Precalc

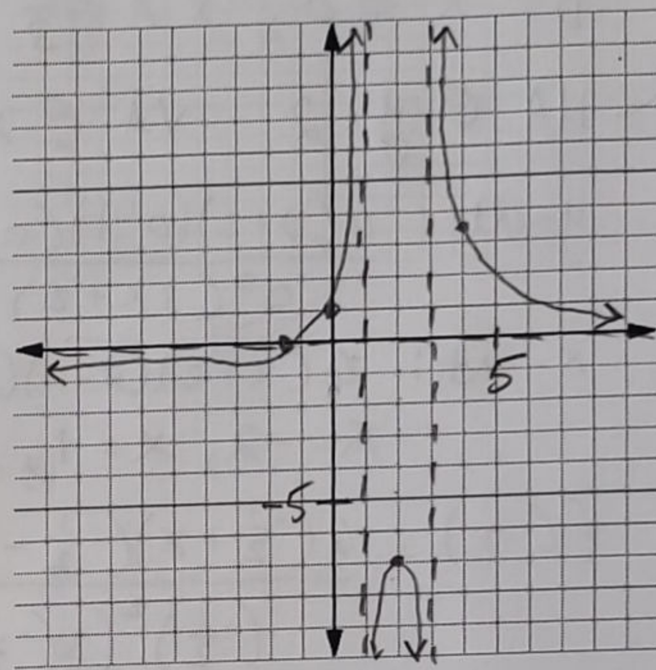
Name: Key

Rational functions review

Find the **domains, vertical asymptotes, horizontal or slant/oblique asymptotes,** and **removable discontinuities/holes** of all the functions, and graph the functions. You can use Desmos on a school-issued Chromebook to help you graph the functions, but you will need to be able to find the domains, asymptotes, and removable discontinuities without a graphing calculator.

1. $f(x) = \dfrac{2x+3}{x^2-4x+3} = \dfrac{2x+3}{(x-3)(x-1)}$

$f(4) = \dfrac{2(4)+3}{(4-3)(4-1)} = \dfrac{11}{3}$



D: $x \neq 3,\ x \neq 1$

R: $y \neq 0$

HA @ $y=0$

VA @ $x=3,\ x=1$

no RDs

x-int: $2x+3=0$
$2x=-3$
$x=-\frac{3}{2}$

y-int: $\dfrac{2(0)+3}{(0-3)(0-1)} = \dfrac{3}{(-3)(-1)} = 1$

$f(2) = \dfrac{2(2)+3}{(2-3)(2-1)} = \dfrac{7}{-1} = -7$

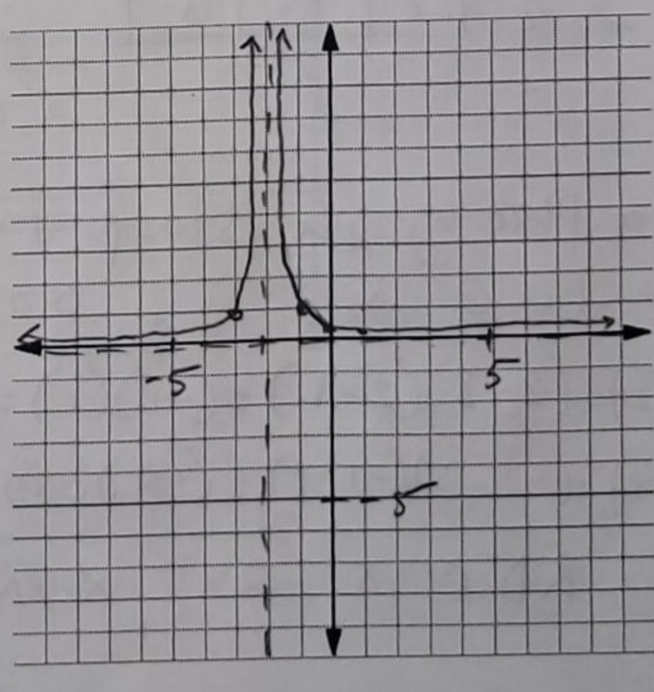
2. $f(x) = \dfrac{x+5}{x^3+9x^2+24x+20}$

