0-4 Counting Techniques

Use the Fundamental Counting Principle to determine the number of outcomes.

2. **QUIZZES** Each question on a five question multiple-choice quiz has answer choices labeled A, B, C, and D. How many different ways can a student answer the five questions?

SOLUTION:

There are 4 ways to answer each of the five questions. The number of different ways a students can answer is 4 × 4 × 4 × 4 × 4 or 1024.

4. **MANUFACTURING** A baseball glove manufacturer makes a glove with the different options shown in the table. How many different gloves are possible?

Option	Number of Choices
sizes	4
types by position	3
materials	2
levels of quality	2

SOLUTION:

There are 4 sizes, 3 position types, 2 types of material, and 2 levels of quality. The number of different possible gloves is 4 × 3 × 2 × 2 or 48.

Evaluate each permutation or combination.

6. $_7P_5$

SOLUTION:

$_nP_r = \frac{n!}{(n-r)!}$ Permutations formula

$_7P_5 = \frac{7!}{(7-5)!}$ $n=7, r=5$

$= \frac{7!}{2!}$ Subtract

$= \frac{7\cdot6\cdot5\cdot4\cdot3\cdot\cancel{2}\cdot\cancel{1}}{\cancel{2}\cdot\cancel{1}}$ Divide

$= 7\cdot6\cdot5\cdot4\cdot3$ Simplify

$= 2520$ Multiply

8. $_{12}C_7$

SOLUTION:

$_nC_r = \frac{n!}{(n-r)!r!}$ Combination formula

$_{12}C_7 = \frac{12!}{(12-7)!7!}$ $n=12, r=7$

$= \frac{12!}{5!7!}$ Subtract

$= 792$ Use a calculator

10. $_9P_5$

SOLUTION:

$_nP_r = \frac{n!}{(n-r)!}$ Permutations formula

$_9P_5 = \frac{9!}{(9-5)!}$ $n=9, r=5$

$= \frac{9!}{4!}$ Subtract

$= 15,120$ Use a calculator

Determine whether each situation involves *permutations* or *combinations*. Then solve the problem.

12. **BALLOONS** How many 4-colored groups can be selected from 13 different colored balloons?

SOLUTION:

Because the order in which the balloons are chosen is not important, this situation involves combinations. Use the combination formula for 13 things taken 4 at a time.

$_nC_r = \frac{n!}{(n-r)!r!}$ Combination formula

$_{13}C_4 = \frac{13!}{(13-4)!4!}$ $n=13, r=4$

$= \frac{13!}{9!4!}$ Subtract

$= 715$ Use a calculator

There are 715 different groups of 4 colored balloons possible.

0-4 Counting Techniques

14. **BANDS** A band is choosing 3 new backup singers from a group of 18 who try out. How many ways can they choose the new singers?

SOLUTION:

Because the order in which the singers are chosen is not important, this situation involves combinations. Use the Combination formula for 18 things taken 3 at a time.

$_nC_r = \frac{n!}{(n-r)!r!}$ Combination formula

$_{18}C_3 = \frac{18!}{(18-3)!3!}$ $n=18, r=3$

$= \frac{18!}{15!3!}$ Subtract

$= 816$ Use a calculator

There are 816 different ways for the band to choose 3 new backup singers.

16. **SOFTBALL** How many ways can the manager of a softball team choose players for the top 4 spots in the lineup if she has 7 possible players in mind?

SOLUTION:

Because the player assigned to each position is different, the order in which they are chosen is important. This situation involves permutations. Use the Permutation formula for 7 things taken 4 at a time.

$_nP_r = \frac{n!}{(n-r)!}$ Permutations formula

$_7P_4 = \frac{7!}{(7-4)!}$ $n=7, r=4$

$= \frac{7!}{3!}$ Subtract

$= 840$ Use a calculator

There are 840 different possible ways to choose players for the top four spots in the lineup.

P15

Determine whether the events are *mutually exclusive* or *not mutually exclusive*. Then find the probability.

