

Bottom

$$\text{E) } \frac{dy}{dx} = -yx - y \quad f(-2) = 1$$

$$dy = (-yx - y) dx$$

$$dy = y(-x - 1) dx$$

$$\int \frac{1}{y} dy = \int (-x - 1) dx$$

$$\ln|y| = -\frac{1}{2}x^2 - x + C$$

$$\ln 1 = -\frac{1}{2}(-2)^2 - (-2) + C \quad 0 = C$$

$$\ln y = -\frac{1}{2}x^2 - x$$

$$y = e^{-\frac{1}{2}x^2 - x}$$

$$y = e^{-\frac{1}{2}x^2 - x} \cdot e^0$$

$$y = e^{-\frac{1}{2}x^2 - x}$$

$t=0$

2011 #5

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dw}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, t is measured in years from the start of 2010.

- a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 ($t = \frac{1}{4}$).

$$\frac{dw}{dt} = \frac{dy}{dx}$$

Point Slope

$$(t, w) \quad \frac{dw}{dt} = \frac{1}{25}(w - 300)$$

$$(0, 1400) \quad \frac{dw}{dt} = \frac{1}{25}(1400 - 300)$$

$$w = 1400 + \frac{1100}{25}(t - 0)$$

$$w\left(\frac{1}{4}\right) = 1400 + \frac{1100}{25}\left(\frac{1}{4} - 0\right)$$

$$w\left(\frac{1}{4}\right) = 1411$$

- b) Find $\frac{d^2w}{dt^2}$ in terms of W . Use $\frac{d^2w}{dt^2}$ to determine whether your answer in part a is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

$$\begin{aligned} \frac{dw}{dt} &= \frac{1}{25}(w - 300) \\ &= \frac{1}{25}w - 12 \end{aligned}$$

$$\frac{d^2w}{dt^2} = \frac{1}{25} \frac{dw}{dt}$$

$$\frac{d^2w}{dt^2} = \frac{1}{25} \left(\frac{dw}{dt} \right) = \frac{1}{25} \left(\frac{1}{25}(w - 300) \right)$$

$$\left. \frac{d^2w}{dt^2} \right|_{w=1400} = \frac{1}{25} \left(\frac{1}{25}(1400 - 300) \right)$$

$$w = 1400$$

UNDERESTIMATE

- c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dw}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

3. $\int_{\pi/2}^{\pi} (\cos^{10} x) (\sin x) dx$ Let $u = \cos x$

$u(\pi) = -1$
 $u(\pi/2) = 0$

$\frac{du}{dx} = -\sin x$
 $\frac{du}{-\sin x} = dx$

$\int_0^{-1} (u^{10}) \sin x dx$

$\int_0^{-1} u^{10} \cancel{\sin x} \frac{du}{-\cancel{\sin x}} = -\int u^{10} du = -\left[\frac{1}{11} u^{11} \right]_0^{-1} = \frac{1}{11}$

4. $\int_0^{\pi/2} \cos x \sqrt{\sin x} dx$ Let $u = \sin x$

$\frac{du}{dx} = \cos x$
 $\frac{du}{\cos x} = dx$

$\int_0^1 \cos x \sqrt{u} dx$

$\int_0^1 \cancel{\cos x} \sqrt{u} \frac{du}{\cancel{\cos x}} = \int_0^1 u^{1/2} du = \left[\frac{2}{3} u^{3/2} \right]_0^1 = \frac{2}{3}$