	x^2	$4x$	4
x	x^3	$4x^2$	$4x$
5	$5x^2$	$20x$	20

$f(x) = \dfrac{x+5}{(x+5)(x^2+4x+4)}$

$f(x) = \dfrac{1}{(x+2)(x+2)}$

y-int: $\dfrac{1}{(0+2)(0+2)}$
$\frac{1}{4}$



no x-intercepts

D: $x \neq -2,\ x \neq -5$

R: $y \neq 0$

HA @ $y=0$

VA @ $x=-2$

RD @ $x=-5$

$f(-3) = \dfrac{1}{(-3+2)(-3+2)}$
$= 1$

$f(-1) = \dfrac{1}{(-1+2)(-1+2)}$
$= 1$

$f(1) = \dfrac{1}{(1+2)(1+2)} = \dfrac{1}{9}$

3. $f(x) = \dfrac{(2x+4)(3x^2-3)}{x^2(3x+2)} = \dfrac{2(x+2)\,3(x^2-1)}{x^2(3x+2)}$

$$\begin{pmatrix} 2x \cdot 3x^2 = 6x^3 \\ x^2 \cdot 3x = 3x^2 \\ \frac{6}{3} = 2 \end{pmatrix} \qquad = \frac{6(x+2)(x-1)(x+1)}{x^2(3x+2)}$$

D: $x \neq 0,\ x \neq -\frac{2}{3}$ R: $y \neq 2$

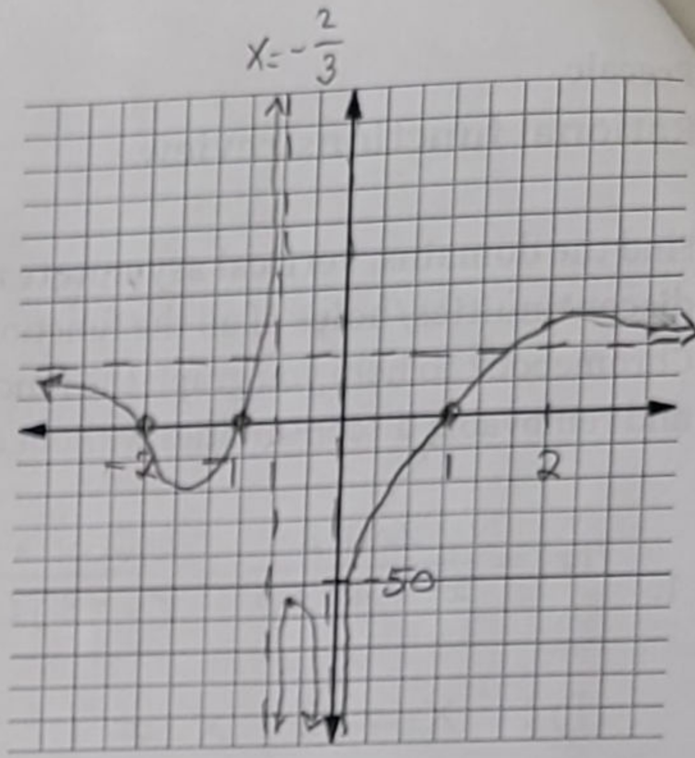
HA @ $y = 2$ VA @ $x = 0,\ x = -\frac{2}{3}$

y-int: $\dfrac{6(0+2)(0-1)(0+1)}{0^2(3x+2)}$ = undefined

x-int: $6(x+2)(x-1)(x+1) = 0$

$x = -2,\ x = 1,\ x = -1$

$f(-\frac{1}{2}) = \dfrac{6(-\frac{1}{2}+2)(-\frac{1}{2}-1)(-\frac{1}{2}+1)}{(-\frac{1}{2})^2(3(-\frac{1}{2})+2)} = -54$



