Parametric and Polar Curves; Conic Sections
"Polar Coordinates"

Section 10.2

All graphics are attributed to:

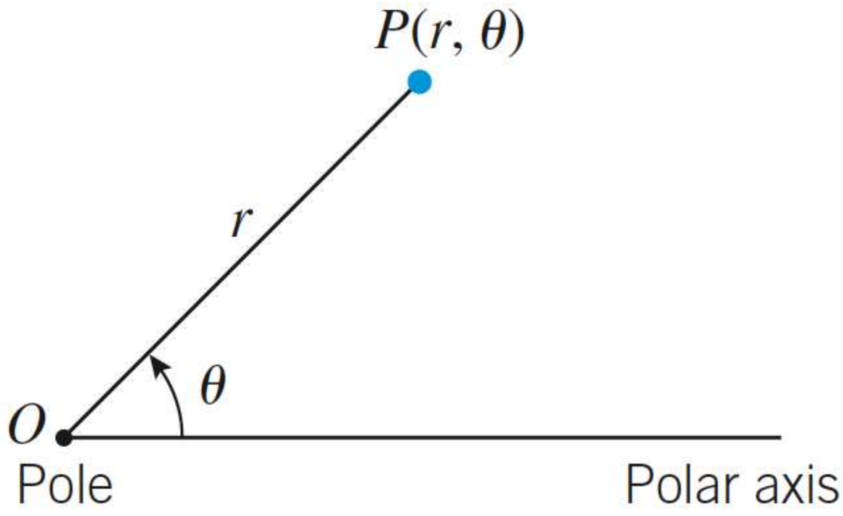
- *Calculus,10/E* by Howard Anton, Irl Bivens, and Stephen Davis

Introduction

- Sometimes, a point has an “affinity” for a fixed point, such as a planet moving in an orbit under the central attraction of the Sun.

- In such cases, the path is best described by its angular direction and distance from the fixed point.

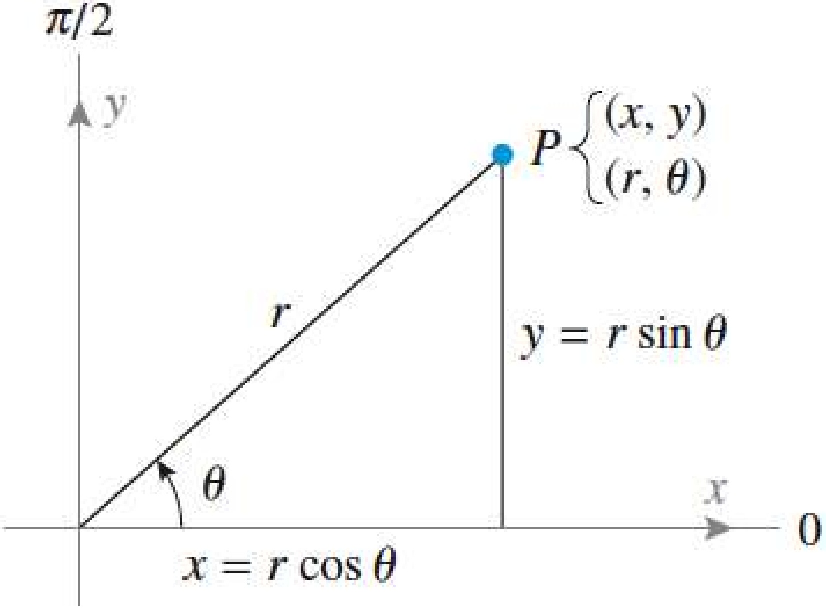
Polar Coordinate Systems



- Origin = (0,0) = the pole
- Ray emanating from the pole = Polar axis
- P(r,) = pair of polar coordinates where:
  - r = radial coordinate
  - = angular coordinate
- Remember, to sweep out clockwise requires - .

