Unit 4.2: Whole Number Multiplication

Big Idea
Multiplicative comparisons compare 2 quantities by showing that one quantity is a given number of times as many as the other.

Unit Objectives
• Students multiply multi-digit numbers using strategies based on place value and properties of operations. • Students compare numbers with multiplicative reasoning. • Students use multiplication to solve real-world and mathematical problems involving measurement. • Students use letters to represent the unknown quantities in equations.

Unit Description
Unit 4.2 extends third grade understanding of multiplication to multi-digit whole numbers (2 digit by 2 digit and up to 4 digit by 1 digit). It connects an area model to a partial product algorithm that relies on decomposition of numbers to their place value components. It introduces the concept of multiplicative comparison with questions such as "*How many times as tall is _____ as _____?*" For more information on models for multiplication, see Multiplication Models. The 4.OA.A.3 standard is a capstone in the development of problem solving skills using the four operations. This standard appears again in Unit 4.3 Whole Number Division, and multiplicative comparisons can be found in Unit 4.6 Factors and Multiples and Unit 4.7 Measurement. This unit highlights multiplicative comparisons in a variety of problem types, some including the use of a letter for unknown quantities. For more information on how students develop their understanding of unknowns across the grade, along with sample problems, read Problems with Unknowns First through Sixth Grades.

CCSS-M Content Standards

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.*

** Note: In this unit, multi-step word problems are limited to addition, subtraction and multiplication with whole numbers.*

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*.

For additional resources to support students' fluency with multiplication facts, see the folder 4.2 Additional Multiplication Fluency Resources.

Progression of Mathematical Ideas

Prior Supporting Mathematics	Current Essential Mathematics	Future Mathematics
In Grade 3, students were formally introduced to multiplication as equal groups. They interpreted unknowns as the missing factor or the missing product in a multiplication problem with 100. They learned to fluently multiply and divide within 100 using strategies such as the relationship between multiplication and division and properties of operations. They used letters and symbols to represent unknown quantities.	In Grade 4, students learn to interpret multiplication with arrays and area models as a precursor to scaling. They learn that quantities can be compared multiplicatively, for example, 35 is 7 times as much or as many as 5. They use this to solve problems involving multiplicative comparison of measurement. They learn how to multiply multi-digit numbers (one digit by up to four digits, and two digits by two digits) using strategies based on place value.	Later in Grade 4, students will multiply whole numbers by unit fractions using repeated addition. In Grade 5, students will learn to fluently multiply whole numbers using the standard algorithm. They will multiply decimals using concepts of place value and properties of operations. Students will multiply whole numbers by fractions and decimals less than 1 and understand the magnitude of the product in terms of scaling. In Grade 6, students will learn to fluently multiply multi-digit decimals using the standard algorithm. In Grade 7, students will learn to apply properties of operations to multiply rational numbers, including integers.

Unit Design

Entry Task: *What do you already know?*

Apprentice Task: *What sense are you making of what you are learning?*
Expert Task: *How can you apply what you have learned so far in a new situation?*
Milestone Task: *Did you learn what was expected of you from this unit?*

Unit Overview (21 days)

	Days	Description	Core Math
Entry Task	1	**Joshua's New Floor** Students find areas of various rectangular rooms with single-digit lengths. They decide which rooms could be covered with 200 square feet of tile. They find some dimensions of an extra room that has an area of 100 square feet.	Multiplication can be represented as the area of a rectangle in which the dimensions (length and width) are the factors and the area is the product.
Lesson Series 1	6	Students expand their understanding of the area model for multiplication to include multi-digit numbers. They learn about multiplicative comparisons and use tape diagrams to solve real-world problems.	Quantities can be compared multiplicatively when we can determine how many times one of the quantities repeats to make another one.
Apprentice Task	1	**Walkathon** Students compare numbers into the tens of thousands, first with additive comparison, and then with multiplicative comparison.	When comparing two quantities, additive comparisons refer to "how much more?" while multiplicative comparisons refer to "how many times as many?"
Lesson Series 2	5	Students learn how to multiply multi-digit numbers using an area model that supports the partial product algorithm.	Numbers can be decomposed by their place value components. These components can be multiplied, then added back together to find the product of two multi-digit numbers. This is called the *partial product method* because the parts are being found separately, then recomposed by adding them together.
Expert Task	1	**Comparing Two Schools** Students compare student populations from two schools using multiplicative comparisons.	Quantities can be compared additively (how many more) or multiplicatively (how many times as many).
Lesson Series 3	6	Students practice solving problems with partial products and work on two tasks that involve using multi-digit multiplication in real-world contexts.	Multiplication problems with two factors can be represented concretely with area models and more abstractly with algorithms based on place value.
Milestone	1	**San Francisco Tour Group**	Multiplication can be used to solve

Task		Students use their knowledge of multi-digit multiplication, addition, and subtraction to solve a problem involving a San Francisco tour group and money.	real-world problems involving equal groups and multiplicative comparison.

Resources

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	Student Pages	Blackline Masters	Materials	Slides
Entry Task S = Spanish C = Chinese	Unit 4.2 Family Letter S C Joshua's New Floor S C	Floor Plan BLM S C	Poster paper Pens Rulers	Entry Task **S**
Lesson Series 1	Extended Facts S C Modeling Multiplication with Area Model S C Multiplication as Comparison S C San Francisco Bay Area Animals S C	One-Digit by Two-Digit Multiplication with Base-10 Blocks BLM S C Equation Cards BLM Multi-Digit Multiplication FAL BLM S C What is Multiplicative Comparison? Teacher	Base-10 blocks Extended Fact Projector Resource Poster paper Markers Projector and screen Giraffe and Photographer Presentation S C	LS1 Day 4 **S** LS1 Day 5 **S** LS1 Day 6 **S**
Apprentice Task	Walkathon S C		Chart paper (optional)	Apprentice Task **S**
Lesson Series 2	How Can I Multiply Two-Digit Numbers? S C 13 x 15 Area Model BLM Multiplication Judo S C How Can I Multiply Two-Digit Numbers with the Box Method? S C	How Can I Multiply Two-Digit Numbers with the Box Method? BLM S C Gray Whale BLM S C From Photo To Poster BLM S C African Animals Photos	Scissors Glue or tape Poster paper Video:: tinyurl.com/jt3my6j Larger paper Markers / colored pencils	LS2 Day 1 **S** LS2 Day 2 **S**
Expert Task	Comparing Two		Poster paper	Expert Task

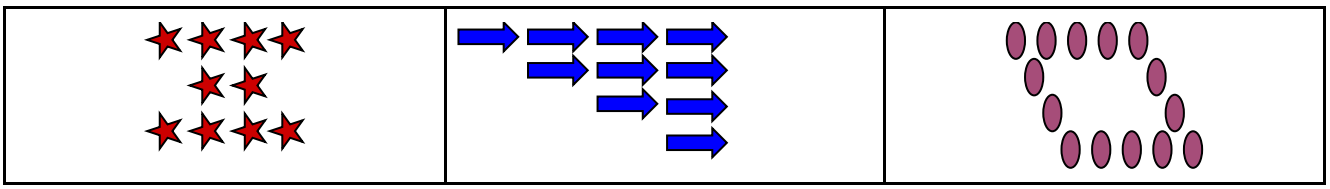
	Schools BLM S C		Markers / Scissors Glue or tape	**S**
Lesson Series 3	Bullseye S C How Can I Multiply with Larger Numbers? S C Re-engagement Reflection S C	Fill It Up BLM S C Fill It Up Game Board BLM S C Kinetic Kingdom BLM S C Four to Go BLM S C Re-engagement Bullseye BLM S C Doubling and Halving BLM S C Multi-Digit Multiplication FAL BLM S C (from LS1 Day 6) Re-engagement Multiplication Judo BLM S Re-engagement Centers Teacher	Dice Poster paper Markers Scissors Glue or tape Sheet protectors Base-10 blocks (optional)	LS3 Day 2 **S** LS3 Day 4 **S**
Milestone Task		San Francisco Tour Group BLM S C		 Milestone Task **S**

Math Talks Bank

How could I use this strategy during this unit?
Below are the Math Talks suggested for this unit. These Math Talks are also listed with each lesson. See the Math Teaching Toolkit section on Math Talks for more information.

Math Talk Visuals are here. The lessons in this unit include four types of Math Talks:

Expression Talk
The Expression Talks in this unit continue with the same process as in Unit 4.1.



Number Talks with Multiplication

This type of Math Talk is intended to bring out a variety of different strategies for multiplication with multi-digit numbers. This can highlight concepts of place value, properties of operations, and strategies such as decomposition/recomposition of numbers, skip counting, and friendly numbers.

For example, students might use the following strategies:

- **12 + 12 is 24 and 24 + 24 is 48, and 48 + 12 is 60.**
- **Or 10 • 5 is 50 and 2 • 5 is 10, and 10 + 50 is 60.**
- **Or skip counting (5, 10, 15, 20, 25 ... 55, 60).**

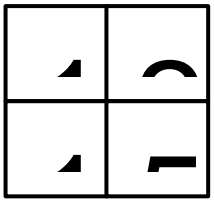
Bring attention to similarities between different strategies, whether they use these or other strategies.

As students share, ask:

- *What questions do you have for ___ about how they solved the problem?*
- *Who thought of it in the same or a similar way?*
- *Who thought of it a different way?*
- *How are these ways similar? How are they similar?*

8 x 15	**4 x 12**	**5 x 16**
12 x 8	**12 x 12**	**15 x 4**
25 x 12	**20 x 13**	**15 x 15**

Which One Does Not Belong?



This type of Math Talk is intended to bring out a variety of different ways we perceive and categorize things. This can highlight concepts of number and geometry, as well as develop categories based on identified attributes.

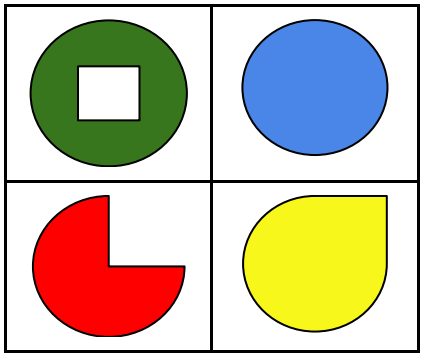
For example, students might use the following reasoning:

- **5 does not belong because it is a single digit number**
- **12 does not belong because it is not divisible by 5**
- **25 does not belong because it is the only square number.**
- **10 does not belong because it is the only one with 0 in the ones place, etc.**

Bring attention to similarities between different categories and their attributes and whether there is more than one way to describe one object as being different from the others.

As students share, ask:

- *What questions do you have for ___ about how they perceived the difference?*
- *Who thought of it in the same or a similar way?*
- *Who thought of it a different way?*
- *How are these ways similar?*



25	**16**
9	**43**

27	**81**
18	**31**

Number Strings

This type of Math Talk is intended to bring out the use of place value understanding in partial product multiplication.

2 • 5
2 • 50
20 • 50

Students might use the following strategies:

I know that 2 • 5 is 10, so I know that 2 • 50 is 100 because 50 is 5 • 10. So the problem is like 2 • 5 • 10. Then I know that 20 • 50 is 1,000 because 100 • 10 is 1,000.

Bring attention to similarities between different strategies, whether they use these or other strategies.

