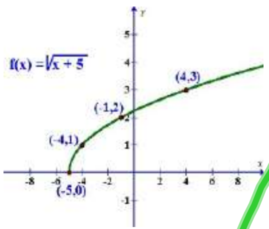
Radicals Notes

Radicals

$y = a\sqrt{x-h} + k$

$(h,k) = vertex$



$Domain: x \mid x \geq h$

$Range: y \mid y \geq k$

Graph

Solve

$7\sqrt{x+4} + 2 = 16$

$7\sqrt{x+4} = 14$

$\sqrt{x+4} = 2$

$x + 4 = 4$

$x = 0$

$$\sqrt[5]{x^3} = x^{\frac{3}{5}}$$

Verify Inverse Functions

$f \circ g = x$

$g \circ f = x$

Find inverse

* Switch x and y
* Solve for y

Notes Notes

Rational Exponents

Rational Notation	Exponential Notation
$\sqrt[Root]{Radicand}$ Radical	$Base^{(\text{Exponent or Power or Root})}$

Radical Notation to Exponential Notation

$$\sqrt{5} = 5^{\frac{1}{2}}$$

$$\sqrt[3]{343} = 343^{\frac{1}{3}} = \left(7^3\right)^{\frac{1}{3}} = 7$$

or

$$\sqrt[3]{343} = \sqrt[3]{7^3} = \left(7^3\right)^{\frac{1}{3}} = 7$$

$$\sqrt[4]{81} = 81^{\frac{1}{4}} = \left(3^4\right)^{\frac{1}{4}} = 3^{\frac{4}{4}} = 3$$

$$\sqrt[4]{5x^6y^3z} = 5^{\frac{1}{4}} x^{\frac{6}{4}} y^{\frac{3}{4}} z^{\frac{1}{4}} = 5^{\frac{1}{4}} x^{\frac{3}{2}} y^{\frac{3}{4}} z^{\frac{1}{4}}$$

Exponential Notation to Radical Notation

$$64^{\frac{2}{5}} = \left(2^6\right)^{\frac{2}{5}} = 2^{\frac{12}{5}} = \sqrt[5]{2^{12}} \; or \left(\sqrt[5]{2}\right)^{12}$$

$$6^{\frac{2}{3}} = \sqrt[3]{6^2} \; or \left(\sqrt[3]{6}\right)^2$$

$$2^{\frac{2}{3}} x^{\frac{1}{3}} y = 2^{\frac{2}{3}} x^{\frac{1}{3}} y^{\frac{3}{3}} = \sqrt[3]{2^2 \, xy^3}$$

$$3125^{\frac{2}{5}} = \left(5^5\right)^{\frac{2}{5}} = 5^{\frac{10}{5}} = 5^2 = 25$$

Notes Notes

Properties of Rational Exponents

- Fractional (Rational) Exponents can be written in radical form, where the numerator represents the power and the denominator represents the root.

$$a^{m/n} = \left(a^{1/n}\right)^m = \left(\sqrt[n]{a}\right)^m \text{ or } a^{m/n} = \left(a^m\right)^{1/n} = \sqrt[n]{a^m}$$

$$9^{3/2} = \left(9^{1/2}\right)^3 = \left(\sqrt{9}\right)^3 = (3)^3 = 27 \text{ or } 9^{3/2} = \left(9^3\right)^{1/2} = \sqrt{9^3} = \sqrt{729} = 27$$

- Negative exponents: Take the reciprocal then write the exponent in radical form

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{\left(a^{1/n}\right)^m} = \frac{1}{\left(\sqrt[n]{a}\right)^m}, a \neq 0 \text{ or } a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{\left(a^m\right)^{1/n}} = \frac{1}{\sqrt[n]{a^m}}$$

$$32^{-2/5} = \frac{1}{32^{2/5}} = \frac{1}{\left(32^{1/5}\right)^2} = \frac{1}{\left(\left(2^5\right)^{1/5}\right)^2} = \frac{1}{2^2} = \frac{1}{4} \text{ or}$$

$$32^{-2/5} = \frac{1}{32^{2/5}} = \frac{1}{\left(32^2\right)^{1/5}} = \frac{1}{\sqrt[5]{32^2}} = \frac{1}{\sqrt[5]{1024}} = \frac{1}{4}$$

- When adding rational exponents, there must be a common denominator.

$$a^m a^n = a^{m+n} \quad 5^{1/2} \bullet 5^{1/4} = 5^{1/2+1/4} = 5^{2/4+1/4} = 5^{3/4}$$

- When multiplying rational exponents, multiply straight across.

$$\left(a^m\right)^n = a^{mn} \quad \left(8^{1/2}\right)^6 = 8^{1/2 \cdot 6} = 8^{1/2 \cdot 6/1} = 8^{6/2} = 8^3$$

- When subtracting rational exponents, there must be a common denominator.

$$\frac{a^m}{a^n} = a^{m-n}, a \neq 0 \quad \frac{7^{1/2}}{7^{1/3}} = 7^{1/2-1/3} = 7^{3/6-2/6} = 7^{1/6}$$

Extra Example

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0 \quad \left(\frac{8}{27}\right)^{1/3} = \frac{8^{1/3}}{27^{1/3}} = \frac{\left(2^3\right)^{1/3}}{\left(3^3\right)^{1/3}} = \frac{2}{3}$$

Power Functions and Function Operations

Addition of Functions – Combine like terms

$h(x)=f(x)+g(x)$

$f(x)=4x,\ g(x)=x-1$

$h(x)=f(x)+g(x)=(4x)+(x-1)=5x-1$

Subtraction of Functions – Combine like terms

$h(x)=f(x)-g(x)$

$f(x)=4x,\ g(x)=x-1$

$h(x)=f(x)-g(x)=(4x)-(x-1)=3x+1$

Multiplication of Functions – Multiply (FOIL, Distribute, etc.), then combine like terms

$h(x)=f(x)\cdot g(x)$

$f(x)=4x,\ g(x)=x-1$

$h(x)=f(x)\cdot g(x)=(4x)(x-1)=4x^2-4x$

Division of Functions – Reduce if possible, then divide using long division or synthetic division.