Relationship Between Polar **P(r,)** and Rectangular (x,y) Coordinates

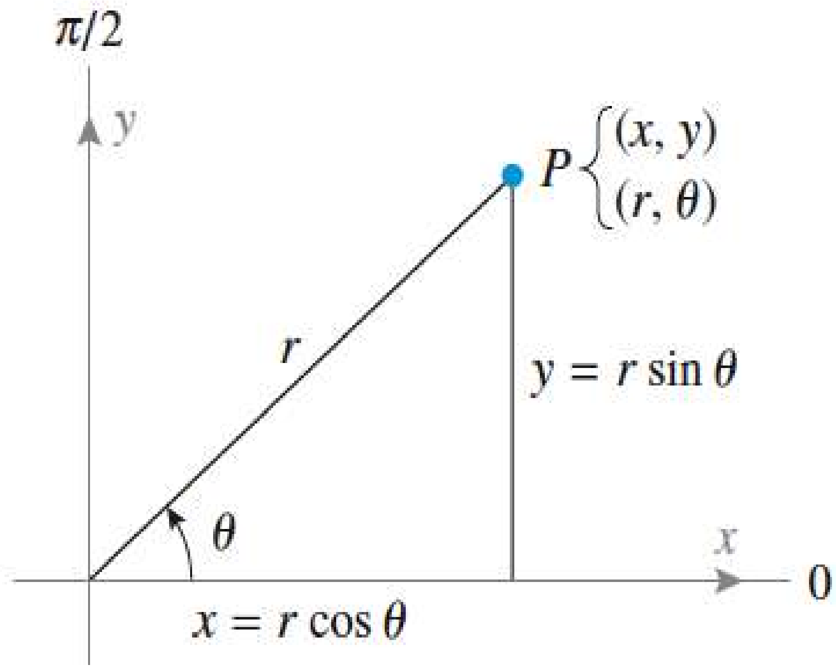
- Sometimes, we may need to switch from one form to the other.
- This can be done by "superimposing" a rectangular (x,y) coordinate plane on top of a polar coordinate plane.



Converting Between Polar P(r,) and Rectangular (x,y) Coordinates

- Polar to Rectangular conversion uses the idea of the unit circle where x is the adjacent leg (cos) and y is the opposite leg (sin). When the radius is not r = 1, then

$$x = r\cos\theta, \quad y = r\sin\theta$$



- Rectangular to Polar conversion uses the Pythagorean Theorem and the fact that tan is the ratio of the opposite leg (y) over the adjacent leg (x).

$$r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x}$$

Graphs in Polar Coordinates

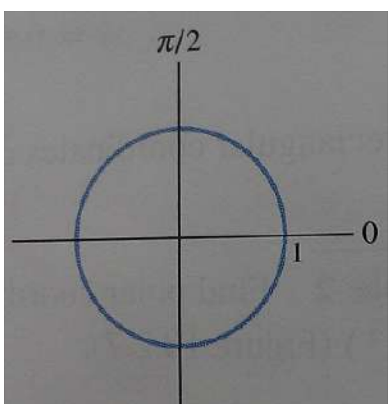
- Given an equation in r and , graph in polar coordinates = all of the points with at least one pair of (r,) that satisfy the equation.

- Some easier examples are:
  - r = constant radius
  - = constant angle

Examples of Graphs in Polar Coordinates

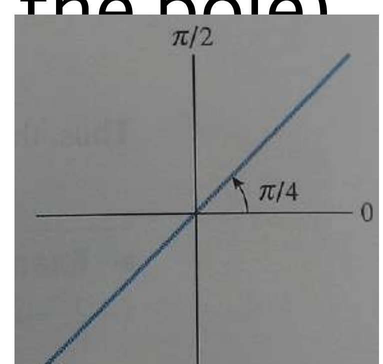
- r = constant radius
  example: r = 1

(means every point that is one away from the pole)



- = constant angle
  example: $= \frac{\ }{4}$

(means every point that has an angular direction of $\frac{\ }{4}$ from the pole)



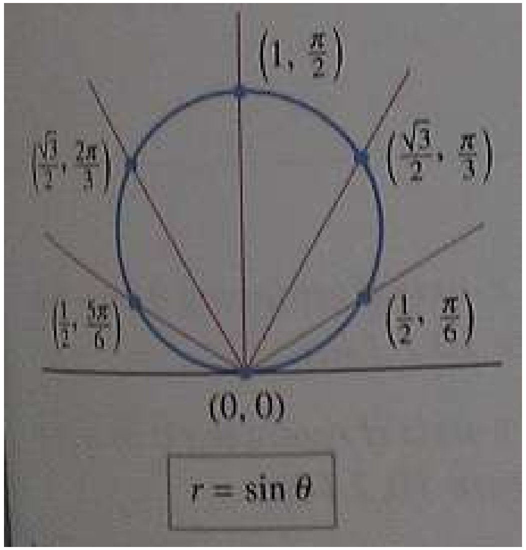
Sketch the graph of r = sin in polar coordinates.

