** • Discuss the meaning of *baked goods*. This is an unusual use of the common word "goods" (from adjective to noun). • Show realia, such as different-sized boxes to give context.

Extensions	Have students write in their math notebooks about other combinations of boxes that hold 60 baked goods.

Lesson Series 2 **Overview**

Description

Students use the area model from multiplication and apply it to division as a missing side length. This leads to an exploration of area models as tools to represent partial quotients, and finally to a more abstract notation for partial quotients.

Standards

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Number and Operations in Base Ten[2]

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

[2.] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

	Day 1	Day 2
Core Math	Area models can be used to visualize and represent both multiplication and division.	Division and multiplication are related and in many contexts are actually interchangeable.
Description	Students revisit the game *Division Rectangles* to reinforce the connection between division and array rectangles.	Students interpret word problems to decide whether to solve them using division or multiplication.
Resources	Division Array Game BLM S C Dice Number cards 6–25 Envelopes	Do I Multiply or Do I Divide? S C
Homework	Day 1 HW S C	Day 2 HW S C

	Day 3	Day 4	Day 5
Core Math	Any dividend can be broken up into smaller parts, which are easier to divide. This division relies on understanding of partial products as well as the relationship between the dimensions of a rectangle and its area.		Division is used to solve many real-world problems in which quantities of things needs to be shared.
Description	Students use rectangular arrays to model division problems (three-digit by one-digit) using the concept of partial quotient.	Students practice using the partial quotient method for doing long division problems. Then they write word problems based on the division problems.	Students apply ideas about division to a real-world problem about a class field trip to Angel Island.
Resources	Partial Quotient and Division Rectangles S C	Practice Partial Quotient and Division Rectangles S C Poster paper Markers *Everyday Mathematics* video: Partial Quotients: http://goo.gl/75G69Y	Angel Island Camping Trip S C Markers Poster paper Angel Island Suggested Answer Guide Teacher
Homework	Day 3 HW S C	Day 4 HW S C	Day 5 HW S C

Lesson Series 2 – **Day 1**

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Core Math	Area models can be used to visualize and represent both multiplication and division.
Description	Students revisit the game Division Array to reinforce the connection between division and array rectangles.
CCSS-M Standard(s)	**Number and Operations in Base Ten[2]** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Resources and Setup	• Division Array Game BLM S C, 1 per student (These will be used several times throughout the unit. Consider placing them in sheet protectors to be reused.)
	• For each pair of students:
	  ○ Number cards 6–25 (from LS1)
	  ○ 1 six-sided die
	  ○ 25 counters (base-10 cubes or something similar)

Homework	Day 1 HW S C

Math Talk

Which One Does Not Belong?

red	**six**
240	**10 + 5**

Lesson Plan	The game *Division Array Game* was first played in Lesson Series 1. This game provides students with an opportunity to practice their **fluency** with basic multiplication and division facts. It also reinforces the concept of division as equal groups in the form of arrays. You may choose to have the whole class play this game with partners and gather small groups of students to work with you, focusing on particular differentiation needs. **Note:** Keep materials for this game—it is played again in Lesson Series 3.
LAUNCH	Tell students that they will be playing the *Division Rectangles* again. Show them Division

1 (5 min)	Array on the document camera and have them look for their previous work in the student workbook. Ask students to remind you of the rules.
EXPLORE **2** (50 min)	Have students play *Division Rectangles* with partners. Either observe their partner work and ask for justification, or call over a small group of students to work with you on some aspect of division or multiplication to support students who require differentiation. **Noticing equitable participation**: Game play is important for all students, but arguably, even more important for students who struggle to retain concepts and/or gain computational fluency because these activities provide them with a fun way to develop fluency. If you gather small groups to work with you on other content, do this for only a small period of time in class so that all students may still benefit from the game play. **→Key Math to Observe** • Are students starting to use mental math instead of manipulatives to find the answer? • Do students think there is a particular strategy with which to win this game?
SUMMARIZE **3** (5 min)	**→ Core Math to Emphasize** • Division is partitioning into equal groups. Because this game is a repeat, focus your debrief on what it was like to play the same game twice in class and what the students think the objective of doing so was in class.  **Notebook Prompt (5 minutes)** One thing I learned today about solving division problems with drawings and larger numbers is _____.

Notes	Universal Support
• Students may struggle with setting up accurate arrays based on the dice. • Students may try to learn the rules of the game without really understanding the underlying concept of arrays in division. • Students may find it difficult to	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. • Have students form teams of two, so that it becomes two against two. Have partners work together to set up the arrays and determine how many points their team should

transition from multiplication to division using the same model.	receive. **Considerations for emerging bilingual students:** ● Provide verbal instructions if needed. ● Provide sentence frames as needed.

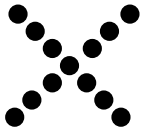
Extensions	Have students write in their math notebooks about how they are improving in this game.

Lesson Series 2 – **Day 2**

Core Math	Division and multiplication are related and in many contexts are actually interchangeable.
Description	Students interpret word problems to decide whether to solve them using division or multiplication.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Resources and Setup	Do I Multiply or Do I Divide? S C
Homework	Day 2 HW S C

Math Talk

Expression Talk



Lesson Plan	
LAUNCH **1** (10 min)	Tell students that just as multiplication and division can both be modeled with number lines and rectangle arrays, they can both be used to solve some kinds of problems. Many times situations can be solved either with multiplication or with division. Sometimes one operation will be easier to use. Today students will be working with partners on a series of word problems that will require them to decide whether to use division or multiplication. Tell students that they will: • Read the problem.

	• Discuss what it means. • Draw a quick diagram or sketch of the situation. • Solve the problem.
EXPLORE **2** (35 min)	Have partners work on the problems together. Observe their work and make every effort to check in with each group, asking them to justify one of their answers. Make note of any patterns of in student ideas that you can address in the Summary. **→Key Math to Observe** • Do students believe that a problem can only be solved with multiplication or only with division? • Are students able to interpret the word problems and find a solution either using division or using multiplication?
SUMMARIZE **3** (10 min)	**→ Core Math to Emphasize** • Some division situations are actually solved using multiplication, such as Problem 1. • These problems can be solved using repeated addition or repeated subtraction. But that is more work than using multiplication or division facts. • When division problems use large numbers, thinking about them as reverse multiplication problems is not always helpful. Based on your observations, choose one problem to go over with the class. Alternatively, you can write the numbers 1–10 (representing the ten questions) and ask the class to tell you whether they chose multiplication or division to solve each one of them. Discuss the disagreements.  **Notebook Prompt (5 minutes)** One thing I learned today about solving division problems with drawings and larger numbers is _____.

