Determine the local extrema of the function

33) $f(x) = x^4 - 4x^{3/2}$

$$f(x) = x^4 - 4x^{3/2}$$ $$x \geq 0$$

1,117

Local max

$x = 0$ Local Max
b/c $f(x)$ dec
$\left(0, \left(\frac{3}{2}\right)^{2/5}\right)$

$x = \left(\frac{3}{2}\right)^{2/5}$ is
a local min
because f' changes
from neg to pos
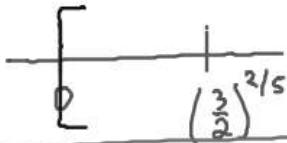
$$f'(x) = 4x^3 - 6x^{1/2}$$

$$0 = 4x^3 - 6x^{1/2}$$

$$0 = 2x^{1/2}\left(2x^{5/2} - 3\right)$$

$$0 = 2\sqrt{x} \qquad 0 = 2x^{5/2} - 3 \longrightarrow \left(x^{5/2}\right)^{2/5} = \left(\frac{3}{2}\right)^{2/5}$$

$$0 = x \qquad \text{Critical Pts} \qquad x = 1.176$$

Real
$f'(-1) = $ No Solution
$f'(1) = -2 < 0$ $f(x)$ dec $\left(0, \left(\frac{3}{2}\right)^{2/5}\right)$
$f(4) = 256 - 12 > 0$ $f(x)$ Inc $\left(\left(\frac{3}{2}\right)^{2/5}, \infty\right)$

$\left(\frac{3}{2}\right)^{2/5}$

36) $f(x) = x^{-2} - 4x^{-1}$ $x > 0$

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Determine the relative extrema of the function

37) $f(x) = \dfrac{1}{x^2 + 1}$

32) $f(x) = x^5 + x^3 + x$

What you'll Learn About
How to find intervals of concavity
How to find local extrema using the second derivative

Concavity

and

Inflection Points

change
concavity

Determine the intervals of concavity and the inflection points

A) $y = x^2$

$y = x^2$
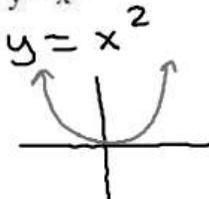
concave up $(-\infty, \infty)$

$y = x^2$ $y'' = 2 > 0$
$y' = 2x$

B) $y = -x^2$
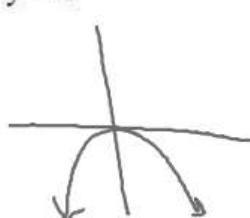
concave down $(-\infty, \infty)$

$y = -x^2$ $y'' = -2 < 0$
$y' = -2x$

5) $f(x) = 10x^3 - x^5$

$f(x) = 10x^3 - x^5$

$f'(x) = 30x^2 - 5x^4$

$f''(x) = 60x - 20x^3$

$-\sqrt{3} \quad 0 \quad \sqrt{3}$

Possible
Inflection
Points
$f''(x) = 0$
$f''(x)$ UND

$0 = 60x - 20x^3$

$0 = 20x(3 - x^2)$

P.I.P.S

$20x = 0 \qquad 3 - x^2 = 0$
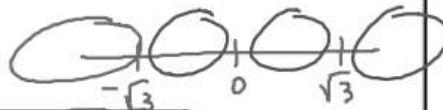
$x = 0 \qquad x = \pm\sqrt{3}$

$f''(-2) = -120 + 160 > 0$
$f(x)$ concave up $(-\infty, -\sqrt{3})$
$f''(-1) = -40 < 0$
$f(x)$ concave down $(-\sqrt{3}, 0)$
$f''(1) = 40 > 0$
$f(x)$ concave up $(0, \sqrt{3})$
$f(2) = -40 < 0$
$f(x)$ concave down $(\sqrt{3}, \infty)$

$x = 0, \pm\sqrt{3}$ are Inflection
points b/c the sign of
f'' changes