3. A card is drawn at random from a standard deck of cards.
   a. P(club or diamond)
   b. P(ace or spade)
   c. P(jack or red card)

SOLUTION:

a. The events are mutually exclusive, because a card cannot be both a club (c) and a diamond (d).
P(club or diamond) = P(c) + P(d)

$P(\text{club or diamond}) = \frac{13}{52} + \frac{13}{52}$ or $\frac{1}{2}$.

b. Because there is an ace (A) of spades (S), these events are not mutually exclusive.
P(ace or spade) = P(A) + P(S) – P(A and S)

$P(\text{ace or spade}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$ or $\frac{4}{13}$.

c. Because 2 jacks (J) are red (R), these events are not mutually exclusive.
P(jack or red) = P(J) + P(R) – P(J and R)

$P(\text{jack or red}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52}$ or $\frac{7}{13}$.

5. There are 40 vehicles on a rental car lot. All are either sedans or SUVs. There are 18 red vehicles, and 3 of them are sedans. There are 15 blue vehicles, and 9 of them are SUVs. Of the remaining vehicles, all are black and 2 are SUVs. A vehicle is selected at random.
   a. P(blue or black)
   b. P(red or SUV)
   c. P(black or sedan)

SOLUTION:

There are 18 red (R), 15 blue (Bl), and 7 black (Bk) vehicles of which 14 are sedans (S) and 26 are SUVs.
a. The events are mutually exclusive, because a vehicle is not both blue and black.
P(blue or black) = P(Bl) + P(Bk)

$P(\text{blue or black}) = \frac{15}{40} + \frac{7}{40}$ or $\frac{11}{20}$.

b. Because there are red SUVs, these events are not mutually exclusive.
P(red or SUV) = P(R) + P(SUV) – P(R and SUV)

$P(\text{red or SUV}) = \frac{18}{40} + \frac{26}{40} - \frac{15}{40}$ or $\frac{29}{40}$.

c. Because there are black sedans, these events are not mutually exclusive.
P(black or sedan) = P(Bk) + P(S) – P(Bk and S)

$P(\text{black or sedan}) = \frac{7}{40} + \frac{14}{40} - \frac{5}{40}$ or $\frac{2}{5}$.

7. **REASONING** Explain why the rule $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ can be used for both mutually exclusive and not mutually exclusive events.

SOLUTION:

When events are mutually exclusive, $P(A \text{ and } B)$ will always equal 0, so the probability will simplify to $P(A) + P(B)$.

ODDS Another measure of the chance that an event will occur is called *odds*. The odds of an event occurring is a ratio that compares the number of ways an event can occur s (successes) to the number of ways it cannot occur f (failure), or s to f. The sum of the number of success and failures equals the number of possible outcomes.

9. Two fair coins are tossed. Find the odds in favor of both landing on heads. Then find the odds in favor of exactly one landing on tails.

SOLUTION:

For both heads, $s = 1$ and $f = 4 - 1$ or 3. The odds of both landing on heads are 1 to 3.
For exactly one landing on tails, $s = 2$ and $f = 4 - 2$ or 2. The odds of exactly one landing on tails are 2 to 2 or 1 to 1.

0-6 Multiplying Probabilities

Determine whether the events are *independent* or *dependent*. Then find the probability.

2. Yana has 4 black socks, 6 blue socks, and 8 white socks in his drawer. If he selects three socks at random with no replacement, what is the probability that he will first select a blue sock, then a black sock, and then another blue sock?

SOLUTION:

Since the socks are being selected with out replacement, the events are dependent.

$P(\text{blue}) = \frac{6}{18} \text{ or } \frac{1}{3}$

$P(\text{black}|\text{blue}) = \frac{4}{17}$

$P(\text{blue}|(\text{blue and black})) = \frac{5}{16}$

$P(\text{blue and black and blue})$
$= P(\text{black}) \cdot P(\text{blue}|\text{black}) \cdot P(\text{blue}|(\text{blue and black}))$
$= \frac{1}{3} \cdot \frac{4}{17} \cdot \frac{5}{16} \text{ or } \frac{5}{204}$

The probability is $\frac{5}{204}$ or about 0.025.

A die is rolled twice. Find each probability.

