Precalc **Name:**

Review: M3 2.4, 2.5

Historical Problem—Products by Logarithms Before there were calculators (b.c.?!), logarithms provided an efficient way to multiply several numbers together, such as

$$(317)(22.4)(7810)(4.9).$$

In the early 1600's, the English mathematician Henry Briggs compiled a table of base 10 logs (called *common logs*). With the table, it was possible to look up the logs of the factors and *add* them. Pencil-and-paper addition can be done all at once, column-wise, whereas multiplication must be done just two numbers at a time. The computation looked like this:

Factors:	Logs:
317	2.5011
22.4	1.3502
7810	3.8927
4.9	0.6902
sum	8.4342

The answer equals $10^{8.4342}$. Using the tables backwards, one could find that $10^{0.4342}$ is about 2.72. So the answer in scientific notation is approximately 2.72×10^8. (With a calculator, you can easily check by direct multiplication that the answer is $2.717405... \times 10^8$.)

Various parts of the logarithm are given specific names. For instance, in the equation

$$\log 317 \approx 2.5011,$$

the 2 is called the *characteristic* of the logarithm. It is the same as the characteristic of the number when it is written in scientific notation, 3.17×10^2. The .5011 is called the *mantissa* of the logarithm. It is the same as the *logarithm* of the mantissa, the 3.17 in scientific notation. The argument, 317, is called the *antilogarithm* of 2.5011, or simply the *antilog*.

Briggs' table looked something like Table II at the back of this book. Part of this table is shown in Figure 6-10a.

The finger is pointing at the entry for 317. Note that only the mantissa appears in the table, and that the decimal point has been omitted for simplicity. People were expected to realize that the characteristic is 2.

Answer the following questions.

a. Look up in Table II the other three logarithms shown above. Make sure you know where to find the mantissa, and how to figure out what the characteristic will be.

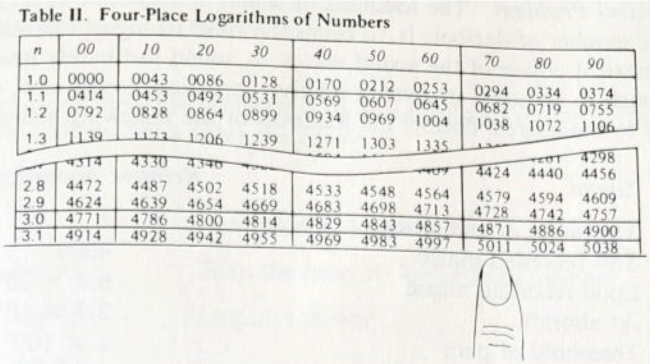

Table II. Four-Place Logarithms of Numbers

n	00	10	20	30	40	50	60	70	80	90
1.0	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
1.1	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
1.2	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
1.3	1139	1173	1206	1239	1271	1303	1335	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4298
[illegible]	4314	4330	4346	[illegible]	[illegible]	[illegible]	[illegible]	4424	4440	4456
2.8	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
2.9	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
3.0	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
3.1	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038

Figure 6-10a

b. Figure out how to use the log table backwards to find that 2.71 is the antilog of .4330.

c. Demonstrate that you understand how to use the log table by using it to approximate (79.2)(37400)(409).

d. The word "logarithm" is a contraction of "*log*ical *arithm*etic." Explain why loarithms *were* a logical way to do multiplication before there were calculators, but are *not* logical for that purpose any more.

e. What seems to be the most important thing that is done with logarithms now?

53. ***Powers Too Large for Calculators Problem*** Most calculators cannot give an answer that has an exponent more than 99. However, you can evaluate such powers by calculator if you find the *logarithm* of the answer. Use this clue to do the following.

a. Evaluate 1776^{53}.

b. Evaluate 2001^{97}.

c. Evaluate 0.007^{105}.

d. If 854^{231} were evaluated exactly, how many digits would the answer have?

e. If 0.2^{1000} were evaluated exactly, how many zeros would there be between the decimal point and the first non-zero digit?

