$3x - 9x^2/2 + 27x^3/3 - 81x^4/4$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (3x)^{n+1}}{n+1} = 3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3} - \frac{(3x)^4}{4} + \cdots \frac{(-1)^n (3x)^{n+1}}{n+1}$$

The Maclaurin series for the function f is given by

$$\cancel{f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1} = 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4} + \cdots + \frac{(2x)^{n+1}}{n+1} + \cdots}$$

(b) error bound $\left| \frac{(3x)^5}{5} \right| \le .005$

$\le \frac{243(.1)^5}{5}$

$\le .000486$

on its interval of convergence.

a) Find the interval of convergence of the Maclaurin series for f.

$$\lim_{n \to \infty} \left| \frac{(3x)^{n+2}}{n+2} \cdot \frac{n+1}{(3x)^{n+1}} \right| = |3x| \qquad -\frac{1}{3} < x \le \frac{1}{3}$$

$x = -\frac{1}{3} \Rightarrow \sum \frac{(-1)^n (-1)^{n+1}}{n+1} = \sum \frac{(-1)^{2n+1}}{n+1}$ compare to $\sum \frac{1}{n}$ Harmonic

Diverges

$$\lim_{n \to \infty} \frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{n+1}\right)} = \frac{n+1}{n} = 1$$

$x = \frac{1}{3}$ $\sum \frac{(-1)^n}{n+1}$ Compare to Alt Harmonic $\lim_{n \to \infty} \frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{n+1}\right)} = \frac{n+1}{n} = 1$

Conditional Convergence

b. Find the first four terms and the general term for the Maclarin series for $f'(x)$

$$f'(x) = 3 - 9x + 27x^2 - 81x^3$$

general term $\frac{(-1)^n \cancel{(n+1)} (3x)^{n+1} \cdot 3}{\cancel{(n+1)}} = (-1)^n 3 (3x)^{n+1}$

c. Find the function that represents the sum of the series in part (b)

$$\text{Sum} = f'(x) = \frac{3}{1 + 3x}$$

d. Use the answer you found in part (c) to find the value of $f'\left(\frac{-1}{2}\right)$

$$f'\left(\frac{1}{3}\right) = \frac{3}{1 + 3\left(-\frac{1}{2}\right)} = \frac{3}{1 - \frac{3}{2}} = \frac{3}{-\frac{1}{2}} = -6$$