Notes	Universal Support
• Students may struggle to determine whether a situation is more easily solved with division or multiplication. • Students may not know enough multiplication facts to feel confident using them to solve division contexts.	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** • Provide verbal instructions if needed. • Provide sentence frames as needed. • Strategic partnerings help students to help translate or explain the problems.

	• Drawing out the situations to show understanding.

Extensions	Have students write in their math notebooks about situations in which they can either use multiplication or division to solve the same problem.
Additional Notes	Avoid teaching key terms in word problems. Not all division problems are actually solved using division, so that the key term approach is misleading and guides students to do computations without making sense of the problem.

Lesson Series 2 – **Day 3**

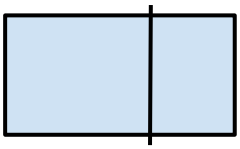
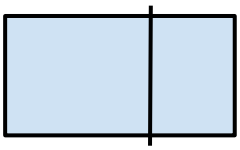
Core Math	Any dividend can be broken up into smaller parts, which are easier to divide. This division relies on understanding of partial products as well as the relationship between the dimensions of a rectangle and its area.
Description	Students use rectangular arrays to model division problems (three-digit by one-digit) using the concept of partial quotient.
CCSS-M Standard(s)	**Number and Operations in Base Ten[2]** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
Resources and Setup	Partial Quotient and Division Rectangles S C
Homework	Day 3 HW S C

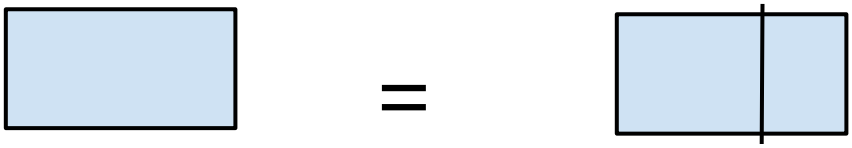
Math Talk

Number Talk with Multiplication

25 x 7

Lesson Plan	
LAUNCH **1** (15 min)	**Homework Debrief:** Have students put last night's homework on their desk. Then have students walk around and read at least two other students' homework and ask themselves: *Does this make sense? Does it represent the equation?* While students are doing this Gallery Walk, pick three or four homeworks to share. Choose examples that will generate good discussion, including at least one in which the situation doesn't match the equation, one that is exceptionally clear, and one that shows divergent thinking. Pay attention to status with this activity, as students are looking at each other's work, and neatness or speed are often associated with 'being

	smart' in math. After the Gallery Walk, share the examples you picked. As you show each one, ask: *Does this situation match the equation? Why or why not? If not, how can we change it to match the equation?* **Lesson:** Tell students that they will learn how to divide larger numbers by breaking them up into smaller parts. They learned about this method in a previous lesson (LS2 Day 1), and today they will be using it as a way to organize intermediate division calculations. Tell students to multiply 8 x 15 using the partial product or box method. (The answer is 120.) If 120 is the dividend (the number to be divided), and you want to find the quotient when it is divided by 8, you can draw a rectangle to represent it as a missing side length. Draw a rectangle and label the inside 120 and one dimension 8.  Think of easy multiples of 8 (**friendly numbers**), such as 80, which is 8 x 10. Dividing up the rectangle you see this:  We know that at least 10 eights are in 120. We have to figure out how many eights "fit" in the remaining part of the rectangle (120 – 80 = 40). We know that 5 eights "fit" in 40.  So, now we know that the missing dimension is 15.



Tell students that they will be working with partners on several division problems. They will represent their work using area models such as the one you just demonstrated.

EXPLORE

2

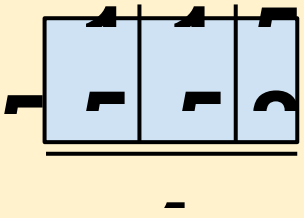
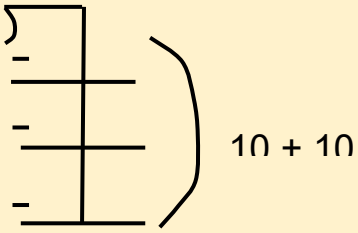
(30 min)

Part 1 (15 minutes)
Have students work with partners to represent the four division problems with area models (division rectangles).

→Key Math to Observe

- Do students understand that there are many ways to divide up the rectangles?
- Do students understand that groups of tens and ones are efficient and "friendly" because they can use their understanding of multiplication facts and extended facts to easily calculate the parts?

Part 2 (15 minutes)
Have students translate their work with division rectangles to a more abstract notation. This is called "partial quotient" long division. Depending on your class, you may want to model this "new" way of notating their thinking. Alternatively, ask students to look at the two ways to show thinking and ask them what **connections** they see between them.

Division Rectangle 125 ÷5	Partial Quotient
	

→Key Math to Observe

- Do students accurately transfer their work to a more abstract form?
- Can students explain what the 10s and 5 on the right side represent in terms of the division rectangle?

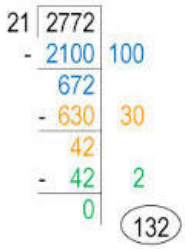
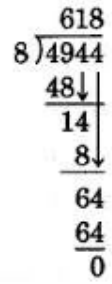
SUMMARIZE

→ Core Math to Emphasize

3 (15 min)	• Division can be represented as a missing dimension of a rectangle. • Rectangles are area models that represent both multiplication and division. • We can break up a larger rectangle into smaller parts to make calculation easier. Have students share their work with representing division with an area model as well as with the partial quotient algorithm. Tell students that the partial quotient algorithm is a way to calculate quotients without having to draw rectangles all the time. It is important to connect the rectangle to the algorithm because it reminds us of the connection between multiplication and division.  **Notebook Prompt (5 minutes)** One thing I learned today about solving division problems with drawings and larger numbers is _____.

Notes	Universal Support
• Students may struggle using an area model for division. This may be a struggle they have from using the same model for multiplication and factors. • The concept of breaking down a number into smaller parts to operate on it (such as partial product and partial quotient) may not be easily understood by some students. • Students who don't yet have a well developed number sense for products or who don't have fluency with single-digit factors may find it difficult to find ways to divide up the rectangle.	**Considerations for students with learning differences:** • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** • Provide verbal instructions if needed. • Provide sentence frames as needed.

Extensions	Have students work on extra problems that you write on the board using this partial quotient method. Alternatively, they can write word problems that use some of the division problems presented in this lesson.
Additional Notes	In order for students to work with division, they need to use their understanding of both multiplication and place value. Partial quotient is directly linked to the partial product method for multiplying, which they learned previously, and relies on a place value understanding. Avoid teaching other algorithms, particularly the standard long division algorithm. There is no specific algorithm called for in the content standards for Grade 4 (or 5) but teachers are encouraged to teach meaning-centered and generalizable strategies and facilitate student learning to use them.