$h(x)=f(x)\div g(x)$

$f(x)=4x,\ g(x)=x-1$

$$h(x)=\frac{f(x)}{g(x)}=\frac{(4x)}{(x-1)}=4+\frac{4}{x-1}$$

Composition of Functions – Substitute one function into every x-value of the other function and simplify by combining like terms.

$h(x)=f(g(x))$

$f(x)=4x,\ g(x)=x-1$

$h(x)=f(g(x))=4(x)=4(x-1)=4x-4$

$h(x)=g(f(x))$

$f(x)=4x,\ g(x)=x-1$

$h(x)=g(f(x))=(x)-1=(4x)-1=4x-1$

Notes Notes

Inverse Functions

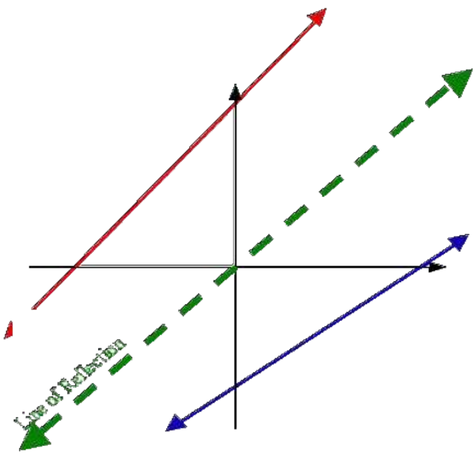
Algorithm: (Graphically)

1. Graph the given function.
2. Label the coordinates of the given function.
3. Switch the x- and y- values for each coordinate.
4. Graph the new coordinates.
5. Draw the new line.
6. Write the equation of the new line.
7. Verify that the two lines are reflected across the line y = mx + b.

$$f(x) = \frac{4}{5}x - 8$$

$$(-5,-12) \longrightarrow (-12,-5)$$
$$(0,-8) \longrightarrow (-8,0)$$
$$(5,-4) \longrightarrow (-4,5)$$
$$(10,0) \longrightarrow (0,10)$$

$$g(x) = \frac{5}{4}x + 10$$



Notes Notes

Inverse Functions

Algorithm: (Algebraically)

1. Switch the x and y variables in the function.
2. Solve the resulting equation for "y=".
3. Write the equation of the new line.
4. Verify that the composites of the two equations are x.

$$f(x) = \frac{4}{5}x - 8$$

$$y = \frac{4}{5}x - 8$$

$$x = \frac{4}{5}y - 8$$

$$x + 8 = \frac{4}{5}y$$

$$5x + 40 = 4y$$

$$\frac{5}{4}x + 10 = y$$

$$g(x) = \frac{5}{4}x + 10$$

Verify

$$f(g(x)) = \frac{4}{5}(x) - 8 = \frac{4}{5}\left(\frac{5}{4}x + 10\right) - 8$$

$$= x + 8 - 8 = x$$

$$g(f(x)) = \frac{5}{4}(x) + 10 = \frac{5}{4}\left(\frac{4}{5}x - 8\right) + 10$$

$$= x - 10 + 10 = x$$

Notes Notes

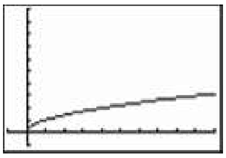
Graphing Radicals

Graphing by hand…

Algorithm:

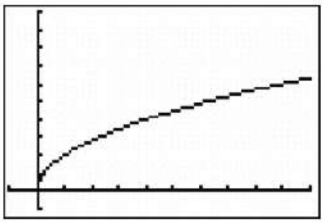
1. Have an idea of what the graph will look like.
2. Make a mini t-chart
3. Plot two or three points
4. Draw a smooth curve

$$f(x) = a\sqrt{x+b} + c$$

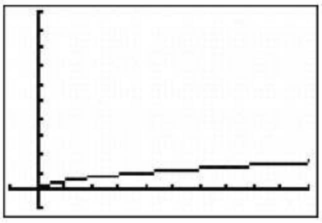


- By changing a the graph of $f(x) = \sqrt{x}$ increases or decreases or goes down

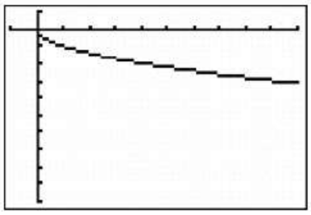
$a = 2$



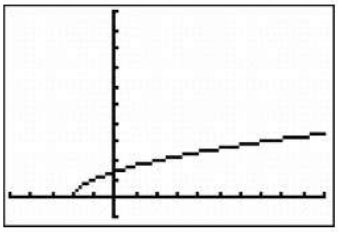
$a = .5$

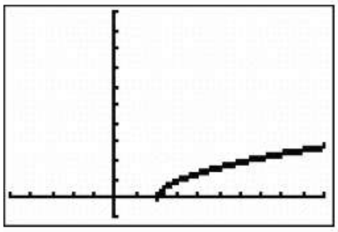


$a = -1$



- By changing b the graph of $f(x) = \sqrt{x}$ is shifted left ($+b$) or right (-b)





- By changing c the graph of $f(x) = \sqrt{x}$ is shifted up ($+c$) and down (-c)