1. **GRADES** Clara got an A on 80% of her first semester Biology quizzes. Design and conduct a simulation using a geometric model to estimate the probability that she will get an A on a second semester Biology quiz. Report the results using appropriate numerical and graphical summaries.

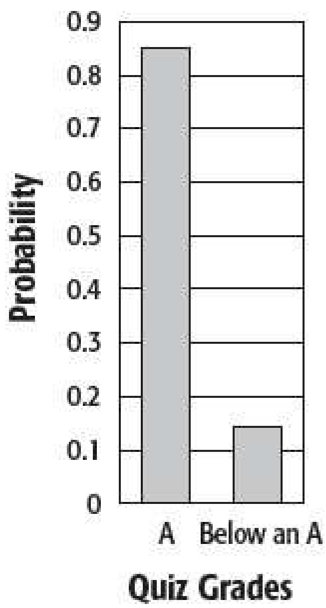
SOLUTION:

Sample answer: While a random number generator would work, a spinner will be easier. Divide the spinner into two sectors, one containing 80% or 288° and the other containing 20% or 72°. Another option would be to roll a six-sided die. Let 1-4 represent an *A*, let 5 represent a non *A*, and roll again for each 6.

Do 20 trials and record the results in a frequency table.

Outcome	Frequency
A	17
Below an A	3
Total	20

Divide the frequency by 20 to get the probabilities.



The probability of Clara getting an A on her next quiz is .85. The probability of earning any other grade is 1 – 0.85 or 0.15.

3. **CARNIVAL GAMES** The object of the game shown is to accumulate points by using a dart to pop the balloons. Assume that each dart will hit a balloon.

a. Calculate the expected value from each throw.
b. Design a simulation and estimate the average value of this game.
c. How do the expected value and average value compare?

SOLUTION:

a. Find the sum of the products (outcome multiplied by the frequency) for each outcome. There are 25 total balloons, so there are 25 total outcomes.

$$\begin{aligned}&\text{Expected Value}\\&=\text{Sum of (Outcome} \times \text{Frequency)}\\&=\left(25\times\frac{16}{25}\right)+\left(50\times\frac{8}{25}\right)+\left(100\times\frac{1}{25}\right)\\&=16+16+4\\&=36\end{aligned}$$

b. With 25 outcomes, 16 will represent 25 points, 8 will represent 50 points, and 1 will represent 100 points.

Use a random number generator to generate integers 1 through 25 where 1–16 represents 25 points, 17–24 represents 50 points, and 25 represents 100 points. Do 50 trials and record the results in a frequency table.

Outcome	Frequency
25	29
50	21
100	0

The average value is calculated the same way as the expected value, except we use the simulation frequencies as opposed to the expected frequencies.

$$\begin{aligned}\text{average value}&=25\cdot\frac{29}{50}+50\cdot\frac{21}{50}+100\cdot\frac{0}{50}\\&=35.5\end{aligned}$$

c. Sample answer: The expected value and average value are very close. While this doesn't give much information about each individual outcome, it does tell us that the average for all of the trials was close to what was expected to happen.

Design and conduct a simulation using a geometric probability model. Then report the results using appropriate numerical and graphical summaries.

5. **VIDEO GAMES** Ian works at a video game store. Last year he sold 95% of the new-release video games.

SOLUTION:

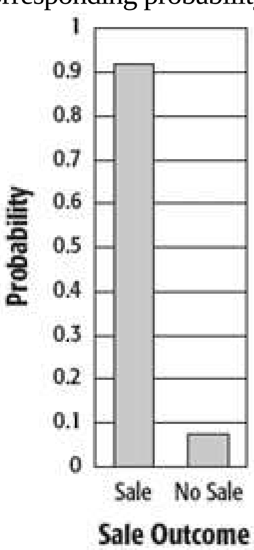
One option would be to create a spinner. Use a spinner that is divided into two sectors, one containing 95% or 342° and the other containing 5% or 18°.

