Agenda

Homework (AP)
Pg.378-380 #37, 55, 56(a)

• Warm Up	10 min
• Checkup	10 min
– *Checked 10/11/18, copies*	
• Linear combinations, w/ example	20 min
• Practice w/ FRQ's	25 min
– *Copies (incl. answers)*	
• Exit Pass	5 min

Warm Up

Only $1 to play my game, what a deal. Let's say you toss three coins. If they're all the same, you immediately double your money! If they're not, you still might win! Pick a card. If it's a Diamond, you double your money!

1. Create a probability distribution for this game.

2. Calculate the expected value of the game.

Warm Up ANSWER

Event	Coins same	Diamond	Lose
$	$1	$1	-$1
P	$2/8$	$6/8 \bullet 13/52 = 3/16$	$9/16$

$$\frac{2}{8}+\frac{3}{16}+\frac{-9}{16}=-0.125$$

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Checkup time

Linear Combinations

AP

Example #1

You work for a company that manufactures refrigerators, and you are examining “surface flaws”, such as dimples and paint chips. You find that your company’s refrigerators have a mean 0.7 dimples, with $\sigma=0.11$, and a mean 1.4 paint chips, with $\sigma=0.2$. The distributions of both dimples and paint chips are Normally distributed.

1. What is the average number of imperfections on a refrigerator?
2. What is the standard deviation of your answer to #1?
3. What is the probability that a randomly selected refrigerator has less than 2 surface flaws?

Notes

Linear Combinations: 3 Rules

1. The average of both random variables is the sum of each of their averages.

   Works whether or not X and Y are independent.

   $\mu_{x+y} = \mu_x + \mu_y$

   $\mu_{x-y} = \mu_x - \mu_y$

2. The standard deviation of both random variables is the square root of the sum of their variances.

   Only works if X and Y are independent.

   $\sigma_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2}$

   $\sigma_{x-y} = \sqrt{\sigma_x^2 + \sigma_y^2}$

3. Any linear combination of normally distributed random variables is also normally distributed.

Example #1

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1. What is the average number of imperfections on a refrigerator?
2. What is the standard deviation of your answer to #1?
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Example #1 ANSWERS

You work for a company that manufactures refrigerators, and you are examining "surface flaws", such as dimples and paint chips. You find that your company's refrigerators have a mean 0.7 dimples, with $\sigma=0.11$, and a mean 1.4 paint chips, with $\sigma=0.2$. The distributions of both dimples and paint chips are Normally distributed.

1. What is the average number of imperfections on a refrigerator?

$$0.7+1.4=2.1$$

2. What is the standard deviation of your answer?

$$\sqrt{0.11^2+0.2^2}=0.228$$

3. What is the probability that a refrigerator has less than 2 surface flaws?

$$\frac{2-2.1}{0.228}=-0.44$$

$$0.33$$

Notes

Linear Combinations: Example #2

Mr. Starnes likes sugar in his hot tea. From experience, he needs **between 8.5 and 9** grams of sugar in a cup of tea for the drink to taste right. While making his tea one morning, Mr. Starnes adds **four randomly selected packets** of sugar. Suppose the amount of sugar in these packets (of the same brand) follows a **Normal distribution** with **mean 2.17** grams and **standard deviation 0.08** grams. What is the probability that Mr. Starnes' cup of tea will taste right?

$$2.17 + 2.17 + 2.17 + 2.17 = 8.68$$

MEAN:

$$\sqrt{0.08^2 + 0.08^2 + 0.08^2 + 0.08^2} = 0.16$$

STANDARD DEVIATION:

$$\frac{8.5 - 8.68}{0.16} = -1.125 \qquad \frac{9 - 8.68}{0.16} = 2$$

$$0.9772 - 0.1292 = 0.848$$

You try

1. WINDOW. The owner of a coffee shop, an amateur statistician, advertises that the price of coffee on any given day will be randomly picked using a **normal distribution** with **mean $1.35** and **standard deviation $0.10**. If a customer buys a cup of coffee for **10 days**, and each price is **independent**, what is the probability that he will pay a total **exceeding $14.00**?

2. DOOR. Last semester, my students **averaged 78.2%** on this upcoming quiz, with a **standard deviation of 8.95%**. If **three** randomly selected students are selected, what is the probability that they average **at least a B**? *Hint: Their sum must be at least 240%.*

Window ANSWER

The owner of a coffee shop, an amateur statistician, advertises that the price of coffee on any given day will be randomly picked using a mean of $1.35 and standard deviation $0.10. If a customer buys a cup of coffee for 10 days, and each price is independent, what is the probability that he will pay a total exceeding $14.00?

MEAN:

$$1.35+1.35+1.35+1.35+1.35+1.35+1.35+1.35+1.35+1.35=13.5$$

STANDARD DEVIATION:

$$\sqrt{.1^2+.1^2+.1^2+.1^2+.1^2+.1^2+.1^2+.1^2+.1^2+.1^2}=0.316$$

$$\frac{14-13.5}{0.316}=1.58$$

$$1-0.9429=0.0571$$

Door ANSWER

Last semester, my students averaged 78.2% on this upcoming quiz, with a standard deviation of 8.95%. If three randomly selected students are selected, what is the probability that they average at least a B? *Hint: Their sum must be at least 240%.*

MEAN: $0.782 + 0.782 + 0.782 = 2.346$

STANDARD DEVIATION: $\sqrt{.0895^2 + .0895^2 + .0895^2} = 0.155$

$$\frac{2.40 - 2.346}{0.155} = 0.35$$

$$1 - 0.6368 = 0.3632$$

FRQ Practice -- linear combinations

- I will give you two problems, Window and Door.
- You may write on the copies.