	This is what is meant by *partial quotient algorithm*:  	This is what is meant by *standard division algorithm*: 
	For more information on the partial quotient algorithm see this video: https://goo.gl/JXolpn	The standard division algorithm relies on place value as well, but it is often hidden both by the language we use to describe the procedure as well as the way we notate it.

Lesson Series 2 – **Day 4**

Core Math	Any dividend can be broken up into smaller parts, which are easier to divide. This division relies on understanding of partial products as well as the relationship between the dimensions of a rectangle and its area.
Description	Students practice using the partial quotient method for doing long division problems. Then they write word problems based on the division problems.
CCSS-M Standard(s)	**Number and Operations in Base Ten[2]** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 2. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
Resources and Setup	• Practice Partial Quotient and Division Rectangles S C • Markers • Poster paper • *Everyday Mathematics* video: Partial Quotients: http://goo.gl/75G69Y
Homework	Day 4 HW S C

Math Talk

Which One Does Not Belong?

63	**500**
150 x 3	**10 ÷2**

Lesson Plan

LAUNCH

1

(10 min)

Tell students that today they will be working with a partner to practice division with larger numbers. Remind them that the previous lesson was about a way to do division that is called *partial quotient*.

Tell students that *partial* means part because we break down the larger number into smaller parts. *Quotient* is the answer to a division problem, which they also saw as the missing side length of a rectangular array.

Write the following problem on the board: 125 ÷ 5.
Ask the class to remind you how they solved problems like this in the previous lesson. They should remember the partial quotient organizer, but may be unclear about all the steps. Have them tell you as much as they can and provide hints if they do not completely remember.

Tell students that they will be working in partners and that you will be doing a Groupwork Feedback, looking for pairs who are working well together and who explain the math to each other.

EXPLORE

2

(40 min)

Circulate as students work on the problems.

→Key Math to Observe

- Are students decomposing numbers in logical ways to help with their calculations or do they feel that there is a "right" way to do it?

SUMMARIZE

3

(10 min)

→ Core Math to Emphasize

- Partial quotient uses a place value understanding of how numbers are composed and decomposed in order to make some of the calculations simpler.

Select student work to illustrate the partial quotient method.

	 **Notebook Prompt (5 minutes)** How is place value useful when doing a partial quotient division problem?

Notes	Universal Support
• Students may struggle using an area model for division. This may be a result of using the same model for multiplication and factors. • The concept of breaking down a number to smaller parts in order to operate on it (such as partial product and partial quotient) may not be easily understood by some students.	**Considerations for students with learning differences:** • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** • Provide verbal instructions. • Provide sentence frames. • Have students write their stories in their native language or provide a simple story frame into which they can write their stories. • The anchor chart for partial quotient is an important support. Use colors and clearly name the parts of the algorithm so students can use the language in subsequent classes.

Extensions	Challenge students to connect partial quotients with partial products more explicitly. For example, 8 • 15 = 120 in partial product is (8 • 10) + (8 • 5). Ask students to show how these same numbers appear in partial quotient.

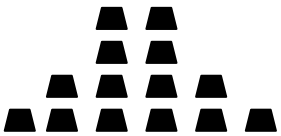
Lesson Series 2 – Day 5

Core Math	Division is used to solve many real-world problems in which quantities of things needs to be shared.
Description	Students apply ideas about division to a real-world problem about a class field trip to Angel Island.
CCSS-M Standard(s)	**Number and Operations in Base Ten²** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.**

	4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. [2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
Resources and Setup	• Angel Island Camping Trip S C • Markers • Poster paper • Sticky notes (optional, for Gallery Walk) • Angel Island Suggested Answer Guide Teacher For more information: http://angelisland.org/history/angel-island-state-park-living-history-program/ Use group roles and Groupwork Feedback for this task to support equitable participation. See Digital Math Teaching Toolkit (http://www.sfusdmath.org/group-roles.html) for additional information. Decide whether to debrief the lesson by doing a Gallery Walk, or presenting student posters using a document camera. If the latter, be sure to provide students with appropriate-sized paper.
Homework	Day 5 HW S C

Math Talk

Expression Talk



Lesson Plan	Although students don't delve formally into remainders until the next lesson series, this task includes situations with remainders.
LAUNCH **1** (5 min)	Ask students if they have visited Camp Reynolds on Angel Island. Tell them that some 4th and 5th grade classes take an overnight field trip to experience what it was like during the U.S. Civil War. Students will be working in groups of four on a problem that involves division, multiplication, and understanding math that arises in real-world situations such as field trips. Tell students that they will be using team roles to create a poster that explains their

	solution to the class. Tell them that you will be taking notes on how well they work as a group and what type of mathematics is shown on their poster.
EXPLORE **2** (40 min)	Circulate to help clear up any confusion and clarify language. Tell students that they will be creating "stand alone" posters that will be shown to the class using a document camera (or in a Gallery Walk). A **stand-alone poster** contains all the work of the group, including questions the group still has. A stand-alone poster is self-explanatory and it describes the work the group did without further oral explanation. It should include numbers, words, and diagrams. It can also include questions the group still has about this problem. Tell the class what you expect of their work on the **stand-alone poster** before they start their work. **→Key Math to Observe** • Do students determine that 124 people are going on the field trip (112 + 12)? • Do students determine that 6 buses will be needed (5 x 25 = 125)? • Do students understand the word "per" and what this means for purchasing the items? • How do students solve the ferry cost question: 7 x 112 and 12 x 12?
SUMMARIZE **3** (15 min)	**→ Core Math to Emphasize** • Multiplication can be used to find repeated groups. Division can be used to determine how a larger group is partitioned out equally. While addition and subtraction can also be used, it is a less efficient way of finding the answer. Have students share their **stand-alone posters** with the class either as part of a Gallery Walk (see Digital Math Teaching Toolkit: http://www.sfusdmath.org/gallery-walk.html) or using a document camera.  **Notebook Prompt (5 minutes)** One thing I learned today about solving division problems with drawings and larger numbers is _____.

Notes	Universal Support
This problem has many components and requires a clear understanding of the context and the quantities being calculated. This may prove challenging for some students.	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it.

Vocabulary: camping trip, items, loaf of bread, per	**Considerations for emerging bilingual students:** • Provide verbal instructions. • Provide sentence frames. • Group students strategically so they can translate as needed.