4. $P(\text{two 4s})$

SOLUTION:

$P(4 \text{ and } 4)$
$= P(4) \cdot P(4)$ Probability of independent events
$= \frac{1}{6} \cdot \frac{1}{6} \text{ or } \frac{1}{36}$ $P(4) = \frac{1}{6} \text{ and } P(4) = \frac{1}{6}$

The probability is $\frac{1}{36}$.

6. $P(\text{two of the same number})$

SOLUTION:

$P(\text{two of the same numbers})$
$= P(2\ 1s) + P(2\ 2s) + P(2\ 3s) + P(2\ 4s) + P(2\ 5s) + P(2\ 6s)$
$= \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}$ $P(2\ 1s) = \frac{1}{6} \cdot \frac{1}{6}$
$= 6\left(\frac{1}{36}\right) = \frac{1}{6}$ Simplify

The probability is $\frac{1}{6}$.

A bag contains 8 blue marbles, 6 red marbles, and 5 green marbles. Three marbles are drawn one at a time. Find each probability.

8. The second marble is red, given that the first marble is green and is replaced.

SOLUTION:

$P(\text{red}|\text{green})$
$= \frac{P(\text{red and green})}{P(\text{green})}$ Conditional probability
$= \frac{\frac{30}{361}}{\frac{5}{19}}$ $P(\text{green}) = \frac{5}{19}$ and $P(\text{red and green}) = \frac{6}{19} \cdot \frac{5}{19}$
$= \frac{30}{361} \cdot \frac{19}{5} = \frac{6}{19}$ Simplify

The probability is $\frac{6}{19}$.

10. The third marble is green, given that the first two are red and are replaced.

SOLUTION:

$P(\text{green}|(\text{red and red}))$
$= \frac{P(\text{green and (red and red)})}{P(\text{red and red})}$ Conditional probability
$= \frac{\frac{180}{19^3}}{\frac{36}{19^2}}$ $P(\text{red and red}) = \frac{6}{19} \cdot \frac{6}{19}$ and

$P(\text{green and (red and red)}) = \frac{5}{19} \cdot \frac{6}{19} \cdot \frac{6}{19}$
$= \frac{180}{19^3} \cdot \frac{19^2}{36} = \frac{5}{19}$ Simplify

The probability is $\frac{5}{19}$.

DVDS There are 8 action, 3 comedy, and 5 drama DVDs on a shelf. Suppose three DVDs are selected at random from the shelf. Find each probability.

12. $P(\text{2 action, then a comedy})$, without replacement

SOLUTION:

$P(\text{action, action, and comedy}) = \frac{3}{16} \cdot \frac{8}{15} \cdot \frac{7}{14} = \frac{1}{20}$

The probability is $\frac{1}{20}$.

14. **HONOR ROLL** Suppose the probability that a student takes AP Calculus and is on the honor roll is 0.0035, and the probability that a student is on the honor roll is 0.23. Find the probability that a student takes AP Calculus given that he or she is on the honor roll.

SOLUTION:

$P(\text{AP Calc}|\text{honor roll})$

$= \dfrac{P(\text{AP Calc and honor roll})}{P(\text{honor roll})}$ Conditional probability

$= \dfrac{0.0035}{0.23}$ $P(\text{honor roll}) = 0.23$ and $P(\text{AP Calc and honor roll}) = 0.0035$

≈ 0.015 Simplify

The probability is about 0.015.

16. **SCHOOL CLUBS** King High School tallied the nu that were members of at least one after school club.

Gender	Clubs	No Clubs
male	156	242
female	312	108

a. A student is a member of a club given that he is m
b. A student is not a member of a club given that she
c. A student is a male given that he is not a member

SOLUTION:

Calculate the column and row totals.

Gender	Clubs	No Clubs	Totals
male	156	242	398
female	312	108	420
Totals	468	350	818

$P(\text{club member}|\text{male})$

$= \dfrac{P(\text{club member and male})}{P(\text{male})}$ Conditional probability

$= \dfrac{\frac{156}{818}}{\frac{398}{818}}$ $P(\text{male}) = \frac{398}{818}$ and $P(\text{club member and male}) = \frac{156}{818}$

a. $= \dfrac{156}{398} = \dfrac{78}{199}$ Simplify

$P(\text{non member}|\text{female})$

$= \dfrac{P(\text{non member and female})}{P(\text{female})}$ Conditional probability

$= \dfrac{\frac{108}{818}}{\frac{420}{818}}$ $P(\text{female}) = \frac{420}{818}$ and $P(\text{non member and female}) = \frac{108}{818}$

b. $= \dfrac{108}{420} = \dfrac{9}{35}$ Simplify

c.