Or you could use a random number generator to generate integers 1 through 20 where 1–19 represents a sale and 20 represents a no-sale. We could generate the integers 1-100 and let 96-100 represent a no-sale.

Do 50 trials and record the results in a frequency table.

Outcome	Frequency
Sale	46
No Sale	4
Total	50

Divide each frequency by 50 to find the corresponding probability.



The probability of Ian selling a game is 0.92. The probability of not selling a game is 1 – 0.92 or 0.08.

7. **BOARD GAMES** Pilar is playing a board game with eight different categories, each with questions that must be answered correctly in order to win.

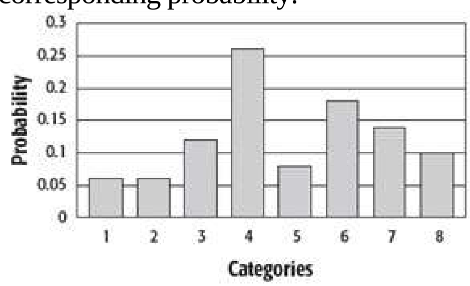
SOLUTION:

Sample answer: Use a spinner that is divided into 8 equal sectors, each 45°. Another option would be to use a random number generator from 1-8 where each value represents a different category.

Do 50 trials and record the results in a frequency table.

Outcome	Frequency
Category 1	3
Category 2	3
Category 3	6
Category 4	13
Category 5	4
Category 6	9
Category 7	7
Category 8	5
Total	50

Divide the frequency by 50 to find each corresponding probability.



The probability of landing on Categories 1 and 2 is 0.06, Category 3 is 0.12, Category 4 is 0.26, Category 5 is 0.08, Category 6 is 0.18, Category 7 is 0.14, and Category 8 is 0.1.

CCSS MODELING Design and conduct a simulation using a random number generator. Then report the results using appropriate numerical and graphical summaries.

9. **BASEBALL** According to a baseball player's on-base percentages, he gets a single 60% of the time, a double 25% of the time, a triple 10% of the time, and a home run 5% of the time.

SOLUTION:

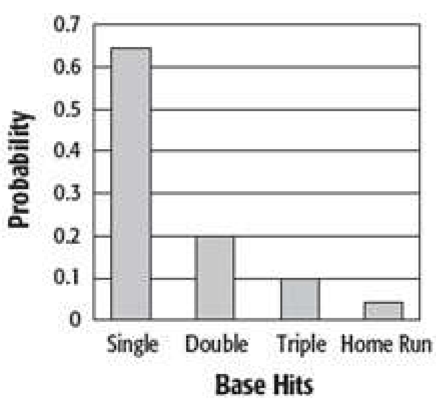
Use a random number generator.

Since the values are given in percents, we could generate the integers 1-100 and let 1-60 represent a single, 61-85 represent a double, 86-95 represent a triple, and 96-100 represent a home run. Simply use the first 60 to represent 60%, the next 25, or 61-85, to represent 25%, and so on. We could also divide all of these values by 5 and generate the integers 1-20.

Do 20 trials and record the results in a frequency table.

Outcome	Frequency
single	13
double	4
triple	2
home run	1
Total	20

Divide each frequency by 20 to get the corresponding probability.



The probability of the baseball player hitting a single is 0.65, a double is 0.2, a triple is 0.1, and a home run is 0.05.

11. **TRANSPORTATION** A car dealership's analysis indicated that 35% of the customers purchased a blue car, 30% purchased a red car, 15% purchased a white car, 15% purchased a black car, and 5% purchased any other color.

SOLUTION:

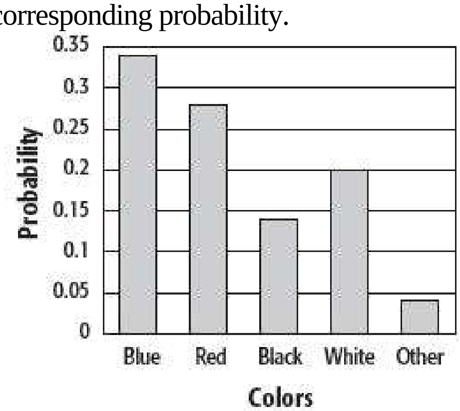
Use a random number generator.