Extensions	Have students change the quantities in the problem or add new items. Have them determine the price of these items and calculate the cost per person.

Expert Task

Open in Google Drive

Core Math	The quotient tells us the quantity in each group when there is equal sharing.
Description	**School Report Cards** Students divide three-digit numbers by one- and two-digit numbers and apply these quotients to find the cost of paper for school report cards.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Number and Operations in Base Ten[2]** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
Resources and Setup	• School Report Cards BLM S C (two per group) • Large paper for poster (or smaller paper if it will be displayed on document camera) • Markers • Scissors (optional) • Tape or glue (optional) • Sticky notes (optional, for gallery walk) • School Report Cards Suggested Answer Guide Teacher
Homework	Expert Task HW S C

Lesson Plan	Although students don't delve formally into remainders until the next lesson series, this task includes situations with remainders.
LAUNCH	Tell students that they will be working in groups of four on a problem about buying

1

(5 min)

paper to print out school report cards.

This task is designed to be done in groups of four. Either use pre-existing groups or create new ones. Consider using role cards for the groups for more equitable participation and learning. For more information on role cards, see the (Digital Math Teaching Toolkit http://www.sfusdmath.org/group-roles.html).

Noticing Equitable Participation
Launch this task using the Multiple Abilities Strategy. See Digital Math Teaching Toolkit http://www.sfusdmath.org/multiple-abilities-strategy.html The goal of this strategy is to communicate to students the variety of strengths and abilities, including skills, understandings, and mathematics practices, that they bring as individuals to a group task. For this Multiple Ability launch, think of specific skills that several of your least participatory or least confident students possess and can bring to the group activity. Also consider giving them a leadership role in the group, such as group reporter or group leader. Finally, when interacting with groups, pay extra attention to these less confident students such as by directing questions and responses to and from them instead of others.

On the board, share a list of strengths you will be looking for students to bring to their work on this task. Your list should include a variety of student mathematical behaviors as well as content-specific strengths.

Tell students:

> "*These are some of the things that will help you with this task. Each of you can do many of these things, although probably none of you can do all of these things, really well. But by working in a group you can combine strengths to learn from this task.*"

Some Suggested Strengths for Multiple Abilities Lists

- Understand the written task and explain it to your partner.
- Ask questions of others and explain your thinking.
- Create tables of information about each grade level at a school.
- Connect calculations to written answers.
- Multiply multi-digit numbers by other numbers.
- Multiply and divide two- and three-digit numbers.
- Explain how your group solved the task.

Tell the class that you will give each group two task sheets (so that partners within a group can easily read the problem) and one piece of poster paper for them to make a poster of their work.

Tell students that each group will be responsible for creating a **stand-alone poster.** A stand-alone poster is self-explanatory and it describes the work the group did without

	further oral explanation. It should include numbers, words, and diagrams. It can also include questions the group still has about this problem. A **stand-alone poster** has all the group work clearly labeled so that it can be read and interpreted without explanation. Tell the class about any additional expectations for their work on the **stand-alone poster** before they start to work. **Stand-alone posters** can either be shared with a Gallery Walk protocol (see Digital Math Teaching Toolkit: http://www.sfusdmath.org/gallery-walk.html) or they can be brought up to a document camera and displayed for the whole class to discuss.
EXPLORE **2** (40 min)	Circulate and ask groups to justify their work. Try responding only to group questions **→Key Math to Observe** • Do students understand that there are six grade levels? • Can students divide the school population by six? Do students round the answer or do they discuss the remainder? • Do students divide to determine the class size? How do they deal with the remainder? • Do students multiply accurately to find the number of sheets of paper? • How do students approach determining the number of paper packs required?
SUMMARIZE **3** (15 min)	**→ Core Math to Emphasize** • Division tells us how many are in each group when we know how many groups there are. • We can either round numbers before dividing them in a problem that says "about," or we can address the remainder (the leftover). Either do a Gallery Walk or have students display their **stand-alone posters** on the document camera for class discussion.  **Notebook Prompt (5 minutes)** One thing I learned today about solving division problems with drawings and larger numbers is _____.

Notes	Universal Support
• Students may be challenged to distinguish the relationships between the quantities and to determine what is being asked of them.	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it.

• Working in groups of four can be challenging for students who are accustomed to individual work and/or are anxious to move faster than the group.	**Considerations for emerging bilingual students:** • Show packs of paper (also called "reams") to clarify the situation. • Group students strategically: Include bilingual students in each group with EL students or newcomers to support their participation.

Lesson Series 3 **Overview**

Open in Google Drive

Description

Students focus on division problems where the divisor doesn't divide the dividend evenly. They learn that the remainder of these problems can be interpreted in a variety of ways that are dependent on context. On the last day of this lesson series, students re-engage with division concepts from throughout the unit.

Standards

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

	Day 1	Day 2
Core Math	Sometimes division situations don't result in equal groups. We call what is left over the remainder. What one does with the whole part and the remainder depends on the situation.	There are patterns in how the remainder is interpreted. We can ask ourselves particular questions that help us determine how to interpret the remainder.
Description	Students explore division situations in which there are not equal groups and think about what the remainder means.	Students work with partners on a variety of word problems with remainders and interpret what to do with them according to the context.
Homework	Day 1 HW S C	Day 2 HW S C
Resources	Division with Remainders S C Division with Remainders Presentation S C (optional) Calculators (optional)	Interpret the Remainder S C Calculators (optional)

	Day 3	Day 4	Day 5
Core Math	What one does with the whole part and the remainder depends on the situation.	Division helps us solve real-world problems in which we want to determine equal groupings. When there is a remainder, it is important to interpret it correctly.	Core math from previous lessons.
Description	Students work in groups to sort a set of word problems according to how the remainder is interpreted.	Students work in groups to determine how many basketball teams can be formed with a given number of players.	**Re-engagement Day:** Students re-engage with concepts and skills they may have found challenging during the unit. There are five activities, including a game, re-engagement with several of the unit's tasks and one digital resource. sfusdmath.org/reengagement.html.
Resources	Rocking Remainders BLM S C Optional: Interpret the Remainder Chart BLM S C Poster paper Scissors Tape or glue	Bay Area Basketball League S C Poster paper Markers Sticky notes (optional) Bay Area Basketball League Suggested Answer Guide Teacher	Re-engagement Division Crossword BLM S C Re-engagement Division Array Game BLM S C Re-engagement Make 24 BLM S C Re-engagement Sharing Tips S C Re-engaging with Division with Whole Numbers Teacher
Homework	Day 3 HW S C	Day 4 HW S C	Day 5 HW S C