As students share, ask:

- *What questions do you have for ___ about how they solved the problem?*
- *Who thought of it in the same or a similar way?*
- *Who thought of it in a different way?*
- *How are these ways similar? How are they different?*

2 • 5 2 • 50 20 • 50	12 x 5 12 x 15 120 x 50	12 x 10 12 x 5 12 x 15

Entry Task

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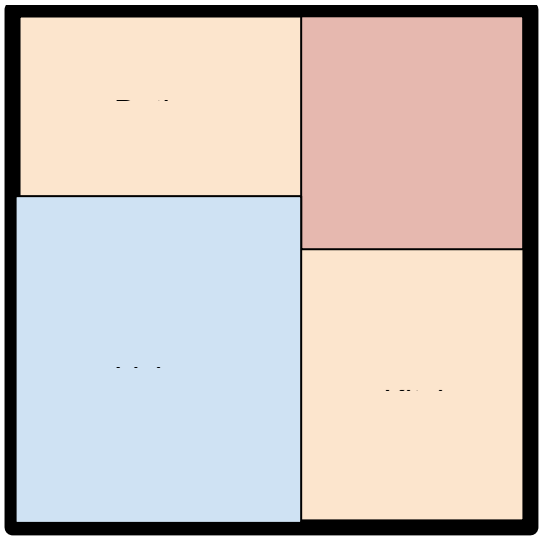
Core Math	Multiplication can be represented as the area of a rectangle in which the dimensions (length and width) are the factors and the area is the product.
Description	**Joshua's New Floor** Students find areas of various rectangular rooms with single-digit lengths. They decide which rooms could be covered with 200 square feet of tile. They find some dimensions of an extra room that has an area of 100 square feet.
CCSS-M Standard(s)	**Measurement and Data** **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** 4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. 4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*.
Resources	• Joshua's New Floor S C • Floor Plan BLM S C • Poster paper and pens • Rulers
Homework	Entry HW S C

Lesson Plan	
LAUNCH **1** (10 min)	

	This Entry Task is designed to assess students' prior knowledge about multiplication represented with an area model. Use the Floor Plan BLM on a document camera to launch the problem. Ask students what they notice about the floor plan. They may notice that this is a house with four rooms, that the rooms are labeled, and that there are measurements of lengths. They may also notice that some lengths are missing.  Tell students that there is a man named Joshua who wants to put tiles in the rooms of his house, but he thinks he does not have enough. Draw a tile on the paper or on the whiteboard to clarify that the tiles in this problem are square with side lengths of 1 foot, so the area of the tile is 1 square foot. Tell students that they will be working with partners on this task. Have them read the problem and ask any clarifying questions before they start working.
EXPLORE **2** (40 min)	As partners work, circulate and observe how they are working together, making sense of the problem, and showing their work. Ask clarifying questions to understand how students are thinking, but avoid reteaching. This Entry Task will provide good insight into how much students remember multiplication from 3rd grade. **→Key Math to Observe** • Are students identifying the correct dimensions of each room to find the areas? • Do students know the multiplication facts needed to solve the problems? • Do students understand that the missing dimensions can be found by calculating the differences of known dimensions? • Do students understand that the sum of the areas of the rooms equals the area of the entire house? • Do students have more than one set of room dimensions for the additional space?
SUMMARIZE	**→ Core Math to Emphasize**

3 (10 min)	• Area is the space inside a shape. • To find the area of a rectangle, multiply one side times the other. (There are no predetermined sides called "length" or "width.") • Rectangles have parallel sides of equal lengths. So if you know the length of one side, you also know the length of the side parallel to it. Based on your observations, select student work that illustrates how multiplication can be represented with an area model as well as how missing dimensions can be determined using properties of polygons. For example, the missing length of the living room can be deduced by noticing that one bathroom wall is 8 feet long and that is the same length as the living room wall in question. Living Room = 72 square feet Bathroom = 48 square feet Bedroom = 49 square feet Kitchen = 56 square feet Total House Area = 225 square feet (which is quite small for a house). The extra space can have up to 100 square feet. Possible dimensions are: 10 x 10 5 x 20 9 x 9 9 x 8 This last problem might be a good source for a classroom discussion about what dimensions the students found, including the meaning of "no more than" as well as a discussion of the commutative property (5 x 20 is the same as 20 x 5 in this context). You could also have a discussion about reasonable dimensions in the real world (1 x 100 is mathematically correct, but in the context of rooms in a house, it is not a practical solution).  **Notebook Prompt (5 minutes)** One thing I remembered about multiplication from this task was _____.

Notes	Universal Support
• Students may not recall how to	**Considerations for students with learning differences:**

multiply one-digit numbers. ● Students may not be fluent with multiplication facts. ● Students may struggle making sense of the floor plan in terms of multiplication. ● Students may not understand how to find the missing side lengths. ● Students may not know how to find different dimensions for the extra room. **Vocabulary:** methods, representations, product, as many as	● Read aloud directions for students as needed. ● Use the provided BLM on the document camera as you read to make sense of the rooms, their dimensions, and other pertinent information. ● Allow students to draw their solutions/representations without any written explanation. ● Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** ● Brainstorm the names of rooms in a house. ● Define *dimension* as the length of a side of a polygon. *Area* is the space inside of a polygon. ● Provide verbal instructions if needed. ● Provide sentence frames as needed.

Extensions	Early finishers can find possible dimensions for another room with an area of 144 square feet. This may lead them to think about two-digit multiplication, such as 12 x 12.

Lesson Series 1 **Overview**

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Description

Students expand their understanding of the area model for multiplication to include multi-digit numbers. They learn about multiplicative comparison and use tape diagrams to solve real-world problems.

Standards

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

	Day 1	Day 2	Day 3
Core Math	Understanding extended facts helps with decomposition of multi-digit numbers and finding partial products.	Multiplication problems with two factors can be represented with area models.	Multiplication can be represented with base-10 blocks using an area model. This leads to an understanding of partial product multiplication.
Description	Students apply their understanding of single-digit multiplication to extended facts and justify their work with visual models as well as place value and properties of operations.	Students model one-digit by two-digit multiplication with base-10 blocks, sketch their models, and show the answer.	Students extend their understanding of single digit multiplication with area models to two digit by two digit multiplication.
Resources	Extended Facts S C Base-10 blocks Extended Facts Projector Resource	One-Digit by Two-Digit Multiplication with Base-10 Blocks BLM S C Equation Cards BLM Base-10 blocks	Modeling Multiplication with Area Model S C Base-10 Blocks

Homework	Day 1 HW S C	Day 2 HW S C	Day 3 HW S C

	Day 4	Day 5	Day 6
Core Math	Quantities can be compared multiplicatively when we can determine how many times one of the quantities repeats to make another one.	Quantities can be compared multiplicatively when we can determine how many times one of the quantities repeats to make another one.	Multiplication situations can be represented with equations and a variety of visual models.
Description	Students learn about multiplicative comparison as a new way to compare two numbers.	Students compare SF Bay Area animal body lengths to determine how many times as long some animals are compared to others.	Students work in groups to match word problems with multiplication equations and visual models.
Resources	Multiplication as Comparison S C	Giraffe and Photographer Presentation S C San Francisco Bay Area Animals S C Projector and screen	Multi-Digit Multiplication FAL BLM S C
Homework	Day 4 HW S C	Day 5 HW S C	Day 6 HW S C

Lesson Series 1 – **Day 1**

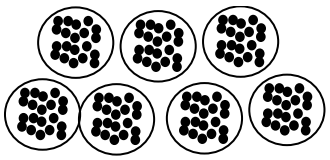
Core Math	Extended facts are a powerful way to multiply larger numbers that are multiples of powers of 10 (10; 100; 1,000; etc.). Understanding extended facts helps with decomposition of multi-digit numbers and finding partial products. **Note:** This lesson addresses concepts from Unit 4.1, specifically, that a digit to the left is 10 times as great as the same digit in its current place.
Description	Students apply their understanding of single-digit multiplication to extended facts and justify their work with visual models as well as place value and properties of operations.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Resources	Extended Facts S C Base-10 blocks, at least 10 each of cubes (ones), longs (tens), and flats (hundreds) for each table group Extended Facts Projector Resource
Homework	Day 1 HW S C

Math Talk

Expression Talk

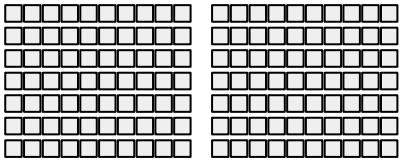
Lesson Plan	
LAUNCH **1** (15 Min)	Pose this problem: *7 apples cost 20¢ each. What is the total cost of the apples?* Ask students to talk to a partner about how they would solve this problem. Ask students to share their strategies. **Possible strategies:**

Equal Group with numbers or visuals:



20 + 20 + 20 + 20 + 20 + 20 + 20 = 140¢ or $1.40

Arrays:



Multiplication: 7 x 20 = 140

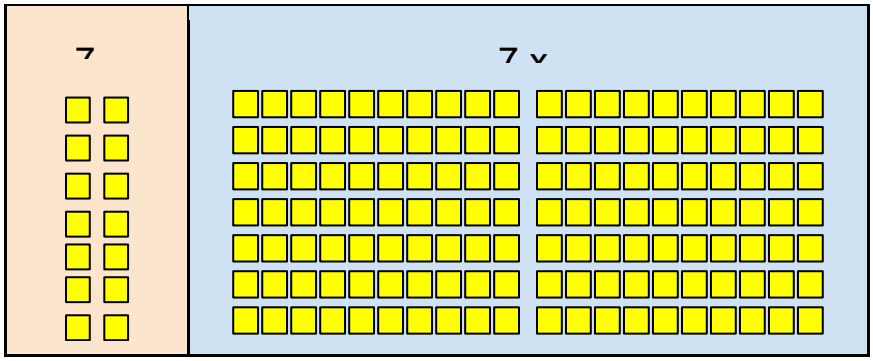
Repeated addition and groupings are strategies that connect multiplication to addition (a 3rd grade concept), while the array model can connect multiplication to the concept of scaling (a 4th grade concept). For example, a student might say that because she knows that 7 x 2 = 14, then she knows that 7 x 20 is 140 because "you just put a zero on the end."

A mathematical explanation would be that 7 x 20 is "ten times as many as 7 x 2."

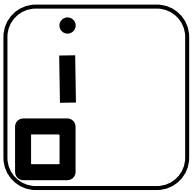
Another way to show this is

7 x 20 = 7 x 2 x 10

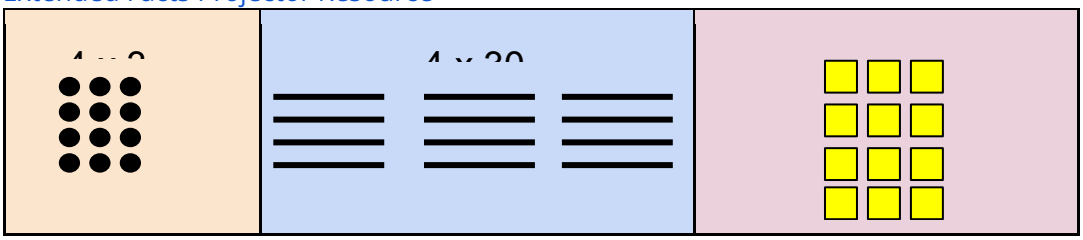
This can be modeled with base-10 blocks:



In this lesson, students are asked to model a series of three problems using base-10 blocks and drawings. Because it can be time-consuming for students to draw their base-10 blocks models, **teach them a short cut.** For example, using the symbols on the right, a series of extended facts can be represented like this:



Extended Facts Projector Resource

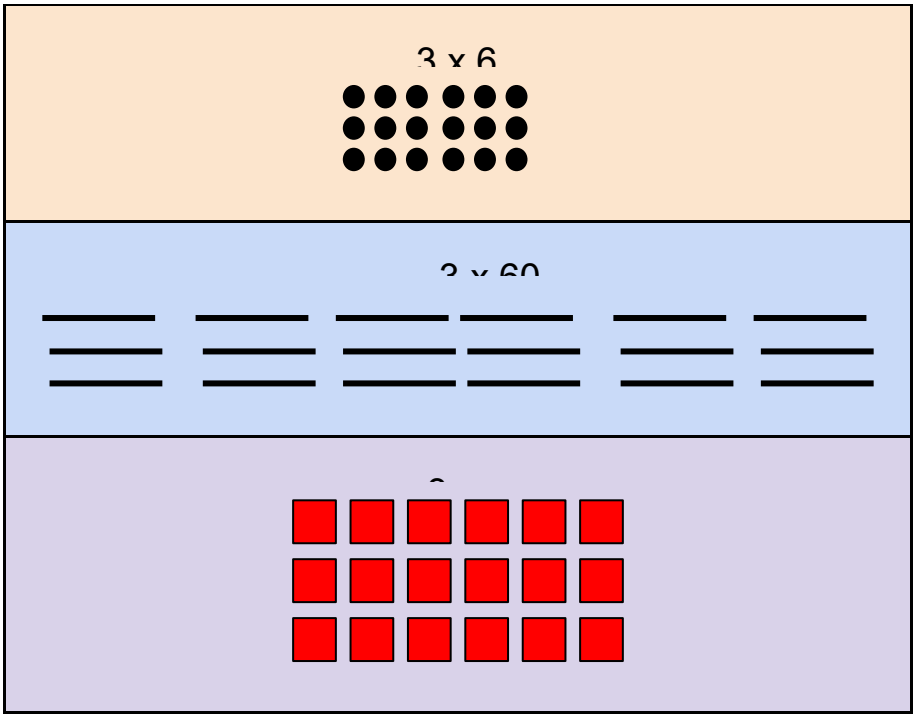


EXPLORE

2

(35 min)

Have students work on **Extended Facts** with partners. Make sure that each partner has enough base-10 blocks to model 3 x 6, 3 x 60, and 3 x 600.



Circulate and observe how students are approaching the word problems. Notice what patterns they are finding so that you may use them in your summary.

Note: Students may notice or already know that when multiplying with multiples of 10, 100, 1000, etc., the product will have the same number of zeros as do the factors. For example, 8 x 3 = 24 and 8 x 300 = 2400. It can be common to say "*just multiply the numbers and add on how many zeros there are.*" This does not make logical sense because when we "add zeros" we do nothing to change the number. What is truly happening is that the place value of digits are moving up by powers of 10. This is a connection that can be made based on work in the previous unit. 8 x 300 = 2400 because it is equal to **8 x 3 x 10 x 10**.