Since the values are given in multiples of 5%, we could generate the integers 1-20 and let 1 - 7 represents blue, 8 -13 represents red, 14 - 16 represents white, 17 - 19 represents black, and 20 represents all other colors.

Do 50 trials and record the results in a frequency table.

Outcome	Frequency
blue	17
red	14
black	7
white	10
other	2
Total	50

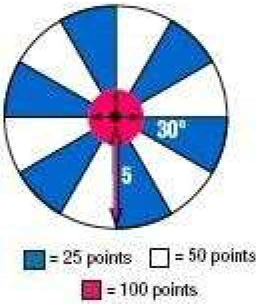
Divide each frequency by 50 to get the corresponding probability.



The probability of a customer buying a blue car is 0.34, buying a red car is 0.28, buying a black car is 0.14, buying a white car is 0.2, and any other color is 0.04.

DARTBOARDS The dimensions of each dartboard below are given in inches. There is only one shot per game. Calculate the expected value of each dart game. Then design a

simulation to estimate each game's average value. Compare the average and expected values.

13. 

SOLUTION:

Find the geometric probability of red, blue and white regions. The area of the big circle is 25π. The area of the red circle is 0.25π.

$$P(\text{red}) = \frac{\text{area(red)}}{\text{area(circle)}} = \frac{0.25\pi}{25\pi} = 0.01$$

$$P(\text{white}) = \frac{\left(\frac{\text{area(circle)} - \text{area(red)}}{2}\right)}{\text{area(circle)}} = \frac{\left(\frac{25\pi - 0.25\pi}{2}\right)}{25\pi} = \frac{12.375\pi}{25\pi} = 0.495$$

$$P(\text{blue}) = \frac{\left(\frac{\text{area(circle)} - \text{area(red)}}{2}\right)}{\text{area(circle)}} = \frac{\left(\frac{25\pi - 0.25\pi}{2}\right)}{25\pi} = \frac{12.375\pi}{25\pi} = 0.495$$

$$E = P(\text{red}) \times \text{value} + P(\text{white}) \times \text{value} + P(\text{blue}) \times \text{value}$$
$$E = (0.01 \times 100) + (0.495 \times 50) + (0.495 \times 25) \approx 38.1$$

Now run a simulation and calculate the average value.

Sample answer:

Outcome	Frequency
red	0
blue	29
white	21
total	50

$$\text{average} = \left(100 \times \frac{0}{50}\right) + \left(25 \times \frac{29}{50}\right) + \left(50 \times \frac{21}{50}\right) = 35.5$$

The expected value is greater than the average value.

15. **CARDS** You are playing a team card game where a team can get 0 points, 1 point, or 3 points for a hand. The probability of your team getting 1 point for a hand is 60% and of getting 3 points for a hand is 5%.

a. Calculate your team's expected value for a hand.

b. Design a simulation and estimate your team's average value per hand.

c. Compare the values for parts a and b.

SOLUTION:

a. The expected value is the sum of the product of each point value and its corresponding probability.

$$\begin{aligned} E &= \text{sum}(\text{point value} \times \text{probability}) \\ &= 0 + 1 \cdot \frac{60}{100} + 3 \cdot \frac{5}{100} \\ &= 0 + 0.6 + 0.15 \\ &= 0.75 \end{aligned}$$

b. Sample answer: Use a random number generator to generate integers 1 through 20, where 1–7 represents 0 points, 8–19 represents 1 point, and 20 represents 3 points. Do 50 trials and record the results in a frequency table.