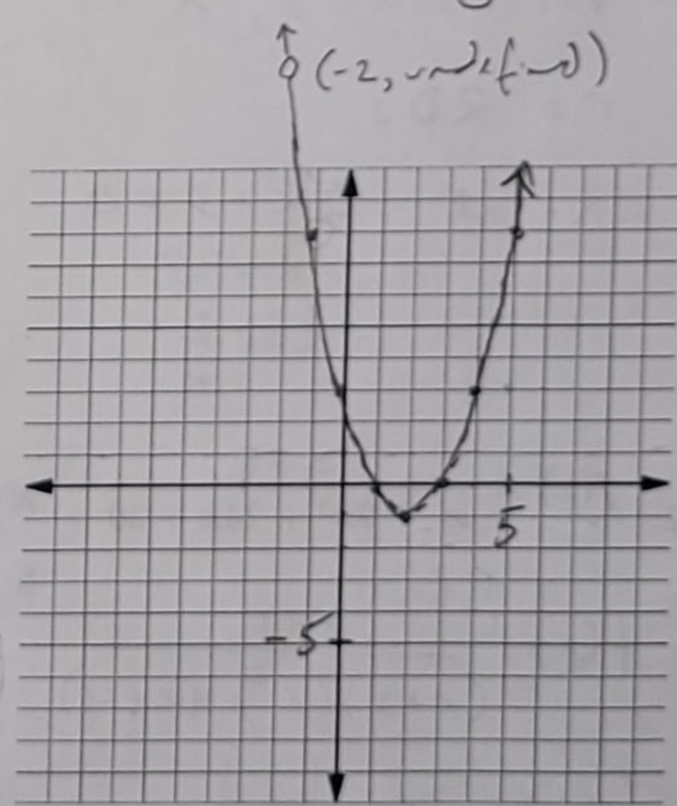
you really need Desmos for this one - so weird, right?!

4. $f(x) = \dfrac{x^3 - 2x^2 - 5x + 6}{x+2} = \dfrac{(x+2)(x^2-4x+3)}{x+2}$

	x^2	$-4x$	3
x	x^3	$-4x^2$	$3x$
2	$2x^2$	$-8x$	6

$= \dfrac{(x+2)(x-3)(x-1)}{(x+2)}$

$= (x-3)(x-1)$

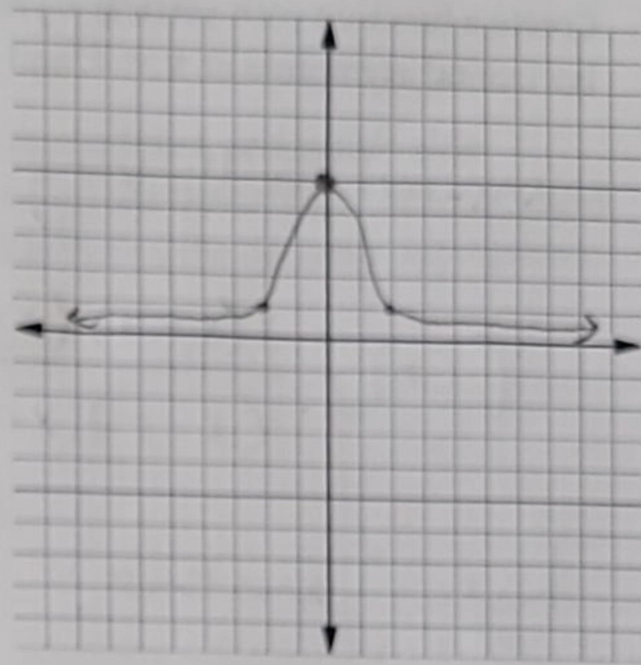


so the graph is a parabola with a hole @ $x = -2$, so RD @ $x = -2$

$f(2) = (2-3)(2-1) = (-1)(1) = -1$ so vertex @ $(2,-1)$
$f(-1) = (-1-3)(-1-1) = (-4)(-2) = 8$ so point @ $(-1, 8)$ } to help you graph the parabola

RD @ $x = -2$ x-ints @ $x = 3,\ x = 1$ no asymptotes!