→Key Math to Observe

	• Do students notice that each product increases by a power of 10? • Can students articulate the concept of "10 times as many" to explain why 3 x 60 = 3 x 6 x10? • Do students use the base-10 block shorthand correctly to represent the problems? • Do students recall their one-digit by one-digit multiplication facts from Grade 3?
SUMMARIZE **3** (10 min)	→ **Core Math to Emphasize** • Extended facts allow us to use what we know of single-digit multiplication to multiply larger numbers. • Extended facts are examples of scaling. 5 x 70 is ten times as many as 5 x 7 because it is (5 x 7) x 10. Based on your observation of student work, **select** several samples to highlight how the models for the colored pencils grew by "*10 times as many*" each time. Another interesting discussion could be about the numerical patterns students noticed. This is interesting because it will help students give a more mathematical justification as to why the zeros are "put on at the end of the number." **Connecting** the diagrams of the base-10 blocks to the equations would be another interesting discussion. **Note:** The last question of the lesson, *Explain how knowing the answer to 9 x 8 can help you find the answer to 90 x 80,* can be used as the summary question to discuss in class or as an exit ticket. Because 9 x 8 is 72 and 90 x 80 is 9 x 10 x 8 x 10, which can be rearranged using the associative property to (9 x 8) x (10 x 10) = 72 x 100.  **Notebook Prompt (5 minutes)** 7 x 5 is connected to 7 x 50 because _____.

Notes	Universal Support
Students may have just learned a rule to "add zeros at the end of the numbers," rather than understanding that they are multiplying by multiples of powers of 10. Students may find it difficult to relate base-10 blocks as representative of the colored	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Use smaller numbers, for example, 2 x 3, 2 x 30, and 2 x 300. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. • Calculators can be used to investigate the place value and the role of zero as a fact family is extended. **Considerations for emerging bilingual students:**

pencils. Students may be challenged by the concept of "10 times as many."	• There are three different ways the pencils are packaged in this problem: set, box, bag. There is an implication that a bag is larger than a box and that a box is larger than a set, but that is not stated explicitly. You may draw out a simple diagram showing the implied size of each word. • To support student descriptions of patterns, you can provide a simple sentence frame: I noticed that _____ changes every time that _____ changes.

Extensions	Have students write in their math notebooks what they have learned about extended facts and practice them with numbers of their choice.
Additional Notes	Extended facts are multiplication problems using multiples of powers of ten. For example, in the apple problem, 7 x 20 can be solved as 7 x 2 x 10. Since we know 7 x 2 as a basic multiplication fact, we can then take its product (14) and multiply it by 10 to get 140.

Lesson Series 1 – **Day 2**

Core Math	All multiplication problems with two factors can be represented with area models. This representation helps with understanding of algorithms for multiplication.
Description	Students model one-digit by two-digit multiplication with base-10 blocks, sketch their models, and show the answer.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Resources	One-Digit by Two-Digit Multiplication with Base-10 Blocks BLM S C Equation Cards BLM Base-10 blocks
Homework	Day 2 HW S C

Math Talk	**Number Talk with Multiplication** *What is the product of 8 x 15?*

Lesson Plan	
LAUNCH **1** (10 Min)	Tell students that they will be representing multiplication problems using base-10 blocks. Remind them that in 3rd grade, they learned how to represent single-digit multiplication ("single digit" means problems like 5 x 8 or 2 x 9). In the previous lesson, they learned how to represent multiplication problems with multi-digit numbers, but always with zeros. Today they will be working on how to understand a multiplication problem that has a single-digit factor multiplied by a two-digit factor. Write this example on the board: **5 x 14** Have partners read this problem and discuss how they might solve it. Ask for volunteers to show ways to solve this problem. This is an informal assessment of how students are thinking about such multiplication problems. Put the One-Digit by Two-Digit Multiplication with Base-10 Blocks BLM S C on the document camera and discuss how they will be doing this activity with partners. Emphasize the need to do all four steps carefully and to share the work between both partners.

Example

1. Pick an equation.	2. Model the problem using base-10 blocks.
5 x 14 = n	
3. Sketch base-10 blocks using lines and dots.	4. Write out a multiplication problem representing your model, then find the solution.
	5 • 10 = 50 and 5 • 4 = 20 50 + 20 = 70 5 • 14 = 70

Pass out one activity sheet and set of equation cards per student.

EXPLORE

2

(40 min)

Circulate and observe how students are working on representing multiplication problems using base-10 blocks, sketches, and equations. Emphasize that all three representations are important and will be used throughout this unit.

→Key Math to Observe

- Are students representing one-digit by two-digit multiplication problems effectively using base-10 blocks?
- Can students easily find the product by grouping base-10 blocks into larger chunks and/or skip counting?
- Do students take up the challenge problem with three digits?

SUMMARIZE

3

(10 min)

→ Core Math to Emphasize

- When multiplying multi-digit numbers, we can first decompose them into place value components, multiply the parts, then recompose.

Have students share the work they did modeling one-digit by two-digit multiplication. Try to pre-select several students whose work highlights a place value understanding, but also show other work that illustrates interesting points about this way to model.

	 **Notebook Prompt (5 minutes)** One smart thing my partner and I did during this activity was _____.

Notes	Universal Support
Students may still be unclear about some place value concepts and might not easily make the connection between base-10 blocks and the numbers. **Vocabulary:** methods, representations, product, bar model	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** • Provide verbal instructions if needed. • Provide sentence frames as needed.

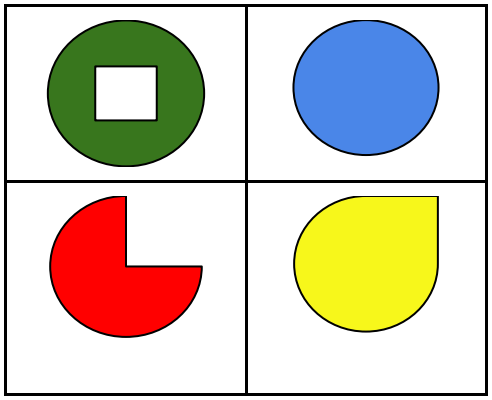
Extensions	Have students write in their math notebooks reflecting on this lesson and/or trying out their own numbers.

Lesson Series 1 – **Day 3**

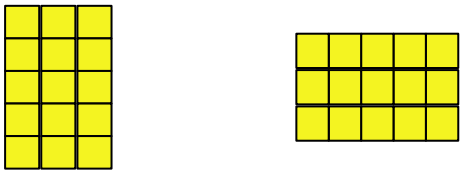
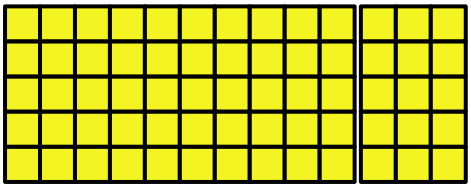
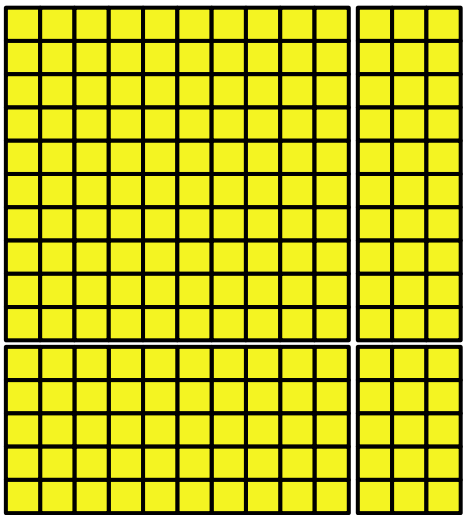
Core Math	Multiplication can be represented with base-10 blocks using an area model. This leads to an understanding of partial product multiplication.
Description	Students extend their understanding of single digit multiplication with area models to two digit by two digit multiplication.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Resources	Area Model for Multiplying Multi-Digit Numbers S C Base-10 Blocks
Homework	Day 3 HW S C

Math Talk

Which One Does Not Belong?

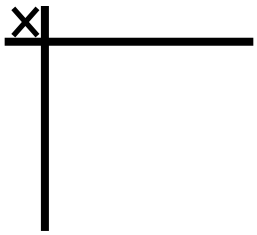


Lesson Plan	
LAUNCH **1** (10 Min)	Write three multiplication expressions on the whiteboard: 5 x 3 5 x 13 15 x 13 Have partners discuss the similarities between these three problems. (Examples: same digits in ones place, all multiplication problems, the last two have digits in tens place) Tell them that they will be working with partners to understand how they can apply what they know about single-digit multiplication to areas of rectangles, and then apply this to multi-digit numbers in ways that can make multiplication problems easier to solve.

	Avoid pre-teaching about the connections. Instead, leave it as a small mystery and as students work on this, use questioning strategies to help students see the connections themselves.
EXPLORE **2** (40 min)	As students work, circulate and observe how they are approaching the problems and sharing their thinking with each other. It is likely that most students will remember from 3rd grade, as well as from previous activities in this unit, how to represent 5 x 3 as a rectangle with dimensions of 5 and 3.  Don't worry about which dimension is labeled as which because it is not important for this lesson. Students may represent 5 x 13 by continuing to use unit blocks or they may notice that they can use 10 block longs to more efficiently build the area model.  Students may represent 15 x 13 by continuing to use unit and ten blocks or they may notice that they can use 100 block flats to more efficiently build the area model. 

	Ask students to explain how the rectangles help them find the answers. They may respond by saying that they have to count all the little squares or they may skip count. Ask them to think of a way they could subdivide the rectangles into smaller ones that are easier to solve, similarly to Joshua's floor plan. Avoid telling students exactly how they should subdivide the rectangles. Instead, observe how they do that to select student work for a summary discussion at the end of class. **→Key Math to Observe** • Are students able to sketch an area model for some or all of the multiplication problems? • How do students subdivide the larger rectangles into smaller ones? • Can students find accurate answers?
SUMMARIZE **3** (10 min)	**→ Core Math to Emphasize** • All multiplication problems with two factors can be represented using a rectangular area model. • The dimensions of this rectangle (the factors), can be broken down into their place value components. • These place value components allow us to multiply with "friendly" one-digit numbers as well as multiples of powers of 10. Based on your observations, select student work that shows how they partitioned the 15 x 13 rectangle (last question). Select work that shows a variety of ways, including, if possible, a place value understanding of decomposition so that 15 x 13 can be seen as (10 + 5) x (10 + 3). Discuss merits of several different ways of decomposition, ending with the place value understanding. Tell them that while all ways are valid, the place value decomposition will allow them to learn new ways of multiplying large numbers that will not require them to draw everything out.  **Notebook Prompt (5 minutes)** When I multiply a 2-digit number by another 2-digit number, I can _____.

Notes	Universal Support
Vocabulary: methods, representations, product, bar model	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without a written explanation. • Provide a multiplication fact chart for students who need it.

	Considerations for emerging bilingual students: • Provide verbal instructions if needed. • Provide sentence frames as needed. For students who confuse the multiplicands with the product in the base-10 blocks array, make a mat like the one pictured on construction paper on which students can place their blocks. 

Extensions	Early finishers can choose other numbers to investigate.

Lesson Series 1 – **Day 4**

Core Math	Numbers can be compared multiplicatively when we can determine how many times one of the numbers repeats to make another one.
Description	Students learn about multiplicative comparison as a new way to compare two numbers. **Note:** This is an introductory lesson on **multiplicative comparison**. This is a central concept of 4th grade and will be developed further throughout this unit as students deepen their conceptual understanding and their procedural fluency.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
Resources	Multiplication as Comparison S C
Homework	Day 4 HW S C

Math Talk

Number Strings

2 • 5
2 • 50
20 • 50

Lesson Plan	
LAUNCH **1** (10 Min)	Write the following two numbers on the whiteboard: **6 and 12** Ask students to compare these two numbers and tell you what they notice. (Examples: 6 is one digit, 12 is two, 12 is more than 6, 12 is double 6) Tell them that up until 4th grade, when they have been asked to compare two numbers, they have been able to answer one of two questions:

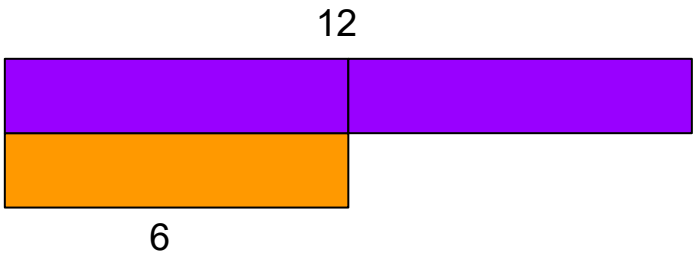
How much more is 12 than 6? (6)
Or
How much less is 6 than 12? (6)

In 4th grade, students will spend much more time comparing numbers using multiplication rather than addition/subtraction. When we compare the two numbers using multiplication, one question we can ask is

How many times as great is 12 as 6? (2 times as great)

There are many ways to represent this problem, but one way we will be focussing on in 4th grade is the tape diagram.

Draw the following diagram on the whiteboard.



When we draw this as a tape diagram, we can see that 12 is 2 times as great as 6.

Write **12 is 2 times as many as 6** on the whiteboard.

Tell students that they will be working with partners to think about comparing numbers using multiplication and using the following three phrase variations:

_____ times as many as _____

_____ times as much as _____

_____ times as long as _____

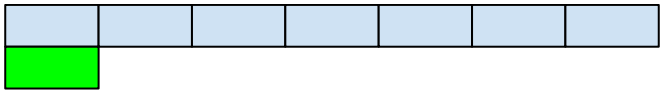
EXPLORE

2

(40 min)

Tell students to read and discuss the three examples on the first two pages of the lesson before moving on to solve the last three questions.

Circulate and observe how students are understanding the problems and representing their work.