DOOR (FRQ 2002, #3)

The team has 4 runners. They are going to compete in a race, and each runner will run a mile. The team time is the sum of individual times for all 4 runners. Assume individual runner times are independent. The individual times (min.) of the runners in similar races are approximately normally distributed, with these means and standard deviations.

	Mean	σ
Runner 1	4.9	0.15
Runner 2	4.7	0.16
Runner 3	4.5	0.14
Runner 4	4.8	0.15

a. Runner 3 thinks he can run a mile in less than 4.2 minutes in the next race. Is this likely? Explain.

b. The distribution of possible team times is approximately normal. What are the mean and standard deviation of this distribution?

c. Suppose the team's best time to date is 18.4 minutes. What is the probability that the team will beat its own best time in the next race?

WINDOW

Suppose men's weights (in pounds) are normal with a mean of 170 and a standard deviation of 40 lb., and women's weights are also normal but with a mean of 135 and a standard deviation of 30 lb.

1. If a man and woman are selected at random….
   a. What is the probability that the sum of their weights is at least 300 pounds?
   b. What is the probability that their total weight is at most 320 pounds?
   c. *(Optional, difficult)* What is the probability that the woman is heavier than the man?

2. If you were to select 3 men at random, what is the probability that the sum of their weights is at least 500 pounds?

DOOR (FRQ 2002, #3) ANSWERS

	Mean	σ
Runner 1	4.9	0.15
Runner 2	4.7	0.16
Runner 3	4.5	0.14
Runner 4	4.8	0.15

a. Runner 3 thinks he can run a mile in less than 4.2 minutes in the next race. Is this likely? Explain.

$$\frac{4.2-4.5}{0.14}=-2.14$$

$$0.0162$$

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b. The distribution of possible team times is approximately normal. What are the mean and standard deviation of this distribution?

$$4.9+4.7+4.5+4.8=18.9$$

$$\sqrt{.15^2+.16^2+.14^2+.15^2}=0.3003$$

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c. Suppose the team's best time to date is 18.4 minutes. What is the probability that the team will beat its own best time in the next race?

$$\frac{18.4-18.9}{0.3003}=-1.67$$

$$0.0475$$

WINDOW ANSWERS

Men: mean of 170, standard deviation of 40 lb.

Women: mean of 135, standard deviation of 30 lb.

a) What is the probability that the sum of their weights is at least 300 pounds?

$$0.5398$$

$$\frac{300-305}{50}=-0.1$$

b) What is the probability that their total weight is at most 320 pounds?

$$0.6179$$

$$\frac{320-305}{50}=0.3$$

c) *(Optional, difficult)* What is the probability that the woman is heavier than the man?

$$\frac{0-35}{50}=-0.7$$

$$0.2420$$

2. If you were to select 3 men at random, what is the probability that the sum of their weights is at least 500 pounds?

$$\frac{500-510}{69.28}=-0.144$$

$$0.5557$$

Casino Project ideas

- No sharp darts
- No latex balloons
- *Careful* w/ food

✓ Good

✓✓ Great

✓✓✓ One-of-a-kind

? I'm confused, or don't have enough information.

B *Extra credit. Requires binomial probability.*

We'll learn this next Tuesday.

C *Extra credit. Requires confidence intervals.*

We'll learn this next month. It's time-consuming.

Stay after class today or Monday to learn.

Casino Project (AP)

Create, mathematically analyze, and run a simple gambling game which is significantly (but not obviously) in favor of you, “The House”.

1-2 people (2 recommended)
Up to +5% extra credit for binomial probabilities and/or confidence intervals.

Monday March 9th, due end of class:

1. List three possible ideas for your project. For each game, state how to play and how to win.

Wednesday March 18th (or earlier), due end of class:

2. Choose one game and give it a catchy name. Describe your game (including the title), and calculate its probabilities and expected value.

Tuesday March 24th:

3. Bring an attractive poster that explains your game, including prices and prizes. Bring all components needed for your game.

Thursday March 26th: CASINO DAY!

7. Run your game, attract gamblers, and help them lose their money! You must profit at least $500 ($1000 total).

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Homework (AP)
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Exit Pass (AP)

You have two balanced six-sided dice. The first has 1, 3, 4, 5, 6, and 8 spots on its six faces. The second die has 1, 2, 2, 3, 3, and 4 spots on its faces.

1. Find the probability model for the sum of the spots on the up-faces of the two dice.

2. Let's say you pay $1 to roll these dice, and if your sum is greater than 8, you double your money! Calculate the expected value.

Exit Pass (AP) ANSWER

1, 3, 4, 5, 6, and 8 spots.
1, 2, 2, 3, 3, and 4 spots.

2	3	4	5	6	7	8	9	10	11	12
$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$1	$-1
$\frac{10}{36}$	$\frac{26}{36}$

1,1	5,1
1,2	5,2
1,2	5,2
1,3	5,3
1,3	5,3
1,4	5,4
3,1	6,1
3,2	6,2
3,2	6,2
3,3	6,3
3,3	6,3
3,4	6,4
4,1	8,1
4,2	8,2
4,2	8.2
4,3	8,3
4,3	8,3
4,4	8,4