$P(\text{male}|\text{non member})$

$= \dfrac{P(\text{male and non member})}{P(\text{non member})}$ Conditional probability

$= \dfrac{\frac{242}{818}}{\frac{350}{818}}$ $P(\text{non member}) = \frac{350}{818}$ and $P(\text{male and non member}) = \frac{242}{818}$

$= \dfrac{242}{350} = \dfrac{121}{175}$ Simplify

P27 - P28

0-9 Measures of Center, Spread, and Position

Find the mean, median, and mode for each set of data.

2. height in centimeters of bean plants at the end of an experiment:
14.5, 12, 16, 11, 14, 11, 10.5, 14, 11.5, 15, 13.5

SOLUTION:

To find the mean divide the sum of all the heights divided by the total of plants.

mean

$= \frac{14.5+12+16+11+14+11+10.5+14+11.5+15+13.5}{11}$

$= \frac{143}{11}$

$= 13$ cm

To find the median, arrange the heights in order.
10.5, 11, 11, 11.5, 12, 13.5, 14, 14, 14.5, 15, 16.

Because there is an odd number of plants, the median is the middle number. So, the median is 13.5 cm
The mode is the value that occurs the most often in the data set. In this case, it is 11 cm and 14 cm.

State whether the data in sets A and B represent *sample* or *population* data. Then find the range, variance, and standard deviation of each set. Use the standard deviations to compare the variability between the data sets.

4.

Wait Times (min)					
Ride A			Ride B		
45	22	40	35	50	32
48	11	51	31	35	45
36	55	60	45	49	40
32	24	37	43	37	45

SOLUTION:

12 wait times for Ride A and Ride B are given. For an amusement park ride, there must be more than 12 rides a day, so the data must be a sample.

The range is the difference between the greatest and least values in the set.
Ride A: $60 - 11 = 49$ min
Ride B: $50 - 31 = 19$ min

In order to find the variance and standard deviation, the mean is needed.

mean Ride A

$= \frac{45+22+40+48+11+51+36+55+60+32+24+37}{12}$

$= \frac{461}{12}$

≈ 38.4 min

X	$X-\bar{X}$	$(X-\bar{X})^2$
45	$45 - 38.4 = 6.6$	$6.6^2 = 43.56$
22	$22 - 38.4 = -16.4$	$(-16.4)^2 = 268.96$
40	$40 - 38.4 = 1.6$	$1.6^2 = 2.56$
48	$48 - 38.4 = 9.6$	$9.6^2 = 92.16$
11	$11 - 38.4 = -27.4$	$(-27.4)^2 = 750.76$
51	$51 - 38.4 = 12.6$	$12.6^2 = 158.76$
36	$36 - 38.4 = -2.4$	$(-2.4)^2 = 5.76$
55	$55 - 38.4 = 16.6$	$16.6^2 = 275.56$
60	$60 - 38.4 = 21.6$	$21.6^2 = 466.56$
32	$32 - 38.4 = -6.4$	$(-6.4)^2 = 40.96$
24	$24 - 38.4 = -14.4$	$(-14.4)^2 = 207.36$
37	$37 - 38.4 = -1.4$	$(-1.4)^2 = 1.96$
		Sum = 2314.92