Question 1 Basketballs	

	Question 2 Giraffe 
	Question 3 Book 

As students draw their tape diagrams, notice how they are talking about them. **It is important that they understand that the smaller length is repeating some number of times to make the larger length**, indicating how many times as long one is as the other. Ask questions to determine whether students understand this comparison and are not just going through the steps and/or understanding the lengths in terms of additive difference rather than multiplication.

→Key Math to Observe

- Are students able to represent the problems using a tape diagram?
- Can students use the language frames effectively?

SUMMARIZE

3

→ Core Math to Emphasize

- Numbers can be compared multiplicatively when we can determine how many times one of the numbers repeats to make another one.

Based on your observations, choose one example of each of the problems. Start with problem 2 because it has the simplest tape diagram representation. Move to problem 1. Problem 3 might be challenging for students to understand because of the money context and a potential struggle to understand wholes and parts since 50¢ is half of a dollar.

Notebook Prompt (5 minutes)

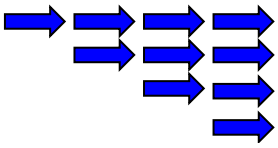
If a father is 3 times as tall as his daughter, that means _____.

Notes	Universal Support
• Comparing numbers multiplicatively will likely be a challenging concept for Grade 4 students, who up until now have been asked to compare numbers additively (20 is 15 more than 5, vs. 20 is 4 times as much as 5). • Students may not understand how to make their own comparisons. **Vocabulary:** methods, representations, product, tape diagram	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/ representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** • Provide verbal instructions if needed. • Provide sentence frames as needed. • The sentence "_____ is _____ times as large as _____" is very important in this task and in this unit.

Extensions	Encourage early finishers to write their own multiplicative comparison problems.
Additional Notes	**Multiplicative comparisons are an important aspect of deepening students' ideas about multiplication.** They are the precursor to work with fractions, division, and algebra in subsequent years. For a one-page description of the importance of multiplicative comparisons, see Multiplicative Comparisons Teacher.

Lesson Series 1 – **Day 5**

Core Math	Numbers can be compared multiplicatively when we can determine how many times one of the numbers repeats to make another one.
Description	Students compare SF Bay Area animal body lengths to determine how many times as long some animals are compared to others. **Note:** This is the second lesson on **multiplicative comparison**. Students may still be working to understand how a tape diagram can represent a multiplicative comparison situation.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
Resources	• San Francisco Bay Area Animals S C • Multiplicative Comparison Giraffe and Photographer Presentation
Homework	Day 5 HW S C

Math Talk 	**Expression Talk** How many arrows are there and how do you know? 

Lesson Plan	
LAUNCH	Remind students of the multiplicative comparison problem they worked on in the

1 (5 Min)	previous class about the giraffe and the photographer. Show the Multiplicative Comparison Giraffe and Photographer Presentation to illustrate what it means to say that the giraffe is about 4 times as tall as the photographer. Write the question on the whiteboard: **How many times as tall is the giraffe as the photographer?** Then write the answer: **The giraffe is about 4 times as tall as the photographer.** Note that both the question and the answer use a similar (and mathematically important) grammatical structure of ____ *times as tall as* ____. Also discuss the meaning of the word "about" in the solution. (It allows for approximation. Note on the presentation that the positioning of the photographer still has a little space between the iterations.) Remind students how a problem can be represented by a tape diagram by showing them the last slide and asking them to explain the tape diagram first to a partner and then to the whole class. Tell them that they will be comparing body lengths of different animals that live in our own region, the SF Bay Area. They will be working with partners, finding information in a table, and representing their work with tape diagrams and words.
EXPLORE **2** (45 min)	Circulate and observe how partners are interacting, talking about the math, and solving the problems. They may have questions about how accurate their tape diagrams need to be. Absolute accuracy is not necessary because the tape diagrams are representing the problem but not truly solving them. Observe how they make their choices to draw the shorter and longer lengths because that might be indicative about how they are approaching the topic of multiplicative logic. **→Key Math to Observe** • Are students able to model the word problems with tape diagrams and words in order to solve them? • Do students use multiplicative or additive reasoning? If multiplicative, they are expressing how many times one number fits in the other. If additive, they may be adding up lengths. If it's the latter, encourage them to use the sentence frames provided and think multiplicatively.
SUMMARIZE **3** (10 min)	**→ Core Math to Emphasize** • We can determine how many of the smaller quantities make up the larger one by comparing them multiplicatively. Based on what you observed students do, choose 1 or 2 of the comparisons to summarize. Select student work that perhaps shows a couple of different ways they approached the tape diagrams. For example, the discussion of precision might be

	interesting. Some students may divide up a longer length into shorter ones, while other students may take a shorter length and simply copy it repeatedly.  **Notebook Prompt (5 minutes)** One thing I learned today was _____.

Notes	Universal Support
• There are some problems that seem to require division, which students might not be very familiar with. • Multi-step problems might prove challenging to both understand and then to solve. **Vocabulary:** methods, representations, product, tape diagram	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** • Provide verbal instructions if needed. • Provide sentence frames as needed (though they are already provided on the student page.

Credits	Opossum Photo: Wikipedia:User:Cody.pope Own Work (CC-BY-SA-2.5)

Lesson Series 1 – **Day 6**

Core Math	Multiplication situations can be represented with equations and a variety of visual models.
Description	Students work in groups to match word problems with multiplication equations and visual models.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
Resources	Multi-Digit Multiplication FAL BLM S C, 1 complete set per group of four (Copy on colored cardstock and cut out into sets.) Consider how you will place your students into groups of four. How will they support one another with the mathematics, with the language, and with the social dynamics?
Homework	Day 6 HW S C

Math Talk

Number Talk with Multiplication

What is the product of 4 x 12?

Lesson Plan	
LAUNCH	Tell students that they will be working in groups of four on a card sort about the

1
(10 Min)
	different models for multi-digit multiplication that they have been learning this year as well as in Grade 3.
	
	This lesson will take two class periods.
	
	Sort a few cards from the first two card sets as a class:
	
	• Demonstrate how to lay out Card Set A, the first set of cards, on the table.
	• Deal out Card Set B, the second set of cards, so that everyone has some cards.
	• Tell students that their job is to agree or question the choices by saying, e.g.,
	  ○ *Why do you think _____ matches _____ ?*
	  ○ *How do these two cards match?*
	  ○ *I am not sure I agree with this because I think....*
	  ○ *I agree with that match because ...*
	• Finally, demonstrate how to take turns, passing from one group member to the next so that everyone is participating..
	
	Avoid demonstrating how to match the entire set of cards.
	
	Noticing equitable participation: In group activities with card sorts such as this, it is very common for one or two students to take over and do most of the work. This may be unintentional on their part, as sometimes two students will be talking together and doing the match, while the others watch passively. To avoid this as a default, tell the students that each member must have a couple cards from each set and that they are responsible for sorting them for the group.
	
	Additionally, if you as the teacher rely on group questions and group answers, you can purposefully or randomly call on any one student in the group to describe their work at any time. This makes it imperative that group members not only share the work, but also share the justifications for their work.
	
	Tell students that their task is to match sets of cards as they are handed out by you.
	
	They will start with just two sets. Tell them that most cards have a partner in each of the sets. If students find there isn't one, they will need to show the missing representation on the blank card.

EXPLORE **2**	Tell students that once their group has matched the two sets, the group needs to raise their hands so that you may come by and hear their justifications. If you are convinced that every member understands the work, hand them Set C. If you feel that they still need to convince one another, tell them that you will return once they are all ready.

While students are working on this task, ask questions to help them explain their thinking.

→Key Math to Observe

- Do students match the cards correctly?
- Do they understand the various visual models for multiplication?
- Can students relate the word problem to a matching multiplication problem?

Common Issues	Suggested Questions and Prompts
Student doesn't match the cards correctly because he or she doesn't have a conceptual understanding of multiplication.	• If you are multiplying 27 x 4, what does the 2 represent? The 7? • What would happen if you multiplied 20 x 4 and 7 x 4? Could you use those answers to solve 27 x 4?
Student doesn't understand the distributive property.	• How can the number(s) we are multiplying be broken apart? • What could you do with those numbers to solve this problem?
Student doesn't understand the area model for multiplication.	• In the problem 27 x 14, let's look at the number 27. How many tens are in 27? How many ones? How could you model 27? Now let's look at 14? How many tens? ones? How could you model 14? • Is there a way to take those two models and fit them on a rectangle to discover 27 x 14 without doing any calculations?

Suggestions to Support Students During the Card Sort:

If one student has placed a particular card, challenge their partner to provide an explanation.

- *Maria placed this equation card with this Model. Martin, do you agree? Why?*

Use the questions from the Common Issues table to support students in articulating the reasons for their decisions.

If the whole class is struggling on the same issue, write a couple of questions on the board and organize a whole class discussion.

SUMMARIZE

→ Core Math to Emphasize

3 (10 min)	• Multiplication situations can be represented with equations and a variety of visual models. Have all groups gather their matched sets and keep them together with paper clips or envelopes. Any card sets that are not completed today can be matched when students return to the activity in the re-engagement center. As a summary, have students share pairings that were challenging.  **Notebook Prompt (5 minutes)** One thing I learned today was _____.

Notes	Universal Support
• This activity offers opportunities to uncover student conceptions about how the various models for multiplication relate to each other and, in turn, how they represent a situation. • Students may not understand how the area model connects to the tape diagram. • There are several situations in which division seems to be called for. But since the students are not required to find the answers to the multiplication problems, all these situations can be connected using multiplicative thinking. **Vocabulary:** methods, representations, product, to sort	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/ representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** • Provide verbal instructions if needed. • Provide sentence frames as needed.

Apprentice Task

Open in Google Drive

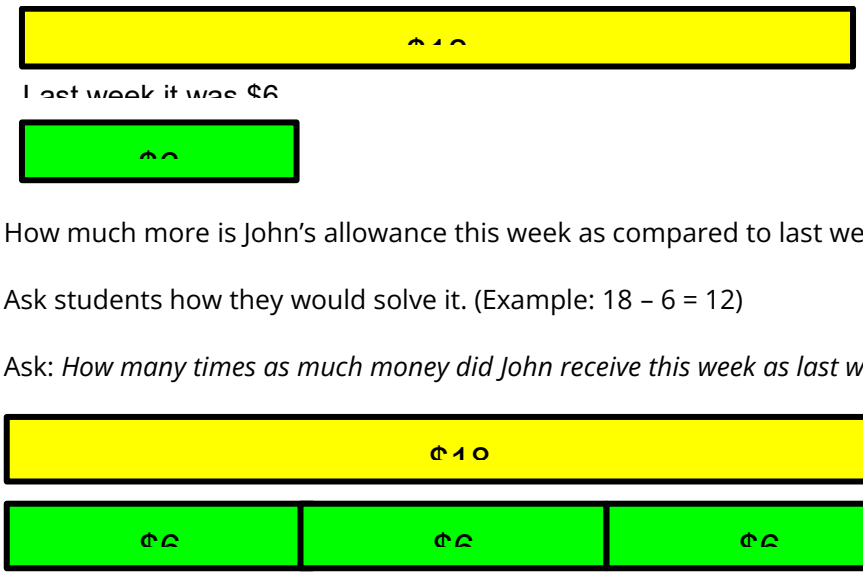
Core Math	When comparing two quantities, additive comparisons refer to "how much more?" while multiplicative comparisons refer to "how many times as many?"
Description	**Walkathon** Students compare numbers into the tens of thousands, first with additive comparison, then with multiplicative comparison.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
Resources	• Walkathon S C • Chart paper (optional) • Walkathon Suggested Answer Guide Teacher
Homework	Apprentice HW S C

Math Talk

Number Talk with Multiplication

What is the product of 5 x 16?

Lesson Plan	
LAUNCH	Tell students that today they will be working with partners to complete the Apprentice Task.This task is about raising money from a walkathon. Discuss what a walkathon is.
1 (10 Min)	Review how a bar model may be used as a representation by looking at a problem similar to the first problem on the task. For example: John's allowance this week is $18.



How much more is John's allowance this week as compared to last week?

Ask students how they would solve it. (Example: 18 – 6 = 12)

Ask: *How many times as much money did John receive this week as last week?*

The answer to this question is 3 times as much.

Tell students that the first comparison is called *additive* because we want to see how much more one number is than the other.

The second comparison is *multiplicative* because we want to see how many times one number fits into another. Both can be modeled with tape diagrams.

Tell students that they will be working with partners on this task and that tape diagrams will be an important tool they will use.

Tell students that today there will also be Groupwork Feedback. Tell them that you will focus on sharing talk time and looking for both people in a partnership to take turns talking. Tell students that if one of the partners is not talking, the other person can ask them questions such as, *What do you think should be done next? Can you explain how you know ____?*

EXPLORE

2

(40 min)

Circulate and notice how students are approaching the problems and sharing their thinking with their partners. Pay particular attention to how they are interpreting the problems using tape diagrams. This is an opportunity to check in with students who you noticed have been struggling to use them effectively.

→Key Math to Observe

- Are students able to distinguish between additive and multiplicative comparisons in the problems?
- Can students find the difference between two numbers in the ten thousands?
- Are students able to show their work using a tape diagram?
- Can students write equations for these problems?