- Solution: We can either do this by using substitution or by plotting points. I find substitution to be more efficient.
- 1. Substitution
  - Given r = sin — multiply both sides by r
    $r^2 = x^2 + y^2$
  - ^{2} = r sin — substi $y = r \sin \theta$
  - $^2 + \ ^2 = y$ — and
  - $^2 + \ ^2 - y = 0$ — subtract y from both sides
  - $^2 + \ ^2 - y + \frac{1}{4} = 0 + \frac{1}{4}$ — complete the square w/$(\frac{\ }{2})^2$
  - $^2 + (\ - \frac{1}{2})^2 = \frac{1}{4}$ — factor

Sketch the graph of r = sin in polar coordinates by plotting points.

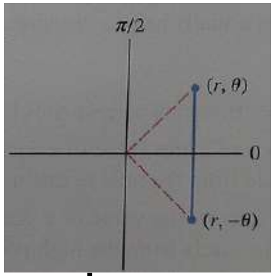
θ (RADIANS)	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$r = \sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0
(r, θ)	$(0, 0)$	$\left(\frac{1}{2}, \frac{\pi}{6}\right)$	$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{3}\right)$	$\left(1, \frac{\pi}{2}\right)$	$\left(\frac{\sqrt{3}}{2}, \frac{2\pi}{3}\right)$	$\left(\frac{1}{2}, \frac{5\pi}{6}\right)$	$(0, \pi)$	$\left(-\frac{1}{2}, \frac{7\pi}{6}\right)$	$\left(-\frac{\sqrt{3}}{2}, \frac{4\pi}{3}\right)$	$\left(-1, \frac{3\pi}{2}\right)$	$\left(-\frac{\sqrt{3}}{2}, \frac{5\pi}{3}\right)$	$\left(-\frac{1}{2}, \frac{11\pi}{6}\right)$	$(0, 2\pi)$

- When you plot these points, they form a circle like we found on the previous slide through substitution.

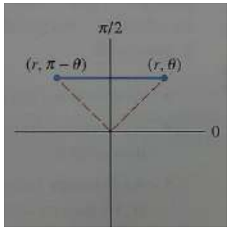


Symmetry Tests

- Testing for symmetry in polar form is similar to testing for symmetry in rectangular form, using substitution.
  - Polar (x) axis symmetry is similar to x-axis symmetry

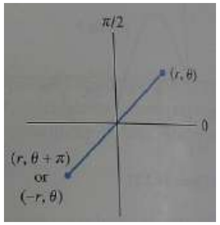


    - If f( ) = (−), then the curve is symmetric about the polar axis.



  - Y-axis symmetry: f( ) = f(-)
  - Pole (origin) symmetry: f( ) = f(+)

or (r,) = (-r,)



Formal Symmetry Tests Theorem

10.2.1 THEOREM (*Symmetry Tests*)

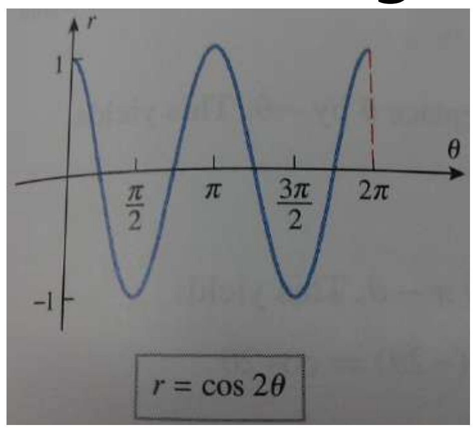
(*a*) *A curve in polar coordinates is symmetric about the x-axis if replacing θ by $-\theta$ in its equation produces an equivalent equation* (Figure 10.2.14*a*).