The variance of Ride A is $\frac{2314.92}{11} = 210.45$ min

The standard deviation of Ride A is $\sqrt{210.45} = 14.5$ min

mean Ride B

$= \frac{35+50+32+31+35+45+45+49+40+43+37+45}{12}$

$= \frac{487}{12}$

≈ 40.6 min

X	$X-\bar{X}$	$(X-\bar{X})^2$
35	$35 - 40.6 = -5.6$	$(-5.6)^2 = 31.36$
50	$50 - 40.6 = 9.4$	$9.4^2 = 88.36$
32	$32 - 40.6 = -8.6$	$(-8.6)^2 = 73.96$
31	$31 - 40.6 = -9.6$	$(-9.6)^2 = 92.16$
35	$35 - 40.6 = -5.6$	$(-5.6)^2 = 31.36$
45	$45 - 40.6 = 4.4$	$4.4^2 = 19.36$
45	$45 - 40.6 = 4.4$	$4.4^2 = 19.36$
49	$49 - 40.6 = 8.4$	$8.4^2 = 70.56$
40	$40 - 40.6 = -0.6$	$(-0.6)^2 = 0.36$

0-9 Measures of Center, Spread, and Position

43	$43 - 40.6 = 2.4$	$2.4^2 = 5.76$
37	$37 - 40.6 = -3.6$	$(-3.6)^2 = 12.96$
45	$45 - 40.6 = 4.4$	$4.4^2 = 19.36$
		Sum = 464.92

The variance of Ride B is $\frac{464.92}{11} = 42.3$ min

The standard deviation of Ride B is $\sqrt{42.3} = 6.5$ min

Since the sample standard deviation of Ride A is greater than that of Ride B, there is more variability in the sample wait times for Ride A than Ride B.

6.

Number of Days Each Student Missed This Year							
Class A							
10	8	5	9	7	3	6	8
5	13	0	15	9	7	9	10
14	11	8	4	7	8	2	
9	11	14	8	12	10	1	
Class B							
5	8	13	7	9	4	10	2
12	6	7	8	11	12	8	9
12	9	6	11	3	8	5	
3	10	5	13	9	1	6	

SOLUTION:

The data given is the of days absent for each student in Class A and Class B. Since there is data for each student, we are given data for the population.

Class A: 15, ≈13.7, 3.7;
Class B: 12, ≈10.5 , ≈3.2;

The range is the difference between the greatest and least values in the set.
Class A: $15 - 0 = 15$ days
Class B: $13 - 1 = 12$ days

In order to find the variance and standard deviation, the mean is needed.

mean Class A

$= \frac{[illegible]}{30}$

$= \frac{243}{30}$

≈ 8.1 days

x	$x - \bar{x}$	$(x - \bar{x})^2$
10	$10 - 8.1 = 1.9$	$(1.9)^2 = 3.61$
8	$8 - 8.1 = -0.1$	$(-0.1)^2 = 0.01$
5	$5 - 8.1 = -3.1$	$(-3.1)^2 = 9.61$
9	$9 - 8.1 = 0.9$	$(0.9)^2 = 0.81$
7	$7 - 8.1 = -1.1$	$(-1.1)^2 = 1.21$
3	$3 - 8.1 = -5.1$	$(-5.1)^2 = 26.01$
6	$6 - 8.1 = -2.1$	$(-2.1)^2 = 4.41$
8	$8 - 8.1 = -0.1$	$(-0.1)^2 = 0.01$
14	$14 - 8.1 = 5.9$	$(5.9)^2 = 34.81$
11	$11 - 8.1 = 2.9$	$(2.9)^2 = 8.41$
8	$8 - 8.1 = -0.1$	$(-0.1)^2 = 0.01$
4	$4 - 8.1 = -4.1$	$(-4.1)^2 = 16.81$
7	$7 - 8.1 = -1.1$	$(-1.1)^2 = 1.21$
8	$8 - 8.1 = -0.1$	$(-0.1)^2 = 0.01$
2	$2 - 8.1 = -6.1$	$(-6.1)^2 = 37.21$
5	$5 - 8.1 = -3.1$	$(-3.1)^2 = 9.61$
13	$13 - 8.1 = 4.9$	$(4.9)^2 = 24.01$
0	$0 - 8.1 = -8.1$	$(-8.1)^2 = 65.61$
15	$15 - 8.1 = 6.9$	$(6.9)^2 = 47.61$
9	$9 - 8.1 = 0.9$	$(0.9)^2 = 0.81$
7	$7 - 8.1 = -1.1$	$(-1.1)^2 = 1.21$
9	$9 - 8.1 = 0.9$	$(0.9)^2 = 0.81$
10	$10 - 8.1 = 1.9$	$(1.9)^2 = 3.61$
9	$9 - 8.1 = 0.9$	$(0.9)^2 = 0.81$
11	$11 - 8.1 = 2.9$	$(2.9)^2 = 8.41$
14	$14 - 8.1 = 5.9$	$(5.9)^2 = 34.81$
8	$8 - 8.1 = -0.1$	$(-0.1)^2 = 0.01$
12	$12 - 8.1 = 3.9$	$(3.9)^2 = 15.21$
10	$10 - 8.1 = 1.9$	$(1.9)^2 = 3.61$
1	$1 - 8.1 = -7.1$	$(-7.1)^2 = 50.41$
		Sum = 410.7

