5.2 I can find domains, vertical asymptotes, and removable discontinuities of rational functions. (Geometry)

- Every root of the denominator represents an x-value that is excluded from the domain. Find those roots by factoring the denominator. Write the domain simply as: $x \neq \#$, $x \neq \#$, ...
- If a factor of the denominator is also a factor of the numerator, the factors will cancel. There will be a **removable discontinuity** or **hole** in the graph at the x-value that makes the canceled root zero. Graph them like this:

- If the numerator is *not* divisible by a factor of the denominator, there is a **vertical asymptote** at $x = \#$, where the # the x-value that makes the factor zero.

Example: $f(x) = \dfrac{x^2 + 8x + 15}{x^2 + 9x + 18}$

factor the numerator and denominator:

	x	5
x	x^2	$5x$
3	$3x$	15

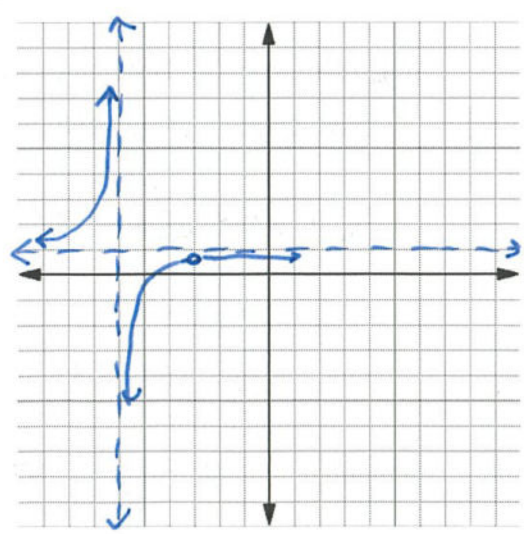
	x	6
x	x^2	$6x$
3	$3x$	18

$$\frac{(x+5)(x+3)}{(x+3)(x+6)}$$

cancel factors that match:

$$\frac{(x+5)\cancel{(x+3)}}{\cancel{(x+3)}(x+6)}$$

graph:



domain: $x \neq -3$, $x \neq -6$

removable discontinuity at $x = -3$

vertical asymptote at $x = -6$

Example: $f(x) = \dfrac{x^3 - 5x^2 - 12x + 36}{x^2 - 2x - 15}$

factor the denominator:

	x	-5
x	x^2	$-5x$
3	$3x$	-15

$(x-5)(x+3)$

divide to find out whether numerator is divisible by one of the factors:

	x^2	-12	rem
x	x^3	$-12x$	-24
-5	$-5x^2$	60	

not divisible

	x^2	$-8x$	12	rem
x	x^3	$-8x^2$	$12x$	0
3	$3x^2$	$-24x$	36	

divisible

divide to find out whether the numerator is divisible by the other factor:

$$\frac{x^3-5x^2-12x+36}{x^2-2x-15} = \frac{(x^2-8x+12)\cancel{(x+3)}}{(x-5)\cancel{(x+3)}}$$

divide after simplifying to find oblique asymptote:

	x	-3	
x	x^2	$-3x$	-3
-5	$-5x$	15	

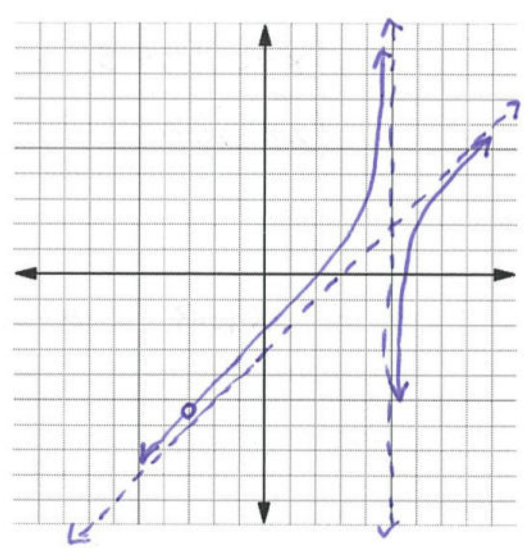
asymptote at $y = x-3$

domain: $x \neq 5$, $x \neq -3$

removable discontinuity at $x = -3$

vertical asymptote at $x = 5$

graph:



5.3 I can find horizontal and oblique asymptotes of rational functions. (Geometry)

- Simplify your fraction first, to find removable discontinuities.
- If the degree of the numerator is less than the degree of the denominator, there is a **horizontal asymptote** at $y = 0$.
- If the degree of the numerator is equal to the degree of the denominator, there may be a **horizontal asymptote** at $y = \#$. Get the # by dividing the leading coefficients of the numerator and denominator.
- If the degree of the numerator is greater than the degree of the denominator, there may be an **oblique asymptote**, a.k.a. **slant asymptote**. Find it by dividing the polynomials. The oblique/slant asymptote is $y =$ the quotient (ignoring the remainder.)