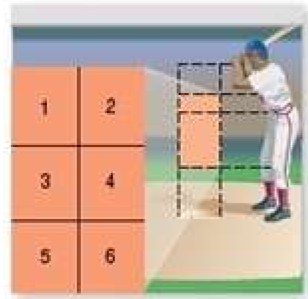
Outcome	Frequency
0	16
1	32
3	2

The average value is the sum of the product of each point value and its corresponding frequency.

$$\begin{aligned} \text{average} &= 0 \cdot \frac{16}{50} + 1 \cdot \frac{32}{50} + 3 \cdot \frac{2}{50} \\ &= 0 + 0.64 + 0.12 \\ &= 0.76 \end{aligned}$$

c. Sample answer: The two values are almost equal.

17. **BASEBALL** Of his pitches thrown for strikes, a baseball pitcher wants to track which areas of the strike zone have a higher probability. He divides the strike zone into six congruent boxes as shown.



a. If a strike is equally likely to hit each box, what is the probability that he will throw a strike in each box?

b. Design a simulation to estimate the probability of a strike being thrown in each box.

c. Compare the values for parts a and b.

SOLUTION:

a. Total number of congruent boxes = 6

There is a $\frac{1}{6}$ or 16.7% probability of throwing a strike in each box.

b. Sample answer: Since there are six equal outcomes, a die with the numbers 1 through 6 can be rolled to simulate each strike that is thrown. Roll the die 100 times and record the number on the die to indicate in which of the six boxes each strike was thrown. Record the percent of times a strike is thrown in each area.

Strike Area	Accuracy (%)
1	15
2	17
3	19
4	22
5	19
6	8
Total	100

c. Sample answer: Some of the values are higher or lower, but most are very close to 16.7%.

19. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate expected value.

a. CONCRETE Roll two dice 20 times and record the sum of each roll.

b. NUMERICAL Use the random number generator on a calculator to generate 20 pairs of integers between 1 and 6. Record the sum of each pair.

c. TABULAR Copy and complete the table below using your results from parts a and b.

Trial	Sum of Die Roll	Sum of Output from Random Number Generator
1		
2		
...		
20		

d. GRAPHICAL Use a bar graph to graph the number of times each possible sum occurred in the first 5 rolls. Repeat the process for the first 10 rolls and then all 20 outcomes.

e. VERBAL How does the shape of the bar graph change with each additional trial?

f. GRAPHICAL Graph the number of times each possible sum occurred with the random number generator as a histogram.

g. VERBAL How do the graphs of the die trial and the random number trial compare?

h. ANALYTICAL Based on the graphs, what do you think the expected value of each experiment would be? Explain your reasoning.

SOLUTION:

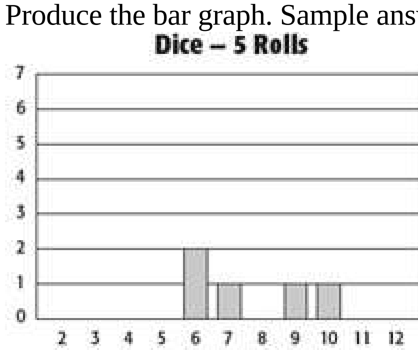
a. Sample answer: 9, 10, 6, 6, 7, 9, 5, 9, 7, 6, 5, 7, 3, 9, 7, 6, 7, 8, 7

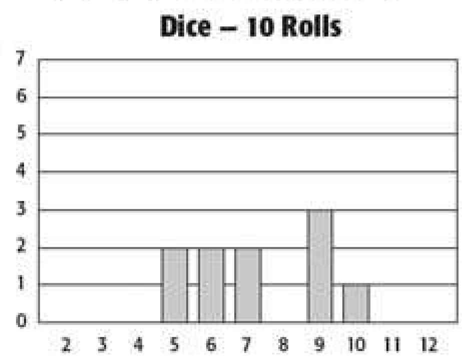
b. Sample answer: 4, 10, 5, 10, 6, 7, 12, 3, 7, 4, 7, 9, 3, 6, 4, 11, 5, 7, 5, 3