Lesson Series 3 – **Day 1**

Core Math	Sometimes division situations don't result in equal groups. We call what is left over the remainder. What one does with the whole part and the remainder depends on the situation.
Description	Students explore division situations in which there are not equal groups and think about what the remainder means.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Resources and Setup	• Division with Remainders S C • Division with Remainders Presentation S C (optional) • Calculators (optional)
Homework	Day 1 HW S C

Math Talk

Number Talk with Multiplication

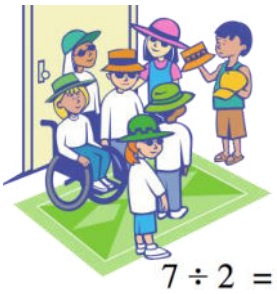
72 ÷7

Lesson Plan	
LAUNCH **1** (10 min)	Tell students that today they will be learning about remainders in a division problem. **Tell them that "remainder" means what is left over when you can't divide into equal groups.** Not all division problems have remainders, but many do have them and we have to be thoughtful about what we do with them. **Note:** There have been remainders in some of the division work students have been doing in this unit. This lesson series is devoted to how we think about and interpret division when there is a remainder in the context of a word problem. Remind students that, in 3rd grade, all the division situations they encountered made nice, equal groups. Tell them that today they will look at a few different problems that

may not divide neatly, and that they will be making decisions about how many groups there are, how many are in each group, and what the remainder is. Then they will need to decide which of those pieces of information help them answer the question.

Introduce Mr. Portillo's example (optionally, use the Division with Remainders Presentation S C). Say:

*Mr. Portillo wants to create **2** equal teams with these **7** children.*



Ask: *What can he do?*

Have students talk with a partner about options.

Ask students these questions about Mr. Portillo's problem:

- Is the unknown the number of groups or the number in each group?
- Do we need to include everything?
- What should we do with the groups?
- What should we do with the remainder?

Some options for this problem include:

- Creating 2 teams of 3 and leaving one person out
- Creating one team of 3 and another of 4
- Having one student be on both teams
- Having the teacher or a student from another class join a team
- etc.

Discuss the merits of each option.

Tell students that they will be working with a partner to solve some division problems and decide what to do if there is a remainder. They will be sharing their solutions with the whole class. Tell them to ask the 4 questions that they asked about Mr. Portillo.

EXPLORE

2

(40 min)

Have students work with partners on **Division with Remainders**. Circulate and ask students how they are identifying and understanding the remainder.

Here are the 4 problems students will work on and some notes about their solution

1. There are 26 tennis balls. Each tennis ball container holds 3 tennis balls. How many containers are needed for all the balls?	*This is a quotitive situation. We must include everything because it says "all the balls" So 2 balls need to go into another container and we need 9 containers.* 26 ÷ 3 = 8 containers with 2 balls left over (because 8 x 3 = 24)
2. Pablo has $65 in tips to share equally among himself and three other	*This is a partitive situation. We want to include everything, if possible. 65 ÷ 4 = 16 and one dollar left.* *There might be debate about what can be done with the*

waiters. How much money does each person get?	*leftover dollar. It can be divided into 4 quarters, or Pablo could keep it.* $1 ÷ 4 = 25¢
3. How many weeks are there in 52 days?	*This is a quotitive situation. It is ambiguous whether or not to include everything. 52 days is 7 full weeks (7 x 7 = 49). You could interpret this problem as needing full weeks, in which case there are 7 full weeks and 3 extra days. Or, you could interpret those extra days as meaning there is another week, so 8 weeks.*
4. Six children are going to share a box of 40 pencils. If they each get the same number of pencils, how many will be left over?	*This is a partitive situation. We don't have to include everything. 6 x 6 = 36, so they'll each get 6 pencils. There will be 4 pencils left over.*

→Key Math to Observe

- Are students making sense of the contexts and dividing accurately?
- Do students make sense of the remainder and think of reasonable ways to deal with it?

SUMMARIZE

3

(10 min)

→ Core Math to Emphasize

- Many division problems do not result in equal groups. The leftover part is called the **remainder**.
- We have to make sense of the context of the situation to decide what part the remainder plays in the solution.

Based on your observations, **select** student work that shows a variety of ways that students interpreted the remainder.

In the first problem, for example, some students may decide that 8 containers are enough, while others may notice that the question states **all the balls** must be used.

The second problem involves division of a whole into fractions, which is a 5th grade idea, but one that students can grapple with.

The third question is open to a variety of interpretations since the context is unclear.

For the fourth question, some students might argue that the pencils can be broken and divided farther.

A class discussion about the relative clarity of each problem might be a productive way to lead into the next day's lesson on interpreting the remainder.

Consider making an anchor chart to show these questions:

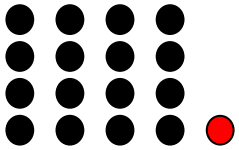
Questions to ask When Solving a Division Problem with Remainders

- Is the unknown the number of groups or the number in each group?
- Do we need to include everything?
- What should we do with the groups?
- What should we do with the remainder?

Notebook Prompt (5 minutes)

One thing I learned today about solving division problems with remainders is _____.

Notes	Universal Support
• Students may not calculate the quotients accurately. • Students may approach this task procedurally and not understand the concept of remainder. **Vocabulary**: quotient, divisor, remainder	**Considerations for students with learning differences:** • Using manipulatives with simpler numbers can help distinguish what the remainder is. For example, 25 ÷4 can be shown with counters like this, making it visually obvious that one is the remainder. **Considerations for emerging bilingual students:** • These problems are highly language dependent because students have to interpret situations. • Problems such as the array problem shown above give students practice with the concept of remainders, which they can apply to more language dependent situations.



Extensions	Have students create their own division problems and solve them in their math notebooks.
Additional Notes	The topic of how remainders in division problems should be written is somewhat controversial. Some of us were taught to write (7 ÷ 2) as 7 ÷ 2 = 3 R1. The problem is that "3 R1" is ambiguous and may cause confusion. Bill McCallum, one of the authors of the Common Core Standards, wrote this on his blog:

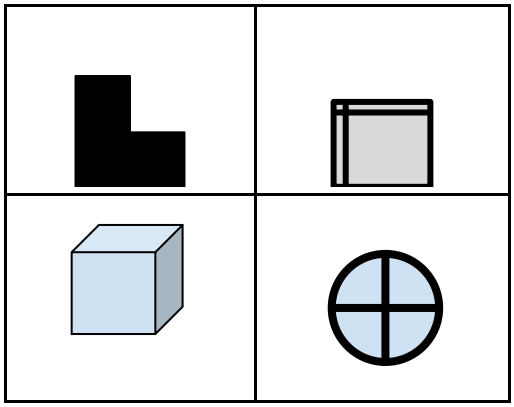
	7 ÷ 3 = 2 1/3 is really a Grade 5 understanding, although 7/3 = 2 1/3 is fine for Grade 4. Bear in mind also that writing something like 7 ÷ 3 = 2 R 1 is simply wrong … the thing on the right is not a number, and this usage leads to strange things like 7 ÷ 3 = 9 ÷ 4 (because they are both "equal" to 2 R 1). Correct usage would either use words like "3 goes into 7 twice with a remainder of 1" or, if you want to write a correct equation, you would write 7 = 2 × 3 + 1. For this reason, we will either write out the solution: *Pablo can make 2 groups of 3 and will have 1 person left over* or write an equation using 7 = (2 x 3) + 1 notation.

