

Consider the curve defined by the equation  $2y^3 + 6x^2y - 12x^2 + 6y = 1$   
with  $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$

b) Write an equation of each horizontal tangent to the curve

c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x and y-coordinates of P.

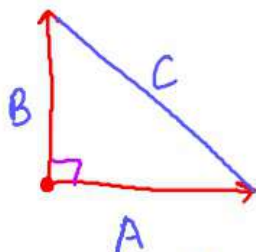
d) Find  $\frac{d^2y}{dx^2}$  in terms of x and y.

$$2(4)(40) + 2(3)(30) = 2(5) \frac{dC}{dt} \rightarrow 500 = 10 \frac{dC}{dt}$$

$$\frac{dC}{dt} = 50 \text{ m/hr}$$

- ① Draw picture  
- only label constants
- ② Label all info
- ③ Find Eq.
- ④ Take derivative

C) Truck A travels east at 40 mi/hr. Truck B travels north at 30 mi/hr. How fast is the distance between the trucks changing 6 minutes later?



$$A^2 + B^2 = C^2$$

$$2A \frac{dA}{dt} + 2B \frac{dB}{dt} = 2C \frac{dC}{dt}$$

$$\frac{dA}{dt} = 40 \text{ mph}$$

$$\frac{dB}{dt} = 30 \text{ mi/hr}$$

Find  $\frac{dC}{dt}$  when  $t = 6 \text{ min}$

$$A = 4 \text{ m} \quad B = 3 \text{ m}$$

$$t = \frac{6}{60} = \frac{1}{10} \text{ hr}$$

$$4^2 + 3^2 = C^2$$

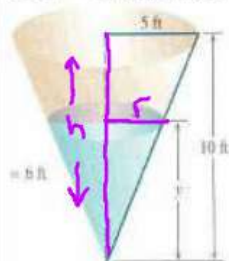
$$16 + 9 = C^2$$

$$25 = C^2$$

$$5 = C$$

Don't use until derivative has been taken.

D) Water runs into a conical tank at the rate of 9 ft<sup>3</sup>/min. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?



$$\frac{dV}{dt} = 9 \text{ ft}^3/\text{min}$$

If full  
 $h = 10 \quad r = 5$

Find  $\frac{dh}{dt}$  when  
 $h = 6$

$$\frac{r}{h} = \frac{5}{10}$$

$$\frac{10r}{10} = \frac{5h}{10}$$

$$r = \frac{1}{2}h$$

$$h = 2r$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{1}{2}h\right)^2 h$$

$$V = \frac{1}{12} \pi h^3$$

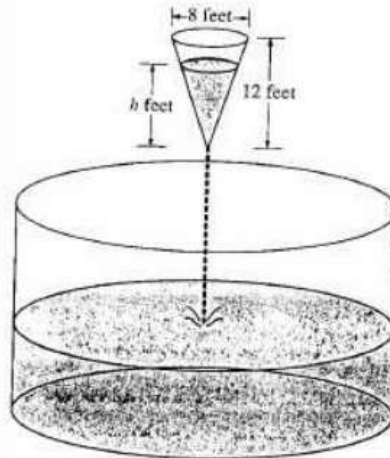
$$\frac{dV}{dt} = \frac{1}{4} \pi h^2 \frac{dh}{dt}$$

$$9 = \frac{1}{4} \pi (6)^2 \frac{dh}{dt}$$

$$9 = 9\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{\pi} \text{ ft/min}$$

21. Water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area  $400\pi$  square feet. The depth,  $h$ , in feet, of the water in the conical tank is changing at the rate of  $(h - 12)$  feet per minute. Volume of a cone:  $V = \frac{1}{3}\pi r^2 h$



- A) Write an expression for the volume of water in the conical tank as a function of  $h$ .
- B) At what rate is the volume of water in the conical tank changing when  $h = 3$ ? Indicate units of measure.
- C) Let  $y$  be the depth, in feet, of the water in the cylindrical tank. At what rate is  $y$  changing when  $h = 3$ ? Indicate units of measure.

What you'll Learn About:  
How to use derivatives to find limits in an indeterminate form

Why L'Hopitals Works

Sketch the graph of two curves with the following characteristic  $f(2) = g(2) = 0$ .

a) Write the tangent line for  $f(x)$

b) Write the tangent line for  $g(x)$

c)  $\lim_{x \rightarrow 2} \frac{f(x)}{g(x)}$

d)  $\lim_{x \rightarrow 0} \frac{2x^2}{x^2}$

2)  $\lim_{x \rightarrow 0} \frac{\sin(5x)}{x}$

$$4) \lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{x - 1}$$

$$49) \lim_{x \rightarrow 1} \frac{x^3 - 1}{4x^3 - x - 3}$$

$$A) \lim_{x \rightarrow \infty} \frac{x^3 - 1}{4x^3 - x - 3}$$

$$27) \lim_{x \rightarrow \infty} \frac{\ln(x^5)}{x}$$

$$35) \lim_{x \rightarrow \infty} \frac{\log_2(x)}{\log_3(x+3)}$$

$$33) \lim_{x \rightarrow 0} \frac{\sin(x^2)}{x}$$