Example: $$f(x) = \frac{2x^2+3}{(2x+5)(2x-5)}$$

degree of numerator: 2

degree of denominator: 2

put the numerator and denominator in standard form:

	$2x$	5
$2x$	$4x^2$	$10x$
-5	$-10x$	-25

$$\frac{2x^2+3}{4x^2-25}$$

vertical asymptotes:

$2x+5=0$
$2x=-5$
$x=-\frac{5}{2}$

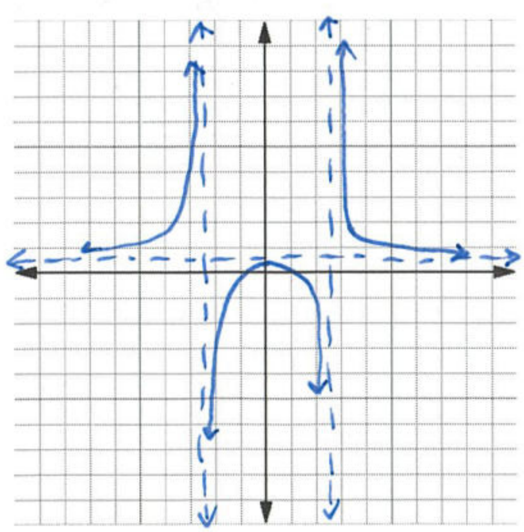
$2x-5=0$
$2x=5$
$x=\frac{5}{2}$

divide coefficients

$\frac{2}{4} = \frac{1}{2}$

horizontal asymptote at $y = \frac{1}{2}$

graph:



Example: $f(x) = \dfrac{x^3 - 10x^2 + 17x + 28}{x^2 - 9x + 14} = \dfrac{x^3 - 10x^2 + 17x + 28}{(x-2)(x-7)}$

Can also use the remainder theorem!

is the numerator divisible by $(x-2)$? if so, then 2 is a root of the numerator. plug in 2:

$2^3 - 10 \cdot 2^2 + 17 \cdot 2 + 28 = 8 - 40 + 34 + 28 = 30$

not zero! so 2 is not a root.

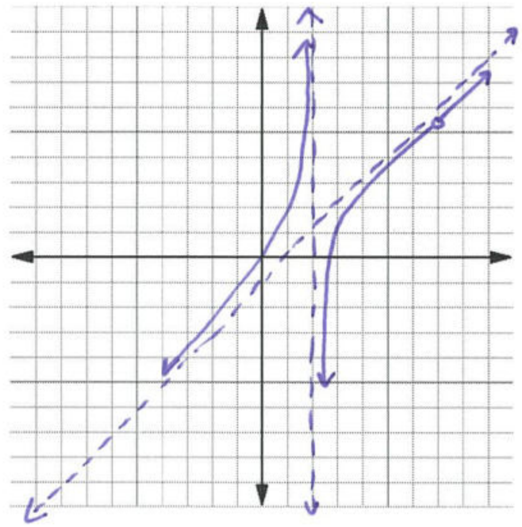
	x^2	$-3x$	-4	rem
x	x^3	$-3x^2$	$-4x$	0
-7	$-7x^2$	$21x$	28	divisible!

$$\frac{\cancel{(x-7)}(x^2-3x-4)}{(x-2)\cancel{(x-7)}}$$

	x	-1	rem
x	x^2	$-x$	-6
-2	$-2x$	2	

oblique asymptote at $y = x - 1$

graph:



Example: $f(x) = \dfrac{x}{x^2 - x - 6} = \dfrac{x}{(x-3)(x+2)}$

$$= \frac{0x^2 + 1x + 0}{1x^2 - 1x - 6}$$

$$\frac{0}{1} = 0$$

horizontal asymptote at $y = 0$

graph: