7.1

Solving Systems of Equations

What You Should Learn

- Use the methods of substitution and graphing to solve systems of equations in two variables

- Use systems of equations to model and solve real-life problems

- You probably do not need to take many notes for 7.1, maybe only the two red sentences on slides 5 and 13? More is fine. ☺

The Methods of Substitution and Graphing

The Methods of Substitution and Graphing

So far in this text, most problems have involved either a function of one variable or a single equation in two variables. However, many problems in science, business, and engineering involve two or more equations in two or more variables.

To solve such problems, you need to find solutions of **systems of equations.** Here is an example of a system of two equations in two unknowns, *x* and *y*.

$$\begin{cases} 2x + y = 5 & \text{Equation 1} \\ 3x - 2y = 4 & \text{Equation 2} \end{cases}$$

The Methods of Substitution and Graphing

A **solution** of this system is an ordered pair that satisfies each equation in the system. Finding the set of all such solutions is called **solving the system of equations.** For instance, the ordered pair (2, 1) is a solution of this system.

To check this, you can substitute 2 for *x* and 1 for *y* in *each* equation.

Check (2, 1) *in Equation 1:*

$$2x + y = 5$$
$$2(2) + 1 \stackrel{?}{=} 5$$
$$5 = 5 \checkmark$$

Check (2, 1) *in Equation 2:*

$$3x - 2y = 4$$
$$3(2) - 2(1) \stackrel{?}{=} 4$$
$$4 = 4 \checkmark$$

The Methods of Substitution and Graphing

In this section, we will study two ways to solve systems of equations, beginning with the **method of substitution.**

The Method of Substitution

To use the **method of substitution** to solve a system of two equations in x and y, perform the following steps.

1. Solve one of the equations for one variable in terms of the other.
2. Substitute the expression found in Step 1 into the other equation to obtain an equation in one variable.
3. Solve the equation obtained in Step 2.
4. Back-substitute the value(s) obtained in Step 3 into the expression obtained in Step 1 to find the value(s) of the other variable.
5. Check that each solution satisfies *both* of the original equations.

The Methods of Substitution and Graphing

When using the **method of graphing,** note that the solution of the system corresponds to the **point(s) of intersection** of the graphs.

The Method of Graphing

To use the **method of graphing** to solve a system of two equations in x and y, perform the following steps.

1. Solve both equations for y in terms of x.
2. Use a graphing utility to graph both equations in the same viewing window.
3. Use the *intersect* feature or the *zoom* and *trace* features of the graphing utility to approximate the point(s) of intersection of the graphs.
4. Check that each solution satisfies *both* of the original equations.

Example 1 – *Solving a System of Equations*

Solve the system of equations.

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

Equation 1

Equation 2

Solution:

Begin by solving for y in Equation 1.

$y = 4 - x$ Solve for in Equation 1.

Next, substitute this expression for y into Equation 2 and solve the resulting single-variable equation for x.

$x - y = 2$ Write Equation 2.

Example 1 – *Solution*

cont'd

$x - (4 - x) = 2$ Substitute $4 - x$ for y.

$x - 4 + x = 2$ Distributive Property

$2x - 4 = 2$ Combine like terms.

$2x = 6$ Add 4 to each side.

$x = 3$ Divide each side by 2.

Finally, you can solve for by *back-substituting* $x = 3$ into the equation $y = 4 - x$ to obtain

$y = 4 - x$ Write revised Equation 1.

Example 1 – *Solution*

cont'd

$y = 4 - 3$ Substitute 3 for *x*.

$y = 1.$ Solve for *y*.

The solution is the ordered pair (3, 1).

Check this as follows.

Check (3, 1) *in Equation 1:*

$x + y = 4$ Write Equation 1.

$3 + 1 \stackrel{?}{=} 4$ Substitute for *x* and *y*

$4 = 4$ Solution checks in Equation 1. ✓

Example 1 – *Solution*

cont'd

Check (3, 1) *in Equation 2:*

$x - y = 2$ — Write Equation 2.

$3 - 1 \stackrel{?}{=} 2$ — Substitute for x and y.

$2 = 2$ — Solution checks in Equation 2. ✓

Because (3, 1) satisfies both equations in the system, it is a solution of the system of equations.

Application

Application

The total cost *C* of producing *x* units of a product typically has two components: the initial cost and the cost per unit.

When enough units have been sold so that the total revenue *R* equals the total cost *C*, the sales are said to have reached the **break-even point.**

You will find that the break-even point corresponds to the point of intersection of the cost and revenue curves.

Example 6 – *Break-Even Analysis*

A small business invests \$10,000 in equipment to produce a new soft drink. Each bottle of the soft drink costs \$0.65 to produce and is sold for \$1.20. How many bottles must be sold before the business breaks even?

Solution:

The total cost of producing x bottles is

$$\boxed{\text{Total cost}} = \boxed{\text{Cost per bottle}} \cdot \boxed{\text{Number of bottles}} + \boxed{\text{Initial cost}}$$

$$C = 0.65x + 10{,}000.$$ Equation 1

Example 6 – *Solution*

cont'd

The revenue obtained by selling x bottles is

$$\boxed{\text{Total revenue}} = \boxed{\text{Price per bottle}} \cdot \boxed{\text{Number of bottles}}$$

$R = 1.20x.$ Equation 2

Because the break-even point occurs when $R = C$, you have

$C = 1.20x$

and the system of equations to solve is

$$\begin{cases} C = 0.65x + 10{,}000 \\ C = 1.20x \end{cases}.$$

Example 6 – *Solution*

cont'd

Now you can solve by substitution.

$C = 0.65x + 10{,}000$ Write Equation 1.

$1.20x = 0.65x + 10{,}000$ Substitute 1.20 for *C*.

$0.55x = 10{,}000$ Subtract 0.65 from each side.

$$x = \frac{10{,}000}{0.55}$$

$x \approx 18{,}182$ bottles. Use a calculator.

Example 6 – *Solution*

cont'd

Note in Figure 7.8 that revenue less than the break-even point corresponds to an overall loss, whereas revenue greater than the break-even point corresponds to a profit. Verify the break-even point using the *intersect* feature or the *zoom* and *trace* features of a graphing utility.

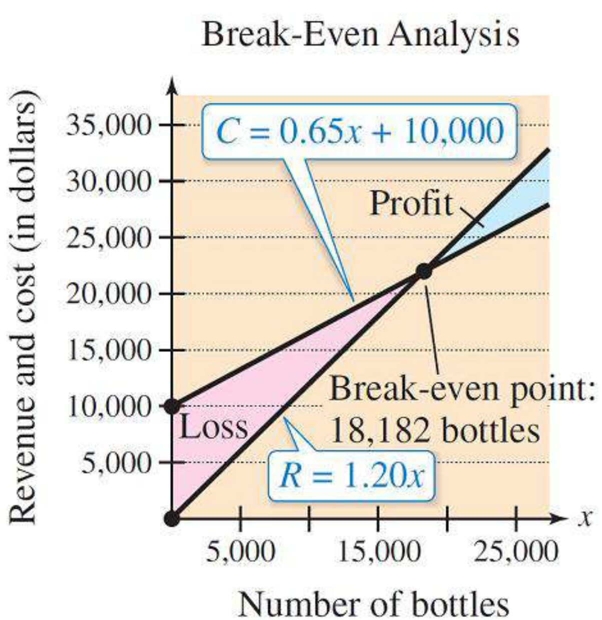

Figure 7.8