Lesson Series 3 – **Day 2**

Core Math	There are patterns in how the remainder is interpreted. We can ask ourselves particular questions that help us determine how to interpret the remainder.
Description	Students work with partners on a variety of word problems with remainders and interpret what to do with them according to the context.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Resources and Setup	• Interpret the Remainder S C • Calculators (optional)
Homework	Day 2 HW S C

Math Talk

Which One Does Not Belong?



Lesson Plan	Note: In this lesson, all the problems use the same numbers. This is intentional, and meant to help students focus on the interpretation of the remainder rather than the computational aspects of division.
LAUNCH **1**	Tell students that today they will think about a number of division situations and what to do with the remainder in each one. Remind students of the questions they can ask themselves in order to decide what to do with the remainder. • Is the unknown the number of groups or the number in each group? • Do we need to include everything? • What should we do with the groups?

(10 min)

- What should we do with the remainder?

Have students work with partners on **Interpret the Remainder**. Circulate around the room and ask students to justify their answers.

EXPLORE

2

(40 min)

As students notice that the problems all use the same number, you may bring the class together and discuss this. Reiterate that their focus is on understanding the context and deciding how to use their results - groups and remainder - to answer the problem.

→Key Math to Observe

- Are students making sense of the word problems and using the contexts of the problems to interpret the remainder?
- Do students divide accurately to find the remainder?
- Do students understand the meaning of each of the four ways to interpret the remainder?

SUMMARIZE

3

(10 min)

→ Core Math to Emphasize

- Depending on the context of a division problem, how we deal with and interpret the remainder is different.

Based on your observations, **select** student work that illustrates the importance of understanding the situation in order to make accurate and useful interpretations of the remainder.

For example, the first problem is a classic dilemma because while students may think the answer is 8 minivans, that would leave one student without transportation, hence they need 9.

See the *Additional Notes* box for details about each problem in the student page.

Leave time for looking at this diagram and classifying some of the problems in this lesson into the right box based on the questions.

Making sense of a division problem with (or without) remainders

Include the remainder		**Leave the remainder out**	
PARTITIVE (number in each group unknown)	**QUOTITIVE** (number of groups unknown)	**PARTITIVE** (number in each group unknown)	**QUOTITIVE** (number of groups unknown)
Use the remainder to make uneven groups*	Use the remainder to make an extra group*	Ignore the remainder	Ignore the remainder

	<table><tr><td>Use only the remainder</td><td>Use only the remainder</td><td>Use only the remainder</td><td>Use only the remainder</td></tr></table>  **Notebook Prompt (5 minutes)** One thing I learned today about solving division problems with remainders is _____.

Notes	Universal Support
• Students may not calculate quotients accurately. • Students may approach this task procedurally and not understand the concept of remainder. **Vocabulary:** quotient, divisor, remainder, interpret	**Considerations for students with learning differences:** • Context is very important in understanding that the remainder may be shown in different ways. Use a simple example, such as *How many cars do we need to go on a field trip if only 3 kids can fit in a car and we have 20 students?* The division problem would give a quotient of 6 with a remainder of 2, but all students should go on the field trip, so we actually need 7 cars. Showing this with a diagram on an anchor chart will support all students. **Considerations for emerging bilingual students:** • Discuss the word **interpret** with your students. Many bilingual students may understand the word "interpret" to mean translation from one language to another. Clarify this. • Act out some of the contexts in the problems so that all students can understand them.

Extensions	Have students create their own division problems and solve them in their math notebooks.

Additional Notes - Here are additional notes about each problem

Problem	Explanation	Notes
50 fourth graders are going on a field trip. The school will rent minivans for the trip. Each minivan holds 8 students. How many vans will the school need?	50 ÷ 8 = 6 , because 6 x 8 = 48, and there will be 2 students left over. We can't leave any students behind, so we need an extra van for the last students.	• Quotitive • Include all • make an extra group In real life, we would probably distribute the students as evenly as possible

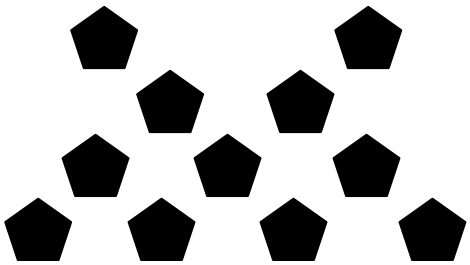
Andrés has 50 baseball cards. He wants to share them equally with his 8 friends. How many cards will each of his friends get?	50 ÷ 8 = 6 with 2 cards left over. No one gets the cards because that wouldn't be fair.	• Partitive • Leave some out • ignore the remainder
A baker has 50 eggs. She needs 8 eggs for each cake she bakes. How many cakes can she bake?	50 ÷ 8 = 6 with 2 eggs left over, not enough to make another cake.	• Quotitive • Leave some out • ignore the remainder
A light bulb factory has 50 light bulbs. They pack the light bulbs so that there are 8 in each box. How many light bulbs will be left over?	50 ÷ 8 = 6 with 2 light bulbs left over.	• Quotitive • Leave some out • Use only the remainder
50 fourth graders are going on a field trip. The school will rent minivans for the trip. There are 8 minivans. How many students will go in each minivan?	50 ÷ 8 = 6 with 2 students left over. They will have to go in one of the vans, so two vans will have 7 students in them (or 1 van will have 8 students)	• Partitive • Include all • Uneven groups

Lesson Series 3 – **Day 3**

Core Math	What one does with the whole part and the remainder depends on the situation.
Description	Students work in groups to sort a set of word problems according to how the remainder is interpreted.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Resources and Setup	• Rocking Remainders BLM S C, one per group • Optional: Interpret the Remainder Chart BLM S C • Poster paper • Scissors • Tape or glue
Homework	Day 3 HW S C