SUMMARIZE

→ Core Math to Emphasize

3 (10 min)	• Multiplicative comparison means finding how many times a smaller number fits into a larger one. • A tape diagram can be a useful tool to visualize and solve a multiplicative comparison problem. • Equations involving multiplication model these situations. Depending on your observations, choose one or two problems to use as a summary. The last two problems address multiplicative comparison with and without a diagram. Have students explain how they solved these two problems.  **Notebook Prompt (5 minutes)** One person earns 4 times as much money as another person. How can we represent this with a tape diagram?

Notes	Universal Support
• Students may not understand the difference between additive comparison (how much more) and multiplicative comparison (how many times as much or as many). • Students may not understand how to use a bar model to show these comparisons. **Vocabulary:** methods, representations, product, bar model	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/ representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** • Provide verbal instructions if needed. • Provide sentence frames as needed. • The sentence "_____ is ____ times as large as _____" is very important in this task and in this unit.

Extensions	Ask students to double all the numbers in the problem to see how that changes the results.

Lesson Series 2 **Overview**

Open in Google Drive

Description

Students learn how to multiply multi-digit numbers using an area model that supports the partial product algorithm.

Standards

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*.

	Day 1	Day 2
Core Math	Numbers can be decomposed into their place value components. These components can be multiplied, then added back together to find the product of two multi-digit numbers. This can be represented with an area model.	
Description	Students represent multi-digit multiplication using base-10 blocks in an area model.	Students find partial products in order to solve two-digit by two-digit multiplication problems.
Resources	How Can I Multiply Two-Digit Numbers? S C 13 x 15 Area Model BLM	How Can I Multiply Two-Digit Numbers with the Box Method? Student S C How Can I Multiply Two-Digit Numbers with the Box Method? BLM S C
Homework	Day 1 HW S C	Day 2 HW S C

	Day 3	Day 4	Day 5
Description	Students practice partial products with two-digit by two-digit multiplication problems by playing a game called *Multiplication Judo*.	Students work with partners on a problem involving multi-digit multiplication to solve a problem about gray whale migration up the Pacific Coast of North America.	Students work in groups of four to find dimensions of posters based on a scaling up in size of photographs.
Core Math	Partial product refers to breaking numbers into the place value "parts", multiplying, then recomposing them.	Multiplication can be used to solve problems involving distance traveled over time.	Multiplication can be used to solve problems involving lengths.
Resources	Multiplication Judo S C	Gray Whale BLM S C Gray Whale Suggested Answer Guide Teacher Scissors Glue or tape Poster paper Video showing a mother gray whale and her baby: http://tinyurl.com/jt3my6j	From Photo To Poster BLM S C African Animals Photos Larger paper for poster Markers or colored pencils Scissors and glue Centimeter ruler
Homework	Day 3 HW S C	Day 4 HW S C	Day 5 HW S C

Lesson Series 2 – **Day 1**

Core Math	Numbers can be decomposed into their place value components. These components can be multiplied, then added back together to find the product of two multi-digit numbers. This can be represented with an area model.
Description	Students represent multi-digit multiplication using base-10 blocks in an area model.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Resources	How Can I Multiply Two-Digit Numbers? S C 13 x 15 Area Model BLM
Homework	Day 1 HW S C

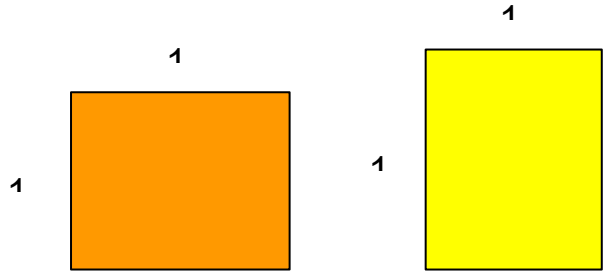
Math Talk

Which One Does Not Belong?

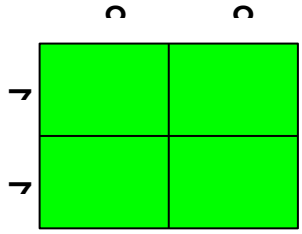
25	**16**
9	**43**

Lesson Plan	
LAUNCH **1** (20 Min)	Tell students that they have learned how to represent multiplication problems using an area diagram. They first learned how to multiply single-digit problems, like 3 x 5, in 3rd grade. They also learned how to multiply problems such as 3 x 50 by thinking of them as 3 x 5 x 10. In the previous lesson series, they learned how to multiply a single digit by a two-digit number by breaking down the two-digit number into tens and ones. Each time, they were asked to draw area models. Now the question becomes: How do we use an area model quickly and easily to solve a multiplication problem with 2 two-digit numbers? For example, how do we solve a

	problem like 13 x 15? Use the 13 x 15 Area Model BLM to record work on overhead projector. Tell students that they will be working with partners to use what they already know about multiplication area models to figure this out.
EXPLORE **2** (30 min)	Circulate as partners start working on the problems. The first problem asks them to notice what the dimensions of the rectangle are. If they do not seem to notice that the dimensions are 10 plus some more, ask questions to focus them on the different colors. The second problem asks them to notice the four smaller rectangles that form the larger one. What is important here is to notice that adding up the areas of all four smaller rectangles, we get the area of the larger one. Help students see the dimension of the smaller rectangles. The third problem asks them to take the visual model and represent it with numbers. It is important that students notice that all the problems represent the same area. While it is not true that decomposing by tens and ones is the only way to find the area, it is a very efficient and organized way. The last problem asks students to apply the previous information to a new multiplication problem. Observe how your students approach this last problem. Select student work on this problem for the summary. **→Key Math to Observe** • Are students able to decompose a multi-digit number into place value components? • Are students able to connect the sketch of the base-10 block representation to the numbers in the box? • Are students able to articulate the relationship between the part (partial products) and the whole (the product of the 2 two-digit numbers)?
SUMMARIZE **3** (10 min)	**→ Core Math to Emphasize** • Partial products are important because they use our place value understanding to make multiplying large numbers easier. • The partial product box method is directly related to the base-10 representation and area model. Based on your observations of student work, focus the summary on the last problem, 14 x 16. There could be valid variations in the work that will support a rich class discussion. For example, because multiplication is commutative, student may draw the rectangle two different ways:



While students may decompose the rectangles into lengths with tens and ones, there are other variations, such as:



A rich discussion could center on how, no matter how we divide up the larger rectangle, the sum of the smaller ones will always equal the area of the larger one.

Finally, it is important that students understand and start to use the **box method**. This method relies on a place value understanding of multi-digit multiplication and will be used extensively for the rest of this unit as well in subsequent 4th grade units.

Notebook Prompt (5 minutes)
For the problem 12 x 13, describe how you can use a diagram to solve it.

Notes	Universal Support
• Decomposing numbers by place value, multiplying them, then recomposing them might be challenging for some students. • Multiplication fact fluency might be an issue for some students. **Vocabulary:** partial (part of a whole), product	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who

(answer to multiplication problem), break up (decompose), place value components	need it. **Considerations for emerging bilingual students:** • Some of the number names may be unfamiliar to students who are learning English. • Product is a word with multiple meanings. Define it with students.

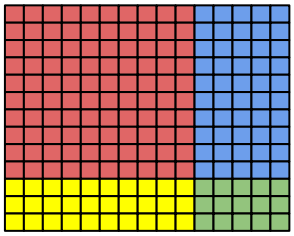
Extensions	Have students create their own 2 by 2 digit number multiplication problems.
Additional Notes	For more information on partial products, watch this video: http://tinyurl.com/zza4hs4

Lesson Series 2 – **Day 2**

Core Math	Numbers can be decomposed into their place value components. These components can be multiplied, then added back together to find the product of two multi-digit numbers. This is called the *partial product method* because the products of each part are calculated separately.
Description	Students find partial products in order to solve two-digit by two-digit multiplication problems.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Resources	How Can I Multiply Two-Digit Numbers with the Box Method? Student S C How Can I Multiply Two-Digit Numbers with the Box Method? BLM S C Dice (one or more per partner)
Homework	Day 2 HW S C

Math Talk	**Number Strings** 12 x 5 12 x 15

120 x 50

Lesson Plan	
LAUNCH **1** (10 Min)	Review the area model and box method learned in the previous lesson. Specifically, write the following problem: 13 x 15 and draw the area model (without the grid)  and box method |	10	5
---	---	---
10	100	50
3	30	15	 **100 + 50 + 30 + 15 = 195** Tell students that they will be doing an activity with partners that will give them more practice on the box method so that they gain confidence in using it. Pass out the number cards and have students read directions with their partners. They record their work in their student notebooks. **Notice that in this lesson students are asked to estimate before they calculate.**
EXPLORE **2** (45 min)	Either circulate around the room observing students as they practice the box method, or call a small group of students to work with you for additional support. Select student work based on observations you make during the lesson. A rich class discussion could arise by talking about different strategies students have for estimating an answer and then calculating a precise one.

	→Key Math to Observe • Can students look at a two-digit number and decompose it into place value components of tens and ones? • How do students interpret this method when applied to three digits? • Do students justify their work using number sense or do they seem to be following a method without really understanding it?
SUMMARIZE **3** (10 min)	**→ Core Math to Emphasize** • Partial products allow us to multiply smaller chunks and often find products using mental math instead of having to write all the steps down. • Partial products work with numbers of any number of digits (even decimals if we like). These problems ask students to make estimations before calculating. There is no tried-and-true way to estimate. As long as a method is logical and leads to a ballpark answer of similar magnitude, accept it. The conversation comparing the estimated answer to the calculated one should be the focus of the summary.  **Notebook Prompt (5 minutes)** For the problem 12 x 13, show how you can solve it using the box method.

Notes	Universal Support
• Decomposing numbers by place value, multiplying them, then recomposing them might be challenging for some students. • Students may not be fluent with their multiplication facts. • Students may be following a procedure (method) instead of an understanding of the underlying math concepts. • Students may find this method tiresome and want to use a more traditional method they learned outside of school. **Vocabulary:** methods, representations,	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. • The temptation might be to teach the traditional algorithm because it is deemed easier by some. In fact, partial product methods are much more aligned to a deeper sense of the meaning of numbers and place value. They also lead to greater mental math fluency over time. **Considerations for emerging bilingual students:** This activity is not as language dependent as others in this unit. Students can show their understanding using numbers and diagrams.

product	

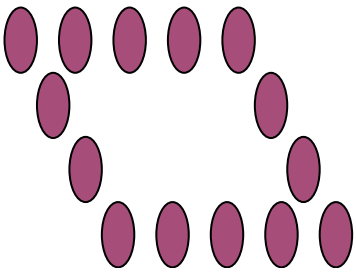
Extensions	Challenge students who master the two-digit box method to apply it to larger numbers. Ask students how they could show 20 • 25 with the box method.

Lesson Series 2 – **Day 3**

Core Math	Numbers can be decomposed by their place value components. These components can be multiplied, then added back together to find the product of two multi-digit numbers. This is called the *partial product method* because the parts are being found separately, then recomposed by adding them together.
Description	Students practice partial products with two-digit by two-digit multiplication problems by playing a game called *Multiplication Judo*.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Resources	Multiplication Judo S C
Homework	Day 3 HW S C

Math Talk

Expression Talk



Lesson Plan	
LAUNCH	*Multiplication Judo* is a game designed to help students practice partial products. In this

1 (10 Min)	game, students use place value understanding to create the largest possible product. Play one round with your students, then have them play with partners. Instructions are on the **Multiplication Judo** student page. **Noticing equitable participation:** Games are an important component of a rigorous math program. They can motivate students to practice concepts and skills with peers. The element of win or lose (competition) is motivating to many, but not all, students equally. It is important to talk about the nature of competition in your classroom and what you expect of your students in terms of behavior as well as mutual support. Be thoughtful about how games are played in class: Are all students learning from the game? Do all students understand how to play the game? Are they all equipped to compete reasonably with their peers? Consider how you set up partners and groups to support equitable participation. In addition, have clear and explicit expectations about game play in class and go over them often. Game days in class offer an excellent opportunity to do Groupwork Feedback. Tell students what you will be looking for, such as equal participation, writing down math work, and interesting approaches to solving problems together.
EXPLORE **2** (35 min)	Have students play *Multiplication Judo* with partners. Circulate, observe, and assist. **→Key Math to Observe** • Do students understand the rules and expectations of this game? • Can students look at a two-digit number and decompose it into place value components of tens and ones? • How do students interpret this method when applied to three digits? • Do students justify their work using number sense, or do they seem to be following a method without really understanding it?
SUMMARIZE **3** (15 min)	**→ Core Math to Emphasize** • Numbers can be broken down to the place value components, multiplied as simpler numbers, and recomposed. • With two-digit by two-digit multiplication, fluency with partial products can be developed to such an extent that it is faster to do this mentally than with paper and pencil. Debrief the Groupwork Feedback notes you took. Have students write a reflection on how the game went for them and their partners in their math notebooks. What was one thing that went well and one thing that could be improved next time?