(*b*) *A curve in polar coordinates is symmetric about the y-axis if replacing θ by $\pi - \theta$ in its equation produces an equivalent equation* (Figure 10.2.14*b*).

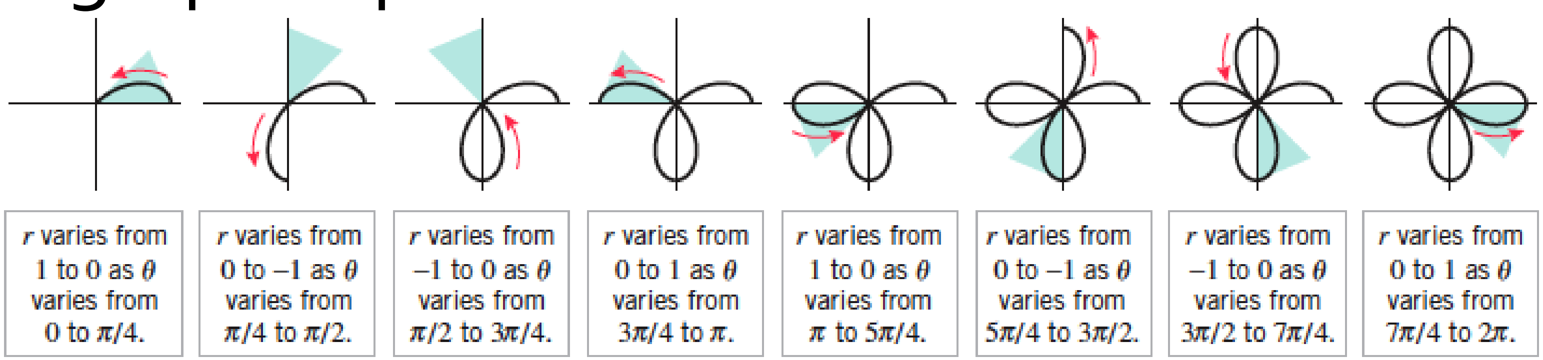
(*c*) *A curve in polar coordinates is symmetric about the origin if replacing θ by $\theta + \pi$, or replacing r by $-r$ in its equation produces an equivalent equation* (Figure 10.2.14*c*).

Sketch the graph of r = cos 2 in polar coordinates.

- First, graph in rectangular coordinates.



- Note how r varies as varies to sketch the graph in polar coordinates.

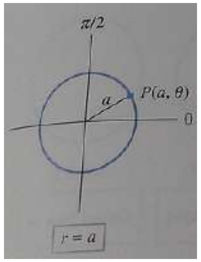


Examples 8 & 9

- Read examples 8 & 9 on pg 711-712 and we will go through it in class tomorrow. Do I need to have a quiz to make sure you did it?
- Steps:
  - 1. Check for symmetry (saves so many steps)
  - 2. Rewrite equation in rectangular form
  - 3. Graph the equation in rectangular coordinates
  - 4. Use #3 to produce polar curve
  - 5. Use symmetry in #1 to reflect appropriately

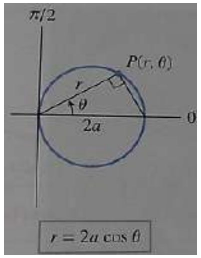
Families of Circles

- If a is a positive constant and the equation is in the following forms, then the graph is a circle.
  - r = a is the basic example from slide #8



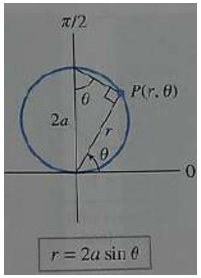
  - r = 2a cos   has polar axis symmetry