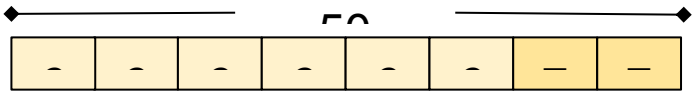
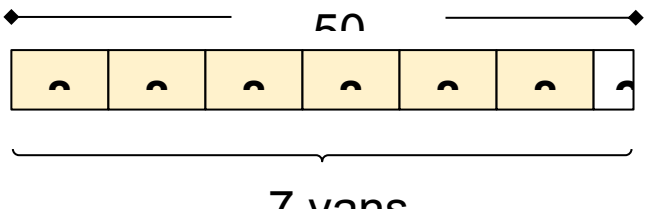
Math Talk

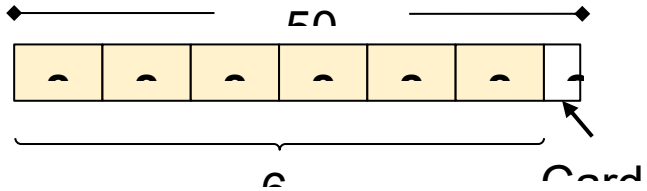
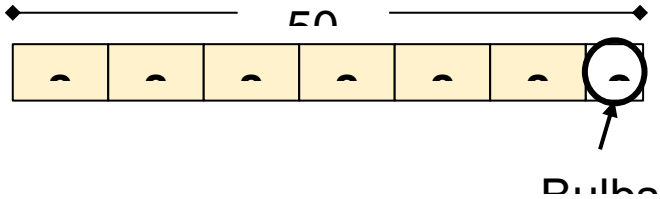
Expression Talk



Lesson Plan	
LAUNCH **1** (10 min)	Remind students that they have been working on how to interpret remainders in division problems. Share the Interpret the Remainder Chart BLM S C with students. It is a summary of the ways remainders are interpreted using the examples from yesterday's lesson.

What do we do with the remainder in a division problem?

Include the remainder **PARTITIVE**	**Use the remainder to make uneven groups**
50 fourth graders are going on a field trip. The school will rent minivans for the trip. There are 8 minivans. How many students will go in each minivan?	 **Some vans will have 6 and some will have 7.** *Another alternative is to put 8 students in one van.*
Include the remainder **QUOTITIVE**	**Use the remainder to make an extra group**
50 fourth graders are going on a field trip. The school will rent minivans for the trip. Each minivan holds 8 students. How many vans will the school need?	 **We will need 7 vans.**

	Other distributions are possible, e.g 8, 8, 7, 7, 7, 7, 6
Leave the remainder out **PARTITIVE or QUOTITIVE** Andrés has 50 baseball cards. He wants to share them equally with his 8 friends. How many cards will each of his friends get?	**Ignore the remainder**  **Each friend will get 8 cards**
Use only the remainder **PARTITIVE or QUOTITIVE** A light bulb factory has 50 light bulbs. They pack the light bulbs so that there are 8 in each box. How many light bulbs will be left over?	**Use only the remainder**  **There will be 2 light bulbs left over.**

Tell students that today they will be working in groups to create a poster in which they sort division problems according to how the remainder is interpreted. They will be reading word problems and deciding how they would interpret the remainder. They will choose four problems (one from each category) to solve on the poster.

This is a good opportunity to do a Groupwork Feedback.

EXPLORE

2

(40 min)

Have students work on **Rocking Remainders—**reading the problems, discussing them, solving them, and interpreting the remainder and determining what needs to be done with it.

→Key Math to Observe

- Are students able to understand a division word problem, solve it, and interpret the remainder logically?

SUMMARIZE

3

(10 min)

→ Core Math to Emphasize

- Depending on the context of a division problem, how we deal with and interpret the remainder is different.

This is a good opportunity to use a **Gallery Walk** protocol. For more information, see Digital Math Teaching Toolkit (http://www.sfusdmath.org/gallery-walk.html). From the

	Gallery Walk, decide on one or two points you want to discuss or reinforce as a class.  **Notebook Prompt (5 minutes)** One thing I learned today about solving division problems is _____.

Notes	Universal Support
• Students may not calculate quotients accurately. • Working in groups of four may be challenging for some students. • Students may approach this task procedurally and not understand the concept of the remainder. • Students may not understand how to interpret the remainder. **Vocabulary:** quotient, divisor, remainder, interpret	**Considerations for students with learning differences:** • Context is critical in understanding how to interpret the remainder. Using smaller numbers is fine as students develop their understanding of this new idea. • Encourage students to draw out each situation and use tape diagrams as needed. • Use the anchor chart to help students relate each problem to a problem they already understand. **Considerations for emerging bilingual students:** • Notice the word **interpret** with your students. Many bilingual students may understand the word "interpret" to mean translation from their home language to English. Clarify this. • Act out some of the contexts in the problems so that all students can understand them. • Asking students to draw what they understand with simple diagrams can help you determine if they are misunderstanding something about the context of the word problems.

Lesson Series 3 – **Day 4**

Core Math	Division helps us solve real-world problems in which we want to determine equal groupings. When there is a remainder, it is important to interpret it correctly.
Description	Students work in groups to determine how many basketball teams can be formed with a given number of players.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Resources and Setup	• Bay Area Basketball League S C • 1 Poster paper per group • Markers • Bay Area Basketball League Suggested Answer Guide Teacher
Homework	Day 4 HW S C

Math Talk

Number Talk with Multiplication

99 x 6

Lesson Plan	
LAUNCH **1** (10–15 min)	If you have a basketball, spin it around and ask students how they feel about basketball. Who is their favorite player? Do any students play on a basketball team?  Tell students that they will be working in groups on a problem about a fictional Bay Area Basketball League. This problem will require them to be able to divide four-digit numbers by one-digit numbers. It will also require them to interpret the remainder because everyone gets a place on a team in this league. **Noticing Equitable Participation:** Students have been working on many group projects during this unit. Discuss strengths you have seen students work in groups: cooperation, sharing work, explaining thinking, etc. For this project, consider using group roles and group questions. Group questions mean that only one person from a group can call you over and ask you questions. Subtly assign this role to a quiet and/or less confident student.
EXPLORE **2** (30–35 min)	Circulate as groups work on the problem. Observe their work and try to only talk to groups who have questions for you and/or need redirection. Avoid the temptation to "reteach" for a group. If you notice obstacles for some groups, call attention to the class and discuss the obstacle or call a **huddle** (one person from each group) in order to give some further instructions to help them. **Marin County** — **1,260 ÷ 9 = 140 teams** **San Francisco County** — **1,578 ÷ 9 = 175 with 3 players left over** **Contra Costa County** — **1,142 ÷ 9 = 126 with 8 players left over**

	→Key Math to Observe • Are students interpreting the context and the larger numbers correctly? • Can students divide four-digit numbers by one-digit numbers using partial quotient or another strategy? • How are students diagramming their work and interpreting the remainders?
SUMMARIZE **3** (10 min)	**→ Core Math to Emphasize** • Depending on the context of a division problem, how we deal with and interpret the remainder is different. In this context, since all players need to be on a team, some teams will have one more person than the others. For a summary, do a **Gallery Walk**, or choose one part of the problem you find most relevant to the concept of interpreting remainders in larger division problems.  **Notebook Prompt (5 minutes)** One thing I learned today about solving division problems is _____.