54. ***Earthquake Problem*** The Richter energy number of an earthquake is the base 10 log of the amplitude (i.e., the severity) of the quake vibrations. The Seattle quake of April 29, 1965, measured 7 on the Richter scale. The San Francisco quake of 1906 measured about 8.25 on the Richter scale. To those who know little about logarithms, 8.25 does not sound much more severe than 7. Using what you know about logarithms, tell how many times more severe the San Francisco quake was than the Seattle one.

1. **Biological Half-Life Problem** You accidentally inhale some mildly poisonous fumes. Twenty hours later you see a doctor. From a blood sample, she measures a poison concentration of 0.00372 milligrams per cubic centimeter (mg/cc), and tells you to come back in 8 hours. On the second visit, she measures a concentration of 0.00219 mg/cc. Assume that the concentration of poison in your blood decays exponentially over time.
   a. The doctor says you might have had serious body damage if the poison concentration was ever as high as 0.015 mg/cc. Was the concentration ever that high?
   b. You can resume normal activities when the poison concentration has dropped to 0.00010 mg/cc. How long after you inhaled the fumes will you be able to resume normal activities?
   c. The "biological half life" of the poison is the length of time it takes for the concentration to drop to half of its present value. Find the biological half-life of the poison.

2. **Carbon 14 Dating Problem** Carbon 14 is an isotope of carbon that is formed when radiation from the sun strikes ordinary carbon dioxide in the atmosphere. Plants such as trees, which get their carbon dioxide from the atmosphere, therefore contain small amounts of carbon 14. Once a particular part of a plant has been formed, no more new carbon 14 is taken in. The carbon 14 in that part of the plant decays, slowly, to form nitrogen 14. The half-life of carbon 14 is 5750 years, which means that after 5750 years, half of the original carbon 14 has turned into nitrogen 14.
   a. Many believe that Christ was crucified about 2000 years ago. If somebody claimed to have a piece of wood from the cross on which he was crucified, what percent of the original carbon 14 would you expect to remain in the wood?
   b. The oldest living trees in the world are the bristlecone pines in the White Mountains of California. 4000 growth rings have been counted in the trunk of one of these, meaning that the innermost ring is 4000 years old. What percent of the original carbon 14 would you expect to find in the oldest ring of the tree?
   c. A piece of wood believed to have come from Noah's Ark has 48.37% of the carbon 14 remaining. The Great Flood is supposed to have occurred in 4004 B.C.E. Is this piece of wood old enough to have come from Noah's Ark?
   d. Coal is supposed to have been formed from trees which lived 100 million years ago. What percent of the original carbon 14 would you expect to find remaining in coal? Is carbon 14 dating a good tool to measure the age of coal?

3. **Rabbit Problem** When rabbits were first brought to Australia last century, they had no natural enemies so their numbers increased rapidly. Assume that there were 60,000 rabbits in 1865, and that by 1867 the number had increased to 2,400,000. Assume that the number of rabbits increased exponentially with the number of years that elapsed since 1865. Write an exponential model (function) that accounts for these numbers. According to your model, when was the first pair of rabbits introduced into Australia?

4. **Air Pressure Problem** The pressure of air in Earth's atmosphere decreases exponentially with altitude above the surface of the Earth. The pressure at the Earth's surface (sea level) is about 14.7 pounds per square inch (psi) and the pressure at 2000 feet is approximately 13.5 psi. Human blood at body temperature will boil if the pressure is below 0.9 psi. At what altitude would your blood start to boil if you were in an unpressurized airplane?

5. **Coffee Cup Problem** After you pour a cup of coffee, it cools off in such a way that the *difference* between the coffee temperature and the room temperature decreases exponentially with time. This model is called "Newton's Law of Cooling." Suppose that you pour a cup of coffee. Three minutes later, you measure its temperature and find that it is 85°C. Five minutes after the first reading, you find that it has cooled to 72°C. The room is at 20°C.
   a. What was the temperature of the coffee when you poured it?
   b. McDonald's serves coffee at 82°C. When was your coffee the temperature of McDonald's coffee?
   c. Your grumpy sibling considers coffee "drinkable" if it is at least 55°C. For how long after it was poured is the coffee drinkable, in their opinion?
   d. A study published in 2008 found that the optimal drinking temperature for coffee is 57.8°C. When was your coffee the optimal temperature?