    since 2a cos   = 2a cos $(-\quad)$

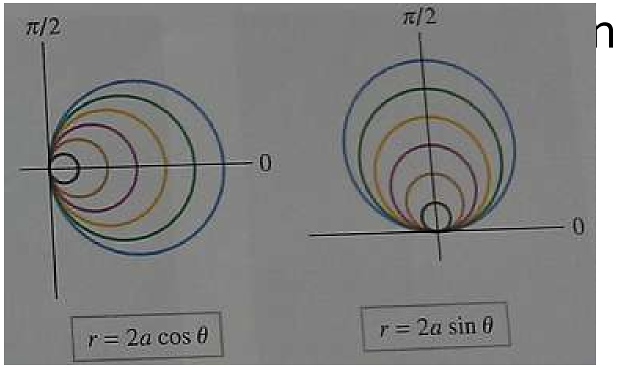


  - r = 2a sin   has y-axis symmetry

    n $(\pi - \quad)$



- Families



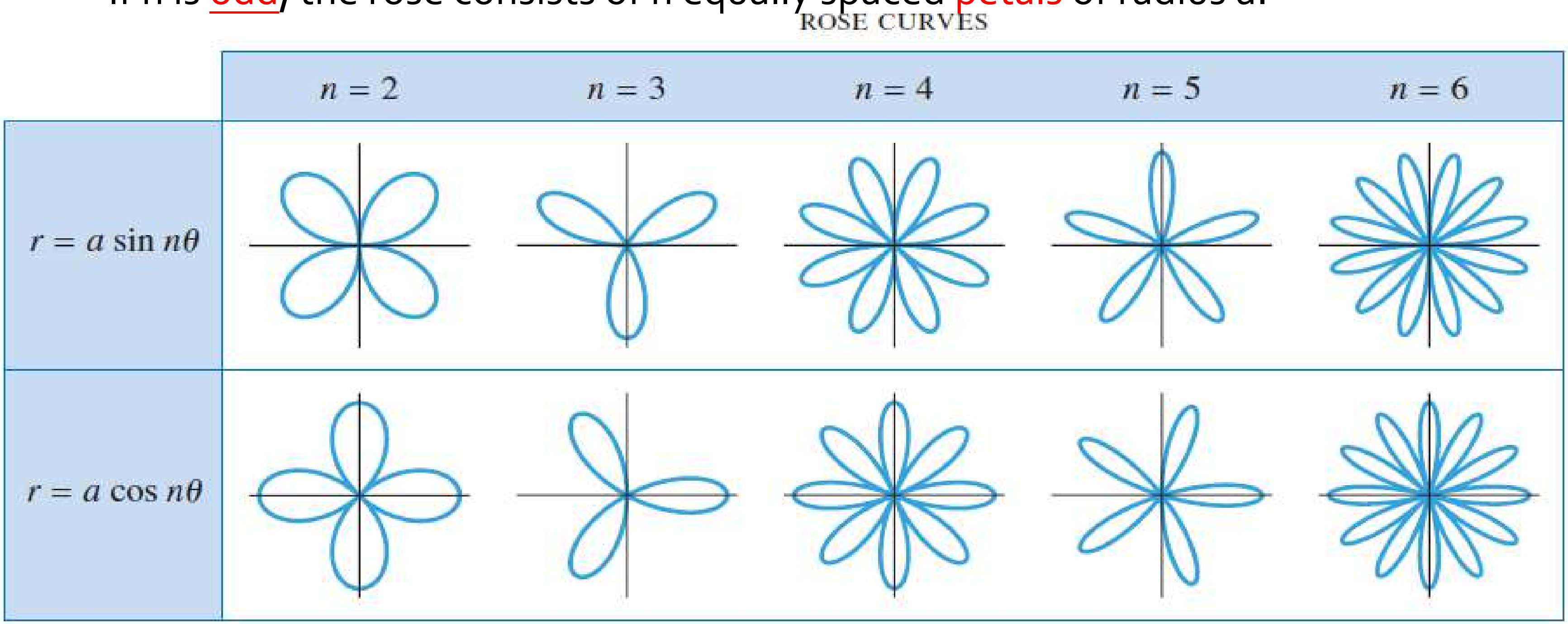
NOTE:
a = radius

Families of Rose Curves

- If a >o and the equation is in the following forms, then the graph is a rose curve.
  - r = a cos n   has polar axis symmetry
  - r = a sin n   has y-axis symmetry

  same reason as circles

- <u>Number of petals</u>
  - If n is <u>odd</u>, the rose consists of n equally spaced petals of radius a.