	 **Notebook Prompt (5 minutes)** One thing I learned by playing *Multiplication Judo* was _____.

Notes	Universal Support
• This is a new game and the rules may confuse students. • Decomposing numbers by place value, multiplying them, then recomposing them might be challenging for students. • Multiplication fact fluency might be an issue for students. **Vocabulary**: wrestling, partial, product	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** • Modeling this game will be important for all students, but particularly for EL students who may struggle to understand the written rules. • Consider how you set up partners. Do you want language models (strong English speakers with ELs) or do you want native language peer support (EL students of the same language background working together)?

Extensions	Have students play this game with one digit times three digit, or any combination of multi-digit numbers.
Additional Notes	See this video with further information on Multiplication Judo (called "Wrestling" in the video): https://www.youtube.com/watch?v=s_A06G4QDW8

Lesson Series 2 – **Day 4**

Core Math	Multiplication can be used to solve problems involving distance traveled over time.
Description	Students work with partners on a problem involving multi-digit multiplication to solve a problem about gray whale migration up the Pacific Coast of North America.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Measurement and Data** **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** 4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
Resources	• Gray Whale BLM S C, 1 per pair • Gray Whale Suggested Answer Guide Teacher • Scissors and Glue or tape • Poster paper, 1 piece per pair • Video showing a mother gray whale and her baby: http://tinyurl.com/jt3my6j
Homework	Day 4 HW S C

Math Talk

Number Talk with Multiplication

What is the product of 12 x 8?

Lesson Plan	
LAUNCH	Show students this short video with a mother gray whale and her baby:

1

(5 Min)

http://tinyurl.com/jt3my6j

Tell students that gray whales travel from Alaska to Baja California, to have their babies, and then return with their babies to Alaska. Tell them they will be working with a partner to solve a problem involving the distances traveled.
This task is designed to be done with partners.

> **Noticing equitable participation:** Launch this task using the Multiple Abilities Strategy. The goal of this strategy is to communicate to students the variety of strengths and abilities, including skills, understandings, and mathematics practices, that they bring as individuals to a group task. By carefully considering the strengths of all your students, including those who may not participate actively, and then naming their strengths explicitly, you are opening up the opportunity for more robust participation by all students.

Share a list of strengths you will be looking for students to bring to their work on this task. Your list should include a variety of mathematical behaviors as well as content-specific strengths.

Tell students: *These are some of the things that will help you with this task. Each of you can do some of these things, although probably none of you can do all of these things, really well. By working in a group you can combine strengths to learn from this task.*

Some Suggested Strengths for Multiple Abilities Lists
• Understand the written task and explain it to your partner. • Ask questions of others and explain your thinking. • Read a map and understand what it is showing. • Understand numerical information in a table. • Multiply multi-digit numbers by other numbers. • Add multiple numbers together. • Explain how your partner and you solved the task.

Note: For more information on how to use Multiple Abilities Lists to launch a task, see the Digital Math Teaching Toolkit http://www.sfusdmath.org/multiple-abilities-strategy.html

Tell students that you will give each pair of students one task sheet, scissors, glue or tape, and one large piece of paper for them to make a poster of their work.

Tell them that they need to cut out the three sections (problem, map, and table) and organize them on the poster first, before they record their other work.

EXPLORE **2** (40 min)	Instruct partners to work on the problem. Circulate and observe. Use your observations to summarize the problem at the end of the lesson. Notice particular strategies students used to find the information. **→Key Math to Observe** • Are students able to interpret the data in the table? • Can students determine the number of days for each period of time? • Can students multiply two-digit numbers together accurately? • Can students add up the distance intervals to find the final distance?
SUMMARIZE **3** (15 min)	**→ Core Math to Emphasize** • The number of days in calendar periods is the difference between two dates. • The number of days times the distance traveled per day results in the distance traveled over the period of time in question. • Add all these distances up and you find the total distance of the trip. Call the class together and have them share the distance this whale traveled. There will likely be several answers due to imprecise counting of days or errors in the multiplication. From your observations, choose two or three sample student work pages to discuss, pointing out merits and areas for growth. If no one found the correct answer, consider solving it as a whole class for clarity. Use the Gray Whale Answers Teacher as a guide, but be sure to work out the individual multiplication problems with the class as well.  **Notebook Prompt (5 minutes)** One thing my group did well on this task was _____.

Notes	Universal Support
• Decomposing numbers by place value, multiplying them, then recomposing them might be challenging for students. • Multiplication fact fluency might be an issue for students. • Reading information from a table may be challenging for students. • Understanding how to interpret calendar dates as numerical data may not be natural for	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/ representations without any written explanation. • Provide a multiplication fact chart for students who need it.

students. **Vocabulary:** There is a lot of vocabulary related to whales, migration, and geography that needs to be clarified in this lesson: whale calf (baby), migration, Baja California, distance per day	**Considerations for emerging bilingual students:** • Video of the baby whale calf with its mother will help contextualize the problem.

Extensions	Have students think about the return trip. What might be the same, and what might be different about the distances traveled? They can create a return trip table similar to the one on the page.

Lesson Series 2 – **Day 5**

Core Math	Multiplication can be used to solve problems involving lengths.
Description	Students work in groups of four to find dimensions of posters based on a scaling up in size of photographs.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Measurement and Data** **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** 4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
Resources	From Photo To Poster BLM S C African Animals Photos BLM Larger paper for poster Markers or colored pencils

	Scissors and glue Centimeter Ruler
Homework	Day 5 HW S C

Math Talk

Number Talk with Multiplication

What is the product of 12 x 12?

Lesson Plan	
LAUNCH **1** (5 Min)	Tell students that they will be working on a problem about a company that takes photos and turns them into larger posters. Draw a rectangle on the whiteboard to approximate a photo. Ask students to tell you the names of the dimensions (length and width). Draw a very large rectangle with the same shape. Tell them that is the poster size of the original photo. They will have to multiply the original side lengths of the photos to find out the new side lengths of the poster. Luckily, the company told us by which numbers to multiply. Students will be working in groups of four. Tell them that you will observe how they share the work and show their group's thinking on the poster.
EXPLORE **2** (40 min)	Pass out the task sheet and the photos of the African animals. Ask clarifying questions of groups that seem stuck on how to start. For example, you may focus them on the first photo to poster and make sure they understand the process. **—Key Math to Observe** • Can students multiply single-digit by double-digit numbers and double-digit by double-digit numbers? • Do groups equitably share the work and the credit for their poster?
SUMMARIZE **3** (15 min)	**→ Core Math to Emphasize** • Multiplication can be used to solve problems involving lengths. Instead of a whole class discussion, reserve the last 10 minutes of class for a Gallery Walk for groups to compare their work with others in the class.  **Notebook Prompt (5 minutes)** One thing my group did well when making our poster was _____.

Notes	Universal Support
• Students may still struggle with multiplying double-digit numbers. • Students may not know all the multiplication facts. • Students may struggle to discern pertinent information from the task due to the type of text involved.	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/ representations without any written explanation. • Provide a multiplication fact chart for students who need it.

Vocabulary: print, service, dimensions, length, width	**Considerations for emerging bilingual students:** • Show an example of a photo and a poster in order to illustrate the idea of scaling into something larger.

Expert Task

Open in Google Drive

Core Math	Numbers can be compared additively (how many more) or multiplicatively (how many times as many).
Description	**Comparing Two Schools** Students compare student populations from two schools using multiplicative comparisons.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Resources	• Comparing Two Schools BLM S C • Poster paper • Markers • Scissors • Glue or tape
Homework	Expert HW S C

Math Talk

Number Talk with Multiplication

What is the product of 15 x 4?

Lesson Plan	
LAUNCH **1** (10 Min)	This task is designed to be done in groups of four students. Either use pre-existing groups you have set up in math class or assign these groups before the lesson. Tell students that this is a problem about comparing the number of students at two different schools in a fictional town. Ask students if they know how many students attend their own school as a way to understand context. Do they think this number has changed over the years? **Noticing equitable participation:** Launch this task using the Multiple Abilities Strategy. The goal of this strategy is to communicate to students the variety of strengths and abilities, including skills, understandings, and mathematics practices, that they bring as individuals to a group task. By carefully considering the strengths of all your students, including those who may not participate actively, and then naming their strengths explicitly, you are opening up the opportunity for more robust participation by all students. Share a list of strengths you will be looking for students to bring to their work on this task. Your list should include a variety of mathematical behaviors as well as content-specific strengths. Tell students: *These are some of the things that will help you with this task. Each of you can do some of these things, although probably none of you can do all of these things, really well. By working in a group you can combine strengths to learn from this task.* **Some Suggested Strengths for Multiple Abilities Lists** • Understand the written task and explain it to your group. • Ask questions of others and explain your thinking. • Draw tape diagrams. • Understand numerical information in a table. • Multiply multi-digit numbers by other numbers. • Explain how your group solved the task. Tell students that you will be using Groupwork Feedback to describe your observations of their work as a group and how each individual is bringing his/her strengths to the group's work. **Note:** For more information on how to use a Multiple Abilities Strategy to

	launch a task, see the Digital Math Teaching Toolkit (http://www.sfusdmath.org/multiple-abilities-strategy.html). For more information on how to use the Groupwork Feedback strategies during a collaborative task, see Digital Math Teaching Toolkit (http://www.sfusdmath.org/participation-quiz--groupwork-feedback.html). Tell students that they will be working in groups of four. Assign each person a role in the group. See Digital Math Teaching Toolkit for more information on group roles (http://www.sfusdmath.org/group-roles.html). Pass out the **Comparing Two Schools BLM** (two per group so that everyone can see the task as partners). Pass out poster paper, markers, scissors, and glue or tape after the groups have read and understood the task. Tell students the amount of time they have to complete the work, including the group poster.
EXPLORE **2** (40 min)	Circulate as the groups work on the task. Instead of answering individual students' questions, require them to ask group questions. In what years did Garfield School only have about three times as many students as Oceanside School? **2000 and 2016** About how many times as many students did Garfield School have as Oceanside School in the years 2008, 2012, and 2016? **2008 = 4, 2012 = 4, 2016 = 3. Note that students will have to make approximations because the numbers are not perfectly rounded.** About how many times as many students did Oceanside School have in 2016 as in 2000? **Note, compare within a column. About 4 times.** About how many times as many students did Garfield School have in 2016 as in 2000? **Note, compare within a column. Either about 4 or about 5, depending on how the approximation is made.** Garfield School and Oceanside School are the only two elementary schools in Springfield. Compare the total elementary student population in Springfield in 2000 to the total population in 2016. About how many times as many students were there in 2016 as in 2000? **2000 = 480 and**

2016 = 1917. So there were approximately 4 times as many students.

The town is expecting to have even more students in both schools by the year 2020. If Garfield School were to have about twice as many students as it did in 2016 and Oceanside School were to have about six times as many students as it did in 2016, how would the two school populations compare?
1,438 x 2 = 2,876 and 479 x 6 = 2, 874. The two schools will have approximately the same population.

→Key Math to Observe

- Are students able to show their work using tape diagrams effectively?
- Are students able to round numbers to solve the problems?
- Are students able to interpret data across the rows and within the columns of the table?

The task asks groups to choose at least two more questions of their choice after the first one, so if groups start to finish earlier, they can work on an additional question from the list.

SUMMARIZE

3

(15 min)

→ Core Math to Emphasize

- Numbers can be compared to each other multiplicatively. This gives a magnitude sense of how the numbers are related.
- In many cases, multiplicative comparisons require some rounding of numbers to make sense of them.
- One way to compare two numbers is using a tape diagram that allows us to quickly see their relationship.

Based on your observations of the math work, decide on one question to debrief with the class. Question #2 involves some approximations and might be a good source of classroom discussion.

Notebook Prompt (5 minutes)
One way I helped my group in this task was _____.

Notes	Universal Support

Students may not be secure in how they round numbers, or whether the rounded numbers represent valid points of comparison for the problems. **Vocabulary:** methods, representations, product, bar model	**Considerations for students with learning differences:** ● Read aloud directions for students as needed. ● Allow students to draw their solutions/representations without any written explanation. ● Provide a multiplication fact chart for students who need it. ● Help students round the numbers so that they may focus on multiplicative comparison. ● Allow students to use calculators if deemed useful. **Considerations for emerging bilingual students:** ● Provide verbal instructions if needed. ● Have EL students work in groups of students with at least one bilingual student as an interpreter. ● Provide sentence frames as needed. ● The sentence "_____ is ____ times as many as _____" can be a difficult grammatical structure for all students, but particularly for EL students. Practicing it with a sentence frame will help make it more normal.

Note	In later grades, numbers can be compared by dividing one number by the other to determine the multiplicative comparison. This strategy is not a Grade 4 standard.

Lesson Series 3 **Overview**

Open in Google Drive

Description

Students practice solving problems with partial products and work on two tasks that involve using multi-digit multiplication in real-world contexts.

Standards

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

	Day 1	Day 2	Day 3
Core Math	Partial product and number sense can help you win in games of mathematical strategy.	Multiplication problems with two factors can be represented concretely with area models and more abstractly with algorithms based on place value.	Multiplication problems can be solved with algorithms based on place value, such as the box method or the standard regrouping algorithm.
Description	Students practice multi-digit multiplication and using number sense while playing a game called *Bullseye*.	Students show how to represent multiplication with multi-digit numbers in a variety of ways, including words, tape diagrams, area models, and equations.	Students play a game called *Fill It Up* in order to practice multi-digit multiplication.