The variance of Class A is $\frac{410.7}{29} = 14.2$ days

The standard deviation of Class A is $\sqrt{14.2} = 3.8$ days

mean Class B

$= \frac{[illegible]}{30}$

$= \frac{234}{30}$
$\approx$ 7.8 days

x	$x - \bar{x}$	$(x - \bar{x})^2$
5	$5 - 7.8 = -2.8$	$(-2.8)^2 = 7.84$
8	$8 - 7.8 = 0.2$	$(0.2)^2 = 0.04$
13	$13 - 7.8 = 5.2$	$(5.2)^2 = 27.04$
7	$7 - 7.8 = -0.8$	$(-0.8)^2 = 0.64$
9	$9 - 7.8 = 1.2$	$(1.2)^2 = 1.44$
4	$4 - 7.8 = -3.8$	$(-3.8)^2 = 14.44$
10	$10 - 7.8 = 2.2$	$(2.2)^2 = 4.84$
2	$2 - 7.8 = -5.8$	$(-5.8)^2 = 33.64$
12	$12 - 7.8 = 4.2$	$(4.2)^2 = 17.64$
9	$9 - 7.8 = 1.2$	$(1.2)^2 = 1.44$
6	$6 - 7.8 = -1.8$	$(-1.8)^2 = 3.24$
11	$11 - 7.8 = 3.2$	$(3.2)^2 = 10.24$
3	$3 - 7.8 = -4.8$	$(-4.8)^2 = 23.04$
8	$8 - 7.8 = 0.2$	$(0.2)^2 = 0.04$
5	$5 - 7.8 = -2.8$	$(-2.8)^2 = 7.84$
12	$12 - 7.8 = 4.2$	$(4.2)^2 = 17.64$
6	$6 - 7.8 = -1.8$	$(-1.8)^2 = 3.24$
7	$7 - 7.8 = -0.8$	$(-0.8)^2 = 0.64$
8	$8 - 7.8 = 0.2$	$(0.2)^2 = 0.04$
11	$11 - 7.8 = 3.2$	$(3.2)^2 = 10.24$
12	$12 - 7.8 = 4.2$	$(4.2)^2 = 17.64$
8	$8 - 7.8 = 0.2$	$(0.2)^2 = 0.04$
9	$9 - 7.8 = 1.2$	$(1.2)^2 = 1.44$
3	$3 - 7.8 = -4.8$	$(-4.8)^2 = 23.04$
10	$10 - 7.8 = 2.2$	$(2.2)^2 = 4.84$
5	$5 - 7.8 = -2.8$	$(-2.8)^2 = 7.84$
13	$13 - 7.8 = 5.2$	$(5.2)^2 = 27.04$
9	$9 - 7.8 = 1.2$	$(1.2)^2 = 1.44$
1	$1 - 7.8 = -6.8$	$(-6.8)^2 = 46.24$
8	$8 - 7.8 = 0.2$	$(0.2)^2 = 0.04$
		Sum = 314.8

The variance of Class B is $\frac{314.8}{29} \approx 10.9$ days

The standard deviation of Class B is $\sqrt{10.9} \approx 3.3$ days

Since the sample standard deviation of Class A is greater than that of Class B, there is more variability in the number of days that students missed during the school year for Class A than for Class B.

Find the minimum, lower quartile, median, upper quartile, and maximum of each data set. Then interpret this five-number summary.