Notes	Universal Support
• Students may not calculate quotients accurately. • Working in groups of four may be challenging for some students. • Students may not yet know how to write fractions accurately. • Students may approach this task procedurally and not understand the concept of remainder. • Students may not understand how to interpret the remainder. **Vocabulary:** quotient, divisor, remainder, interpret, sign up, basketball league	**Considerations for students with learning differences:** • Context is very important in understanding that the remainder may be shown in different ways. Act out the idea of creating teams with your class to illustrate this. **Considerations for emerging bilingual students:** • Discuss the word **interpret** with your students. Many bilingual students may understand it to mean translation from their home language to English. Clarify this. • Act out the context in the problems so that all students can understand them. • Asking students to draw what they understand with simple diagrams can help you determine whether they are misunderstanding something about the context.

Extensions	Have students answer the question, *What if instead of basketball teams, the same numbers of players were joining a baseball league? How would the numbers change?*

Lesson Series 3 – Day 5

Core Math	• Division and multiplication are inverse operations and can often be used interchangeably to solve problems. • Division can be represented with an area model. • Dividends can be decomposed into smaller parts to make division easier. The

	resulting quotient is called "partial quotient." The partial quotients can be recomposed to find the whole quotient.
Description	**Re-engagement Day:** Students re-engage with concepts and skills they may have found challenging during the unit. There are five activities, including a game, re-engagement with several of the unit's tasks and one digital resource.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Number and Operations in Base Ten[2]** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Resources and Preparation	Re-engagement Division Crossword BLM S C Re-engagement Division Array Game BLM S C Re-engagement Make 24 BLM S C Re-engagement Sharing Tips BLM S C Re-engagement with Division with Whole Numbers Teacher S C Dice iPad and/or computers (optional)
Homework	Day 5 HW S C

Lesson Plan	**Noticing Equitable Participation:** Clear and explicit classroom norms for academic behavior help all students, but, in particular, they allow struggling students an opportunity to learn by creating a sense of purpose and productivity in class. This is especially true during centers work. Be very explicit and persistent with norms for group work and centers.

LAUNCH **1** (5 min)	Tell the class that today they will be working with partners in centers. Explain the centers. Options provided in this unit are: 1. Division Arrays 2. Division Crossword 3. Sharing Tips 4. Make 24 5. Sum Sense tinyurl.com/yy6ql5dt (iPad and computer ready) See **Re-engaging with Division with Whole Numbers Teacher** for details. Go over classroom norms for center work and your expectations for behavior and how the students will know to rotate through them. **Note:** For more information on using math centers, see the Math Teaching Toolkit, www.sfusdmath.org/centers.html You may decide to post yourself at one center to offer extra help for students who need it, or you may circulate around the room observing student work and behaviors. Tell students what you will be doing during centers time.
EXPLORE **2** (45 min)	Depending on your role in the centers, look for the following: **→Key Math to Observe** • How are students understanding and solving the division problems? • How flexible are students with the four numbers and operations in Make 24 to find alternative solutions? • How fluent are students with multiplication and division facts?
SUMMARIZE **3** (15 min)	**→ Core Math to Emphasize** • Division and multiplication are inverse operations and can be used interchangeably. Choose to debrief the work from one center.  **Notebook Prompt (5 minutes)** One thing I learned today about solving division problems is _____.

Notes	Universal Support
• Students may find it challenging to do so many activities in one class period. • Students may not know how to do a center and become overly dependent on external help. • The technology-based activity might be difficult for some students.	**Considerations for students with learning differences:** • Students may benefit from either staying at one center for much of the class or staying in close proximity to you. **Considerations for ELs:** • There are many opportunities for students to practice concepts along with language. • Partner language learners with language mode. • Partner newcomers and students with little confidence in English, with bilingual students who can support them.

Extensions	The re-engagement centers are designed to have students practice a variety of skills and concepts. As such, there is no need for extensions for this lesson.

Milestone Task

Open in Google Drive

Core Math	Division helps us solve real-world problems in which we want to create equal groups. When there is a remainder, we have to interpret it based on the context of the problem.
Description	**Field Trip**: Students apply concepts of division and/or multiplication as well as the interpretation of remainders to a real-world context of students and adults on buses.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Number and Operations in Base Ten[2]** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
Resources and Setup	• Field Trip BLM S C, 1 per student • Constructed Response S C, OPTIONAL, 1 per student • Field Trip Answer Guide Teacher • Field Trip Rubric Teacher • Constructed Response Rubric Teacher
Homework	Milestone HW S C

Lesson Plan	
LAUNCH	Tell students that this is a Milestone Task designed to allow them to show you how

1 (5 min)	well they have learned division of whole numbers and interpreting remainders. This is a problem about a field trip to the War Memorial Opera House in San Francisco. Remind students that they have been working on several problems about different field trips throughout the unit.
EXPLORE **2** (50 min)	Have students work individually on the Task. Circulate to notice what they are doing. **→Key Math to Observe** • How are students understanding the context of the buses and the numbers of people going on the field trip? • How are students interpreting the remainder? • How are students justifying their work?
SUMMARIZE **3** (5 min)	**→ Core Math to Emphasize** • Division is partitioning into equal groups. • Multiplication and division are very interrelated. Often a problem can be solved equally efficiently with either operation. • If there is a remainder, it is interpreted logically depending on the context. When the entire class has finished the task, discuss the questions and how they answered them. Highlight strengths, but also discuss any areas that presented a particular challenge.  **Notebook Prompt (5 minutes)** One thing I learned today about solving division problems is _____.

Notes	Universal Support
• Students may not calculate the quotients accurately. • Working individually on a task for an extended time may be frustrating for some students. • Students may become anxious because they think of this as a test. • Students may approach this task procedurally and not understand the concept of the remainder. • Students may not understand how to interpret the remainder.	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** • Allow students to explain their thinking in their native language. • Drawing and numerical responses can give opportunities for students to show their understanding.