Resources	Bullseye S C Dice, 1 die per student	How Can I Multiply with Larger Numbers? S C	Fill It Up BLM S C Fill It Up Game Board BLM S C (optional) 1 die per partner
Homework	Day 1 HW S C	Day 2 HW S C	Day 3 HW S C

	Day 4	Day 5	Day 6
Core Math	Multiplication can help solve real-world problems involving pricing plans.	Some numbers can be the product of different factors or the quotient of different divisors and dividends. Others cannot.	• Extended facts are a powerful way to multiply larger numbers that are multiples of powers of 10 (10, 100, 1,000, etc.). • Numbers can be decomposed by their place value components. These components can be multiplied, then added back together to find the product of two multi-digit numbers. • Multiplication can be used to compare two numbers. When we say "35 is 7 times as great as 5," we are comparing the magnitude of the number 35 to the number 5.
Description	Students use multiplication and addition to determine the best pricing option for a family of four to visit an Amusement Park.	Students play a game called *Four to Go* and determine winning strategies.	**Re-engagement Day:** Students re-engage with concepts and skills they may have found challenging during the unit. There are five activities, including a game, re-engagement with several of the unit's tasks, and two digital resources.
Resources	Kinetic Kingdom BLM S C Poster paper Markers Scissors Glue or tape	Four to Go BLM S C	Re-engagement Centers Teacher Re-engagement Bullseye BLM S C Doubling and Halving BLM S C Multi-Digit Multiplication FAL BLM S C (from LS1 Day 5) Re-engagement Multiplication Judo BLM S C Re-engagement Reflection S C Sheet protectors

			Dice, 2 per pair Base-10 blocks (optional)
Homework	Day 4 HW S C	Day 5 HW S C	Day 6 HW S C

Lesson Series 3 – **Day 1**

Core Math	Partial product and number sense can help you win in games of mathematical strategy.
Description	Students practice multi-digit multiplication and use their number sense while playing a game called *Bullseye*.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Resources	• Bullseye S C • Dice, 1 die per student
Homework	Day 1 HW S C

Math Talk

Number Talk with Multiplication

What is 25 x 12?

Lesson Plan	
LAUNCH **1** (15 Min)	Tell the students they will be playing a game called *Bullseye*. To play this game, they will need to be able to multiply numbers mentally, including numbers with two digits. Have students read the instructions for *Bullseye*. See **Bullseye Student**. Play a sample game with the class before having them play with partners. **Noticing equitable participation:** As noted earlier in this unit, games are an important component of a rigorous math program. But without careful management by you, they may not actually be beneficial for all students. Be thoughtful about how games are played in class: Are all students learning from the game? Do all students understand how to play the game? Are they all equipped to compete reasonably with their peers?

	Consider how you set up partners and groups to support equitable participation. In addition, have clear and explicit expectations about game play in class and go over them often.
EXPLORE **2** (35 min)	Have students play *Bullseye* with a partner. Circulate and notice how students are strategizing to get close to 300. Make note of particular strategies that you want to mention in the summary. **→Key Math to Observe** • How do students interpret the role of chance (die throw) in this game? • If students roll a high number, do they choose a high or low multiple of 10? • Do students try to space out their progress toward 300 for the six rolls or do they try to get to the goal quickly? • How do strategies evolve after several rounds of this game? Do students start to incorporate more number sense and probability sense into their strategies?
SUMMARIZE **3** (10 min)	**→ Core Math to Emphasize** • A roll of a die produces random numbers from 1 to 6. • One strategy is to think of 300 as being divided into 6 more or less equal parts (50 each). • If on each turn, your attempt is to get close to 50, you will have a better chance of winning. Use your observations to inform your summary. You may consider playing another game with the class and discussing the moves. This game will be repeated on Day 5 in Re-engagement Centers, so keep the materials together in preparation for the center.  **Notebook Prompt (5 minutes)** After playing the game two times, how did you change your strategy?

Notes	Universal Support
Students may struggle staying within 300 and determining a productive strategy.	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it.

Vocabulary: random, change, target	• Calculators may help some students feel more confident. **Considerations for emerging bilingual students:** • Playing a sample game will help clarify understanding.

Lesson Series 3 – **Day 2**

Core Math	Multiplication problems with two factors can be represented concretely with area models and more abstractly with algorithms based on place value such as the box method.
Description	Students show how to represent multiplication with multi-digit numbers in a variety of ways, including words, tape diagrams, area models, and equations.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Resources	How Can I Multiply with Larger Numbers? S C
Homework	Day 2 HW S C

Math Talk

Which One Does Not Belong?

27	**81**
18	**31**

Lesson Plan	
LAUNCH **1** (5 Min)	Remind students that up until now they have been drawing area models and using the box method to solve multi-digit multiplication problems. But sometimes we have to multiply numbers that are really too large to be drawn. Write the following problem on the whiteboard: **6 x 586** Ask students to talk with partners about how they can solve this using strategies they have already learned in this unit as well as in 3rd grade. Ask volunteers to describe how they would solve this. If it does not come up, ask students how they think they could solve this problem using the box method. Tell students that in this lesson they will be learning how to use the box method to solve multiplication problems involving one-digit by three-digit numbers, such as the one

you wrote on the whiteboard.

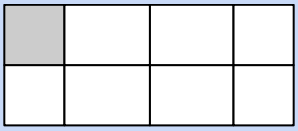
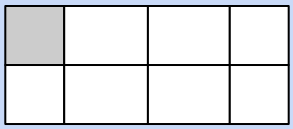
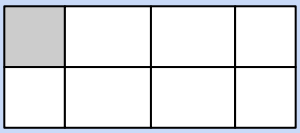
Avoid explicitly teaching the box method with three-digit numbers. Focus instead on questioning strategies that help students apply place value understanding to expand the box to three columns.

EXPLORE

2

(45 min)

Circulate while partners are working on **How Can I Multiply with Larger Numbers?**. Notice how students are setting up their tape diagrams and the box method.

The length of a square city block is 293 feet. **What is the perimeter of this block?**	 Note that students may not think of a city block as having equal sides, but the problem says "square."
A standard calendar year has 365 days. **How many calendar days are in 7 years?**	 Note that every four years the calendar has 366 days, but the problem says "standard" year. Also note that 7 x 300 equals a four-digit number and makes this problem different from the others.
One gallon of water is equal to 128 fluid ounces. **How many fluid ounces are there in 9 gallons of water?**	

→Key Math to Observe

- Can students understand a word problem and represent solutions with tape diagrams, area models, and equations?
- Can students generalize the box method to work for larger numbers using a place value understanding?

SUMMARIZE

3

(10 min)

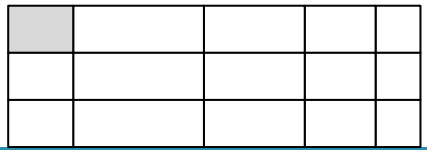
→ Core Math to Emphasize

- The box method can be generalized to be used with numbers of any size.

Based on observations, select student work to highlight in the summary.

Another option would be to pose a different problem, with more digits, and ask students to use what they know about place value and solve with the box method. e.g.

54 x 2,345

	Ask students how they would set this problem up using the box method. *Note that multiplication of a 4 digit by 2 digit number is actually a 5th grade standard, but you may use this problem to lead a discussion on how this box method can be easily generalized to large numbers,*   **Notebook Prompt (5 minutes)** For the problem 9 x 459, how can you break apart the numbers to make the multiplication easier?

Notes	Universal Support
Students may not have mastered the various ways to represent a multiplication problem. **Vocabulary:** tape diagram, area model, equation	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** The language of this lesson is relatively simple. If English is challenging, allow students to write in their native language since the other representations are visual.

Lesson Series 3 – **Day 3**

Core Math	Multiplication problems can be solved with algorithms based on place value, such as the box method or the standard regrouping algorithm.
Description	Students play a game called *Fill It Up* in order to practice multi-digit multiplication.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Resources	Fill It Up BLM S C Fill It Up Game Board BLM S C (optional) 1 die per partner
Homework	Day 3 HW S C

Math Talk

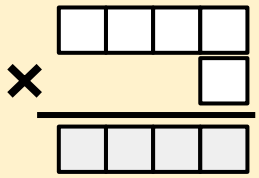
Number Talk with Multiplication

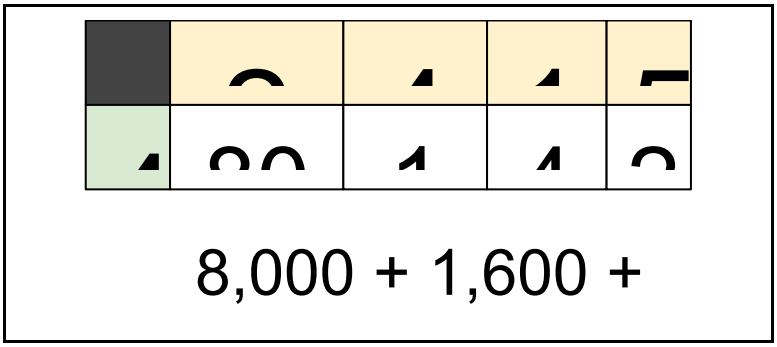
What is the product of 20 x 13?

Lesson Plan	
LAUNCH **1** (5 Min)	Write the following multiplication problem on the whiteboard: **2 x 3,653** Have students discuss with a partner how to set up this problem. Take responses. If it does not come up, ask students how to use the box method learned previously to set this problem up. (Example below)

	3000	600	50	3
2	6000	1200	100	6

6,000 + 1,200 + 100 + 6 = 7,306

	Notice with students that this is a one-digit by four-digit multiplication problem that can still be solved using the box method. It can also be solved using an area model, but as we have seen before, as numbers become large, setting up such problems becomes more difficult/tedious. On the student page, there is a template for the standard regrouping algorithm. Ask students to help you write the problem using that template. (Example below) **Standard Algorithm**  **Box Method** 3000 | 600 | 50 | 3 2 | 6000 | 1200 | 100 | 6 6,000 + 1,200 + 100 + 6 = 7,306 **Note:** The standard algorithm for multiplication is **not a 4th grade standard.** However, many students will be familiar with it from work outside of school and/or possibly previous grades, so it is part of this activity in order to notice the similarity with the box method. **Avoid** teaching this algorithm. Instead, **focus** on the commonalities between how they are both set up to solve multiplication problems. Tell students that they will be playing a game called *Fill It Up*. In this game, they will be trying to create a multiplication problem with a product as close to 9,999 without exceeding it.
EXPLORE **2** (45 min)	Have students read and discuss the rules as well as the example on the student page. Circulate and answer clarifying questions about how to play the game. Notice how students are thinking about ways to get as close to 9,999 as possible. Where are they placing the digits of their numbers and why? **→Key Math to Observe** • Are students using place value understanding as they place the digits in the multiplication problem? • Are students able to accurately calculate one-digit by four-digit multiplication problems? • Are students able to find the difference between the 9,999 and the product of their multiplication problem? • Are students able to accurately keep a running score and find the total after 5 rounds?
SUMMARIZE	**→ Core Math to Emphasize**

3 (10 min)	• When working with multiplication problems with larger products, a place value understanding of the numbers helps us estimate reasonable answers. Estimating a product close to but not exceeding 9,999 may be challenging. It involves several different skills, such as number sense, knowledge of multiplication facts, and properties of multiplication. As a summary, show students the following digits and ask them to help you create a multiplication problem, find the product, and determine how far from 9,999 it is. See the following example: 4, 2, 6, 1, 5  Ask students what strategies can be used to set up the multiplication problem. How can they be sure not to exceed 9,999? In this example, 4 x 2,000 = 8,000, which is close to 9,999. The 5 and 6 can not be used in the thousands place because it would create a product that exceeded 9,999. 1 and 2 would create a product far less than 9,999.  **Notebook Prompt (5 minutes)** One strategy I used to play this game was _____.

Notes	Universal Support
• Students may not yet be confident with the box method. • Students may be more comfortable with procedure and not be as comfortable explaining their reasoning. • Students may not yet be fluent with multiplication facts.	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:**

	• This is a language-rich task. While the context may be familiar to many students, the language may be challenging. • Make your groups of four so that students have language partners to help with the work. Bilingual students will be a great asset in helping everyone in the group communicate and feel integrated into the group.

Lesson Series 3 – **Day 4**

Core Math	Multiplication can help solve real-world problems involving pricing plans.
Description	Students use multiplication and addition to determine the best pricing option for a family of four to visit an Amusement Park.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Measurement and Data** **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** 4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
Resources	Kinetic Kingdom BLM S C, 2 per group Poster paper Markers, Scissors, Glue or tape
Homework	Day 4 HW S C

Math Talk

Number Talk with Multiplication

What is the product of 15 x 15?