8.

State Mean ACT Scores				
20.2	21.3	21.5	20.4	21.6
20.0	21.7	21.3	20.2	21.6
20.8	22.4	21.4	22.2	18.8
20.1	22.3	20.3	21.2	21.4
21.5	20.5	20.3	21.5	22.7
20.3	22.5	21.5	17.8	20.5
22.0	21.6	20.3	19.8	22.6
21.5	21.7	21.2	22.5	21.2
20.6	22.5	21.8	21.9	19.3
20.9	22.5	22.2	21.4	20.7

SOLUTION:

Enter the data into L1.

Press STAT ▸ ENTER to display the 1-Var statistics.

```
1-Var Stats
↑n=50
 minX=17.8
 Q1=20.4
 Med=21.4
 Q3=21.8
 maxX=22.7
```

The minimum is 17.8.
The lower quartile is 20.4.
The median is 21.4.
The upper quartile is 21.8.
The maximum is 22.7.

The lowest mean score for a state is 17.8 and the highest mean score is 22.7. 25% of the states have a mean score that is less than 20.4. 50% of the states have a mean score that is less than 21.4, and 75% of the states have a mean score that is less than 22.7.

Identify any outliers in each data set, and explain your reasoning. Then find the mean,

median, mode, range, and standard deviation of the data set with and without the outlier. Describe the effect on each measure.

10. number of posts to a certain blog each month during a particular year:
25, 23, 21, 27, 29, 19, 10, 21, 20, 18, 26, 23

SOLUTION:

Enter the data into **L1**.

Keystrokes: STAT ▸ ENTER

Use the 1-Var statistics to identify Q_1 and Q_3.

```
1-Var Stats
↑n=12
 minX=10
 Q1=19.5
 Med=22
 Q3=25.5
 maxX=29
```

$Q_1 = 19.5$ and $Q_3 = 25.5$. $IQR = Q_3 - Q_1$ or $25.5 - 19.5 = 6$.

Find and use the IQR to find the values beyond which any outlier would lie.

$Q_1 - 1.5(IQR)$ and $Q_3 + 1.5(IQR)$

$19.5 - 1.5(6)$ $\quad$ $25.5 + 1.5(6)$

10.5 $\quad$ 34.5

The interval beyond which any outliers would lie is $10.5 < x < 34.5$. Since $10 < 10.5$, it is an outlier. There are no outliers on the upper end.

With outlier:

```
1-Var Stats
 x̄=21.83333333
 Σx=262
 Σx²=5996
 Sx=5.006056937
 σx=4.792934615
↓n=12
```

Without outlier:

```
1-Var Stats
 x̄=22.90909091
 Σx=252
 Σx²=5896
 Sx=3.505839285
 σx=3.342686602
↓n=11
```

```
1-Var Stats
↑n=11
 minX=18
 Q1=20
 Med=23
 Q3=26
 maxX=29
```

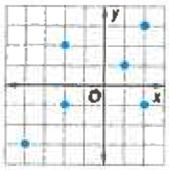
Data Set	Mean	Median	Mode	Range	Standard Deviation
with outlier	≈21.8	22	21	19	≈4.8
without outlier	≈22.9	23	21	11	≈3.3

Removing the outlier did not affect the mode. However, the removal did affect the mean, median, standard deviation, and range. The mean and median increased, and the standard deviation and range decreased.

Chapter 0 Posttest

State the domain and range of each relation. Then determine whether each relation is a function. Write *yes* or *no*. 1. $D = \{0, 4, 5, 7\}$, $R = \{-2, -1, 5, 9, 12\}$; no

1. $\{(4, 5), (5, -1), (0, 12), (0, -2), (7, 9)\}$

2. 

2. $D = \{-4, -2, 1, 2\}$, $R = \{-3, -1, 1, 2, 3\}$; no

Name the quadrant in which each point is located.