Families of Cardiods and Limacons

- Equations with any of the four forms listed below with a>o and b>o represent polar curves called limacons.
  - r = a + bcos and r = a - bcos have polar axis symmetry.
  - r = a + bsin and r = a - bsin have y-axis symmetry.
- Limacons have four possible shapes determined by the
  l

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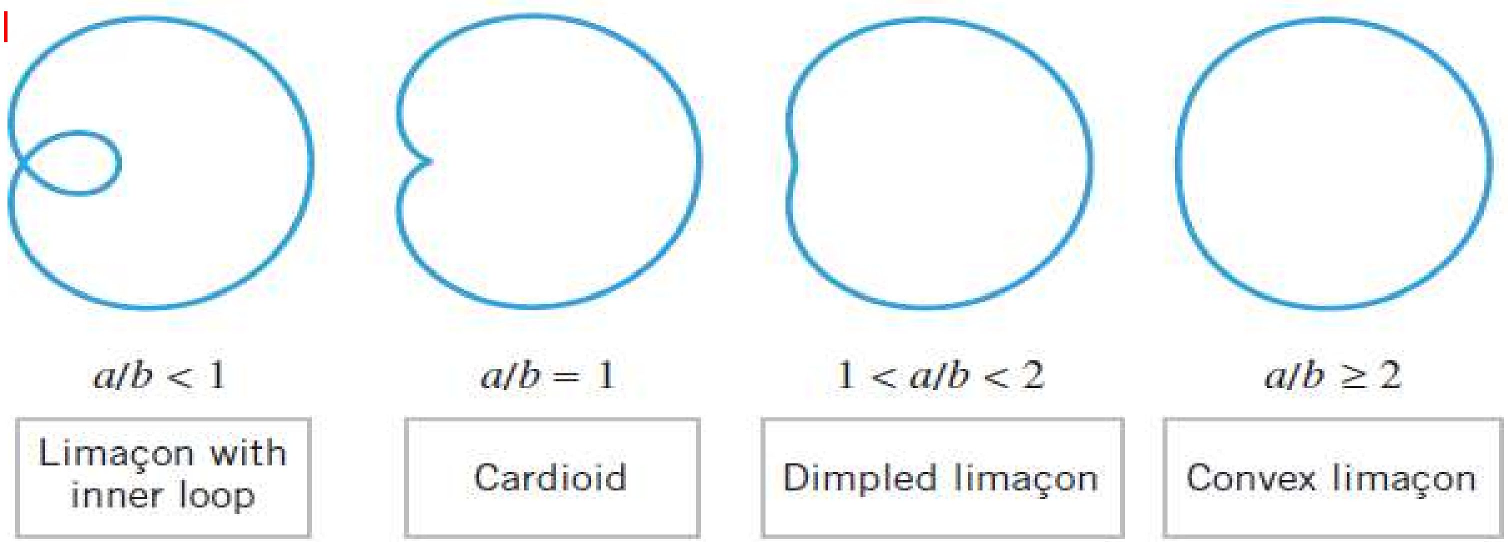
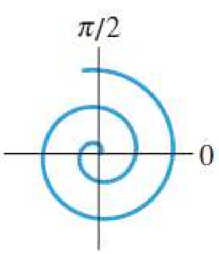
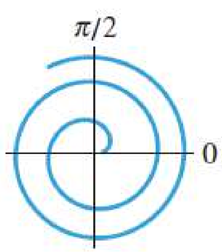


Families of Spirals

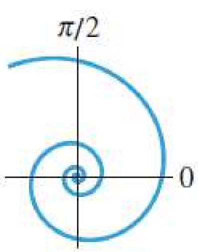
- A spiral is a curve that coils around a central point.
- Spirals generally have "left-hand" and "right-hand" versions that coil in opposite directions depending on the restrictions on the polar angle and the signs of constants that appear in the equations.
- Below are some of the more common types of spirals, but you will not be tested on spirals.