Lesson Plan	
LAUNCH	This task is designed to be done with partners. Either use pre-existing partners you

1 (5 Min)	have set up in math class or assign these partners before the lesson. Ask students about their experiences in amusement parks. Tell them that the problem they will be working on in groups involves figuring out the best pricing plan for a family with four people. **Noticing equitable participation:** Launch this task using the Multiple Abilities Strategy. The goal of this strategy is to communicate to students the variety of strengths and abilities, including skills, understandings, and mathematics practices, that they bring as individuals to a group task. This is the third time in this unit that this strategy has been used. Consider your observations of past group work lessons and decide on one particular participation goal you want to explicitly describe to your class. One suggestion is to review the use of group roles and group questions when you circulate around the room. Share a list of strengths you will be looking for students to bring to their work on this task. Your list should include a variety of mathematical behaviors as well as content-specific strengths. Tell students: *These are some of the things that will help you with this task. Each of you can do some of these things, although probably none of you can do all of these things, really well. By working in a group you can combine strengths to learn from this task.* **Some Suggested Strengths for Multiple Abilities Lists** • Understand the written task and explain it to your partner. • Ask questions of others and explain your thinking. • Create a table of prices and make sense of it. • Understands different pricing plans for amusement parks. • Multiply multi-digit numbers by other numbers. • Add multiple numbers together. • Explain how your partner and you solved the task. **Note:** For more information on how to use the Multiple Abilities Strategy to launch a task, see the Digital Math Teaching Toolkit (www.sfusdmath.org/entry-points.html). Give each pair of students one task sheet, scissors, glue or tape, and one large piece of paper for them to make a poster of their work. Tell them that they need to cut out the pricing plans, paste it to their poster, and record their other work on the same poster.
EXPLORE	Circulate while the students are working on the problem.

2

(45 min)

Kinetic Kingdom Amusement Park Answer Guide

Plan A:
(Total cost: $360 per visit)

Plan B:
(Total cost: $380 for the season)

Plan C:
(Total cost: $225 for family season pass + $95 for another person = $320)

→Key Math to Observe

- Are groups of students able to find the price for one visit?
- Does the group interpret *at least* to mean one or more visits?
- How does the group determine at which point they have found out enough information to decide which plan is best?

SUMMARIZE

3

(10 min)

→ Core Math to Emphasize

- Multiplication and addition help us determine the best pricing plan.
- The phrase *at least* tells us to consider more than one visit.
- At some point, we realize that we do not need to continue to investigate the options because one option is always the best.

Use a Gallery Walk protocol at the end of this task. The protocol can be:

- *We notice* _____.
- *We wonder* _____.

For more on Gallery Walks, see http://www.sfusdmath.org/gallery-walk.html

Notebook Prompt (5 minutes)
One thing our group did well on this task was _____.

Notes	Universal Support
• Students may struggle to understand the constraints of this problem. • Students may not be precise enough with their calculations to make accurate decisions.	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:**

Vocabulary: amusement park, entrance fees, annual pass	• This is a language-rich task. While the context may be familiar to many students, the language may be challenging. • Make your groups of four so that students have language partners to help with the work. Bilingual students will be a great asset in helping everyone in the group communicate and feel integrated into the group.

Lesson Series 3 – **Day 5**

Core Math	Some numbers can be the product of different factors or the quotient of different divisors and dividends. Others cannot.
Description	Students play a game called *Four to Go* and determine winning strategies.
CCSS-M Standard(s)	**Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Resources	Four to Go BLM S C
Homework	Day 5 HW S C

Math Talk — **Number Strings**

12 x 10
12 x 5
12 x 15

Lesson Plan	
LAUNCH **1** (15 Min)	Tell students that they will be playing a new game of strategy. Strategy means that you create a plan to win based on what you understand of the rules. Before learning how to play the game, here are some questions to consider: What good ways do you have to win a game? Does it matter who goes first or second? How are you deciding which number to aim for next? Model how the game is played by projecting it on a document camera and playing with a volunteer. Tell students that they will be playing a game with a partner and that the rules are written on the game.

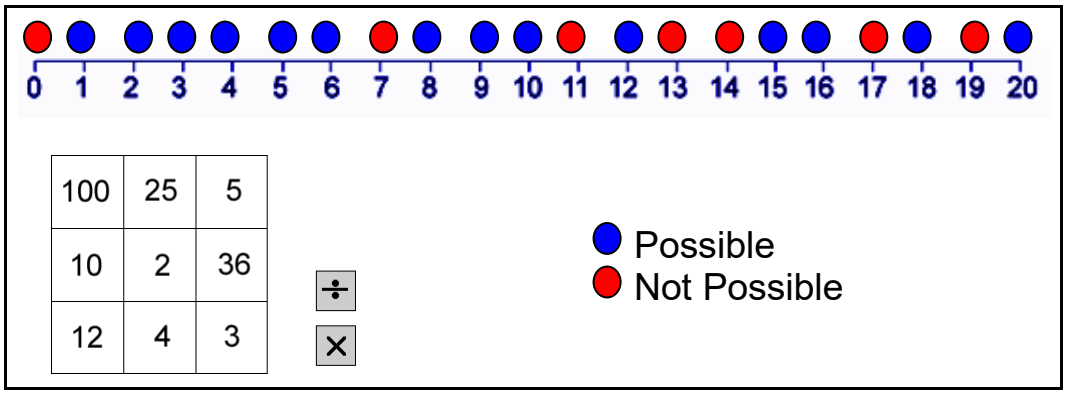
EXPLORE

2

(35 min)

Circulate and observe how they are making sense of the game. Ask questions about how they are choosing their numbers. Are there numbers on the number line that are impossible to create with multiplication and/or division with the numbers in the box? Why?

See the box below for possible and impossible numbers.



→Key Math to Observe

- Are students fluent with multiplication math facts?
- Are students fluent with division math facts?
- Do students notice that some numbers are not possible?
- Do students notice that there is only one section of the number line (1–6) where it is possible to win four in a row?
- Do students notice that if one person has captured 3 and the other 4, for example, that the game will not have a winner?

SUMMARIZE

3

(10 min)

→ Core Math to Emphasize

- Some numbers can be the product of different factors or the quotient of different divisors and dividends. Others cannot.

Reproduce the number line as the box with numbers on the whiteboard. Ask students to say whether a given number is possible or not and why. Use color pens to mark this.

For example, 0 is not possible because there is not a 0 in the box of numbers, which is the only factor that can make a product of 0.

1 is possible because any number divided by itself is 1.

Notice that some numbers have more than one way to create them.
For example, 4 is the product of 2 x 2, and the quotient of 100÷25 and 12÷4.

Avoid explaining much about special numbers, such as prime numbers, because this will be studied more in depth in Unit 4.6 Multiples and Factors.

	 **Notebook Prompt (5 minutes)** Some numbers are impossible to create in this game. What are they and why?

Notes	Universal Support
• Students may be more comfortable with procedure and not be as comfortable explaining their reasoning. • Students may not yet be fluent with multiplication facts. • Students may not have a solid understanding of division yet.	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. **Considerations for emerging bilingual students:** • Define the terms *factors, products, quotients, divisors,* and *dividends* with clear examples using real numbers.

Lesson Series 3 – **Day 6**

Core Math	• Extended facts are a powerful way to multiply larger numbers that are multiples of powers of 10 (10; 100; 1,000; etc.). • Numbers can be decomposed by their place value components. These components can be multiplied, then added back together to find the product of two multi-digit numbers. • Multiplication can be used to compare two numbers. When we say "35 is 7 times as great as 5," we are comparing the magnitude of the number 35 to the number 5.
Description	**Re-engagement Day:** Students re-engage with concepts and skills they may have found challenging during the unit. There are five activities, including a game, re-engagement with several of the unit's tasks, and two digital resources.
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. **Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Resources and Preparation	Re-engagement Centers Teacher Re-engagement Bullseye BLM S C Doubling and Halving BLM S C Multi-Digit Multiplication FAL BLM S C (from LS1 Day 8-9) Re-engagement Multiplication Judo BLM S C Re-engagement Reflection S C Sheet protectors Dice, 2 per pair Base-10 blocks (optional)
Homework	Day 6 HW S C

Lesson Plan	
	Noticing equitable participation: Clear and explicit classroom norms for academic behavior help all students, but, in particular, they allow struggling students an opportunity to learn by creating a sense of purpose and productivity in class. This is especially true during centers work. Be very explicit and persistent with norms for group work and centers.
LAUNCH **1** (5 Min)	Tell the class that today they will be working with partners in centers. Explain the centers. Options provided in this unit are: **Re-engagement Centers** 1. Bullseye (from LS3 Day 1) 2. Doubling and Halving 3. Multi-Digit Multiplication(from LS1 Days 6) 4. Multiplication Judo (from LS2 Day 3) 5. Decimal Plus (iPad and computer ready) **Note:** For more detailed information about these centers, see Re-engagement Centers Teacher Go over classroom norms for center work and your expectations for behavior and how the students will know to rotate through them. **Note:** For more detailed information on using math centers, see the Digital Math Teaching Toolkit (http://www.sfusdmath.org/centers.html). You may decide to post yourself at one center to offer extra help for students who need it, or you may circulate around the room observing student work and behaviors. Tell students what you will be doing during centers time.
EXPLORE **2** (45 min)	Depending on the role you have given yourself during centers work, you will be looking for different things. **→Key Math to Observe** • Can students decompose a multi-digit number into place value components, multiply them, then recompose them to find the product? • Can students justify why the partial product method works? • Do students understand that in a given rectangle, the area is the product of the two side lengths? • Do students understand that multiplicative comparison means how many times as many or as much some number is when compared to another number?

SUMMARIZE	→ Core Math to Emphasize
3 (15 min)	• Multiplication can be used to compare two numbers. When we say "35 is 7 times as great as 5," we are comparing the magnitude of the number 35 to the number 5. • When multiplying multi-digit numbers, you can first break them into place value components, multiply those components together, then recompose them. Depending on time, you can choose to debrief the work from one center (the FAL center is a good choice) or you can have students respond to prompts on **Re-engagement Reflection**.  **Notebook Prompt (5 minutes)** Today I worked at the _____ center. I practiced _____.

Notes	Universal Support
• Students may find it challenging to do so many activities in one class period. • Students may not know how to do a center and become overly dependent on external help. • The technology-based activity might be difficult for some students.	**Considerations for students with learning differences:** • The nature of centers might be challenging and students may benefit from having a clearer path: either by staying at one center for much of the class or working in close proximity to you. **Considerations for ELs:** • There are many opportunities for students to practice concepts along with language. • Partner language learners with language models. • For newcomers and students with little confidence in English, strategic partnering with bilingual students who can support them can be helpful.

Extensions	The re-engagement centers are designed to have students practice a variety of skills and concepts. As such, there is no need for extensions for this lesson.

Milestone Task

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Core Math	Multiplication can be used to solve real-world problems involving equal groups and multiplicative comparison.
Description	**San Francisco Tour Group** Students use their knowledge about multi-digit multiplication, addition, and subtraction to solve a problem involving a San Francisco tour group and money. **This task is the Fall SFUSD Math District Assessment.**
CCSS-M Standard(s)	**Operations and Algebraic Thinking** **Use the four operations with whole numbers to solve problems.** 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Resources	**Copies of the task are provided and delivered by the Achievement Assessments Office upon request. Contact your school's Test Coordinator to order.** • San Francisco Tour Group BLM S C • San Francisco Tour Group Suggested Answer Guide Teacher • San Francisco Tour Group Rubric Teacher • Milestone Task Slides **S** Optional: Find images of the four places named in the task (Alcatraz, California Academy of Sciences, Exploratorium, and Chinatown) and present them to your students. **Student Work Samples and Commentary are available online for this task.**

Homework	Milestone HW S C

Lesson Plan	
LAUNCH **1** (5 Min)	If you found the images, show them one by one to students and ask them to identify the places. Tell students that they will be working individually on a task involving tourists in San Francisco. Students can ask you questions during the task, but tell them that it is an assessment of what they have learned about multiplication during this unit, so it is important that they understand what each question is asking and try to respond to it.
EXPLORE **2** (50 min)	Circulate around the room and observe how students are understanding and working with the problems. Make special note of patterns of difficulty so you can re-engage with these concepts and skills in subsequent units. For example, if students still struggle to decompose numbers into place value components, there will be opportunities in the next unit on division to revisit this. **→Key Math to Observe** • Can students interpret the word problems as multiplication situations and solve accordingly? • Can students multiply 2 two-digit numbers together and find the product accurately?
SUMMARIZE **3** (5 min)	**→ Core Math to Emphasize** • There are many strategies for finding the product of multiplication problems. Some are easier than others to do, while others are easier to explain. Gather students and ask them what the easiest and the hardest part of this task was for them.  **Notebook Prompt (5 minutes)** One thing I did very well on this Milestone Task was _____ because _____. **Note:** Save student work from this Milestone Task as evidence of student learning for report cards and parent conferences.

Notes	Universal Support
• Students may not know how to estimate which of the tours would be the best to eliminate. **Vocabulary:** tour group, eliminate, two for one	**Considerations for students with learning differences:** • Read aloud directions for students as needed. • Allow students to draw their solutions/representations without any written explanation. • Provide a multiplication fact chart for students who need it. • Consider offering calculators to students who would otherwise not be able to work on this task. **Consideration for EL students:** This task is relatively language rich. Context and further explanation of the language is important. Decide if students can express their thinking better in their native language and if that can be allowed in this case.

Extensions	If there are early finishers, have them write in their math notebooks about what they found easy and hard about this task.