3. $(-3, 7)$ II
4. $(10, -11)$ IV
5. $(-15, -3)$ III

Find each product. 6. $8n^2 + 2n - 6$ 7. $30p^2 - 56p + 10$

6. $(4n - 3)(2n + 2)$
7. $(5p - 1)(6p - 10)$
8. $(7x + 4)(7x + 4)$ $49x^2 + 56x + 16$
9. $(3k - 2)(6k + 9)$ $18k^2 + 15k - 18$

10. **GEOMETRY** The height of a rectangle is 3 millimeters less than twice the width.
   a. Write an expression for each measure. w; $2w - 3$
   b. Write a polynomial expression for the area of the rectangle. $2w^2 - 3w$

11. $(2x + y)^2$ 12. $(5a - 2)^2$

Factor each polynomial.

11. $4x^2 + 4xy + y^2$
12. $25a^2 - 20a + 4$
13. $4a^2 + 16ab + 16b^2$ $4(a + 2b)^2$
14. $81t^2 - 36$ $9(3t - 2)(3t + 2)$

15. **STUDENT COUNCIL** A student council has 6 seniors, 5 juniors, and 1 sophomore as members. How many ways can a 3-member committee be formed that includes one member from each class? 30 ways

Determine whether each situation involves *permutations* or *combinations*. Then solve.

16. How many ways are there to select one competitor and one alternate out of 8 students? permutations; 56

17. How many ways are there to form a team of 7 athletes from a group of 15 who try out? combinations; 6435

RESTAURANT There are 24 male and 36 female patrons in a restaurant. Of the 11 patrons under 10 years old, 6 are male. Of the 14 patrons over 55 years old, 9 are female. A patron is selected at random. Determine whether the events are *mutually exclusive* or *not mutually exclusive*. Then find each probability.

18. P(female or under 10) not mutually exclusive; $\frac{7}{10}$
19. P(under 10 or over 55) mutually exclusive; $\frac{5}{12}$

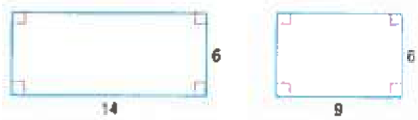
Determine whether the events are *independent* or *dependent*. Then find the probability.

20. Slips of paper numbered 1 through 10 are placed into a bag. What is the probability of drawing the number 10 three times in a row if a slip is drawn at random and then replaced? independent; $\frac{1}{1000}$

21. Two students are selected at random from a class that consists of 13 males and 7 females. What is the probability that both students are female? dependent; $\frac{21}{190}$

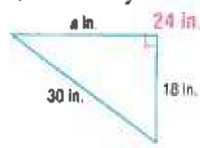
22. **TESTING** Of the students who took both the Mid-Chapter 4 Quiz and the Chapter 4 Test, 56% passed the quiz and 48% passed both the quiz and the test. If a student passed the quiz, find the probability that he or she also passed the test. about 86%

23. Determine whether the rectangles are *similar*, *congruent*, or *neither*. neither



24. **COMPUTERS** A computer image of a painting 320 pixels wide by 240 pixels high. If the actual painting is 42 inches wide, how high is it? 31.5 in.

Find each missing measure. Round to the nearest tenth, if necessary.

25. 

24 in.

26. $a = 33$ cm, $b = ?$ cm, $c = 45$ cm 30.6 cm

The lengths of three sides of a triangle are given. Determine whether each triangle is a right triangle.

27. 6 in., 8 in., 12 in. no
28. 30 m, 34 m, 16 m yes

29. ≈ 36.1 students, 33.5 students, 35 students, 59 students, ≈ 16.8 students; 75 students

Find the mean, median, mode, range, and standard deviation of each data set. Then identify any outliers.

29. number of students present at 8 student council meetings: 23, 45, 16, 75, 32, 35, 28, 35

30. running time in minutes for 17 movies: 95, 102, 148, 140, 110, 103, 107, 104, 99, 111, 109, 124, 109, 90, 92, 110, 129 ≈ 110.7 min, 109 min, 109 and 111 min, 58 min, ≈ 16.1 min; 148 min

Using the Posttest

Use the Chapter 0 Posttest to asse students' understanding of the con after you have presented the lesso Chapter 0. If students are still havi difficulty with one or more concept refer to *Math Triumphs: Foundatio Integrated_Math ii* for strategies fo reteaching.