5. $f(x) = \dfrac{5}{x^2+1}$ not factorable



zeros of denominator: $x^2+1=0$

$x^2=-1$

$x=\pm i$

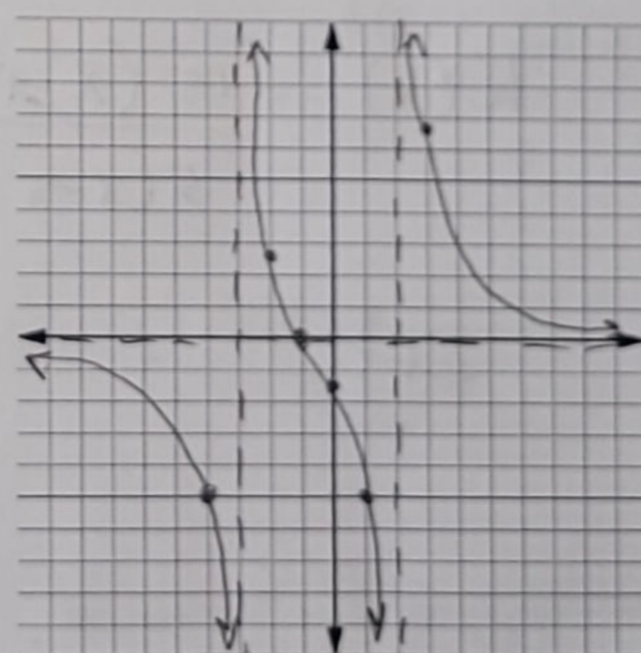
no VAs or RDs, HA @ $y=0$

no x-int, y-int: $\dfrac{5}{0^2+1} = \dfrac{5}{1} = 5$

D: $\mathbb{R}$ R: $(0,5]$

$f(2) = \dfrac{5}{2^2+1} = \dfrac{5}{5} = 1$ $f(-2) = \dfrac{5}{(-2)^2+1} = \dfrac{5}{5} = 1$

6. $f(x) = \dfrac{7x+5}{x^2+2x-3} + \dfrac{3x}{x^2-3x+2}$



$f(x) = \dfrac{7x+5}{(x+3)(x-1)} + \dfrac{3x}{(x-2)(x-1)}$

$f(x) = \dfrac{(7x+5)(x-2)}{(x+3)(x-1)(x-2)} + \dfrac{3x(x+3)}{(x+3)(x-1)(x-2)}$

$f(x) = \dfrac{7x^2-14x+5x-10+3x^2+9x}{(x+3)(x-1)(x-2)}$

$f(x) = \dfrac{10x^2-10}{(x+3)(x-1)(x-2)} = \dfrac{10(x^2-1)}{(x+3)(x-1)(x-2)} = \dfrac{10(x-1)(x+1)}{(x+3)(x-1)(x-2)} = \dfrac{10(x+1)}{(x+3)(x-2)}$

whew!

RD @ $x=1$, VAs @ $x=-3, x=2$, HA @ $y=0$

D: $x \neq 1, 2, -3$ R: $y \neq 0$

$f(-4) = \dfrac{10(-4+1)}{(-4+3)(-4-2)} = -5$

$f(3) = \dfrac{10(3+1)}{(3+3)(3-2)} = \dfrac{40}{6} = \dfrac{20}{3}$

x-int: $10(x+1)=0$

$x+1=0$

$x=-1$

y-int: $\dfrac{10(0+1)}{(0+3)(0-2)} = \dfrac{10}{-6} = -\dfrac{5}{3}$

$f(1) = \dfrac{10(1+1)}{(1+3)(1-2)} = \dfrac{20}{-4} = -5$

$f(-2) = \dfrac{10(-2+1)}{(-2+3)(-2-2)} = \dfrac{-10}{-4} = \dfrac{5}{2}$

7. Create a function with vertical asymptotes at $x = 5$ and $x = -4$, a removable discontinuity at $x = -2$, a horizontal asymptote at $y = 3$, and x-intercepts 1 and -6.

answers will vary, but here's an example:

$$f(x) = \frac{3(x-1)(x+6)(x+2)}{(x-5)(x+4)(x+2)}$$

here's another:

$$g(x) = \frac{9(x-1)(x+6)(x+2)}{3(x-5)(x+4)(x+2)}$$

here's another:

$$h(x) = \frac{15(x-1)(x-6)(x+2)}{5(x-5)(x+4)(x+2)}$$

etc.