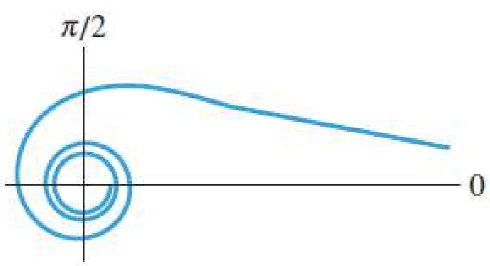
Archimedean spiral
$r = a\theta$



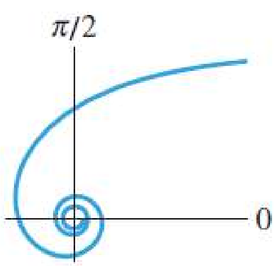
Parabolic spiral
$r = a\sqrt{\theta}$



Logarithmic spiral
$r = ae^{b\theta}$



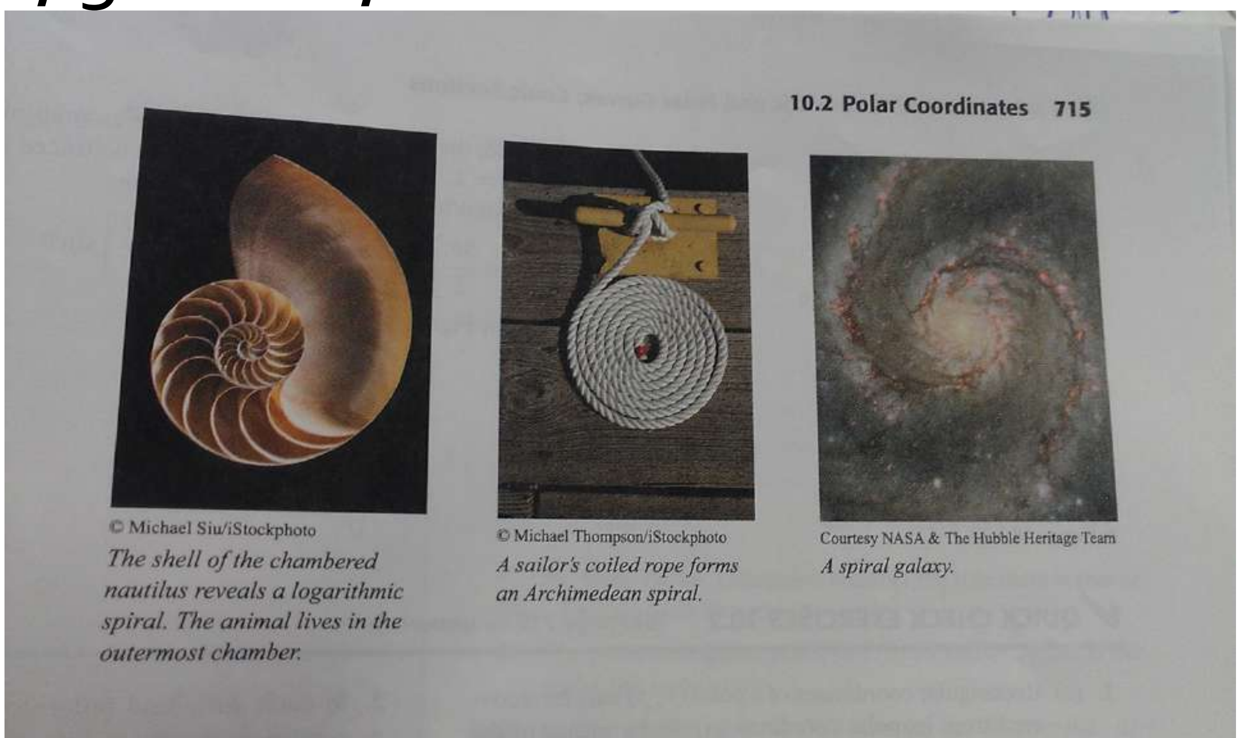
Lituus spiral
$r = a/\sqrt{\theta}$



Hyperbolic spiral
$r = a/\theta$

Spirals in Nature

- Spirals of many kinds occur in nature: the shell of a nautilus, sailor's rope, flowers, tusks, galaxies, etc.



Golden Gate Bridge

- I recently biked across the Golden Gate bridge from San Francisco to Sausalito and Tiburon. This picture is on the ferry back to San Francisco.