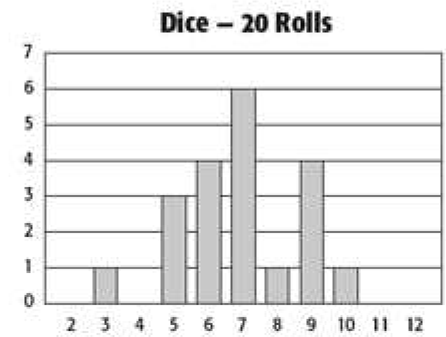
c. Transfer the values from parts a and b.
Sample answer:

Trial	Sum of Die Roll	Sum of Output from Random Number Generator
1	9	4
2	10	10
3	6	5
4	6	10
5	7	6
6	9	7
7	5	12
8	9	3
9	5	7
10	7	4
11	6	7
12	5	9
13	7	3
14	3	6
15	9	4
16	7	11
17	6	5
18	7	7
19	8	5
20	7	3

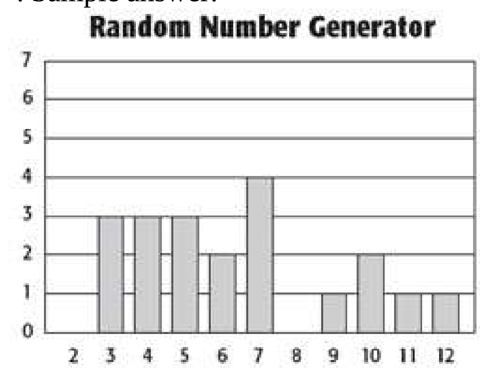
d. Enter the data values into L1 of your calculator. Produce the bar graph. Sample answer:







e. Sample answer: The bar graph has more data points at the middle sums as more trials are added.
f. Sample answer:



g. Sample answer: They both have the most data points at the middle sums.
h. Sample answer: The expected value in both experiments is 7 because it is the sum that occurs most frequently.

21. **REASONING** Can tossing a coin *sometimes*, *always*, or *never* be used to simulate an experiment with two possible outcomes? Explain.

SOLUTION:

Sometimes; sample answer: Flipping a coin can be used to simulate and experiment with two possible outcomes when both of the outcomes are equally likely. If the probabilities of the occurrence of the two outcomes are different, flipping a coin is not an appropriate simulation.

For example, flipping a coin can be used to simulate who will get the ball first at the beginning of a football game, while flipping a coin cannot be used to simulate who will win the game.

23. **REASONING** When designing a simulation where darts are thrown at targets, what assumptions need to be made and why are they needed?

SOLUTION:

Think about throwing darts. How would this be different than flipping a coin?

Sample answer: We assume that the object lands within the target area, and that it is equally likely that the object will land anywhere in the region. These are needed because in the real world it will not be equally likely to land anywhere in the region.

25. **WRITING IN MATH** How is expected value different from probability?

SOLUTION:

Consider an event such as rolling a die. Think of different probabilities and the expected value and compare the terms.

The probability of rolling any number is $\frac{1}{6}$. The probability of rolling an even number is the same as the probability of rolling an odd number. We also know that their sum must be 1, so the probability of rolling an even number is $\frac{1}{2}$.

The expected outcome of rolling a die is the sum of all possible outcomes multiplied by their probability:

$$E(\text{Rolling a die}) = \frac{1}{6}(1) + \frac{1}{6}(2) + \frac{1}{6}(3) + \frac{1}{6}(4) + \frac{1}{6}(5) + \frac{1}{6}(6)$$

$$\begin{aligned} E(\text{Rolling a die}) &= \frac{1}{6}(1) + \frac{1}{6}(2) + \frac{1}{6}(3) + \frac{1}{6}(4) + \frac{1}{6}(5) + \frac{1}{6}(6) \\ &= \frac{1}{6}(1+2+3+4+5+6) \\ &= \frac{1}{6}(21) \\ &= 3.5 \end{aligned}$$

From this we can see that the probability of any given outcome is always less than or equal to 1, with the sum of probabilities for all possible outcomes equal to 1. The expected value can be greater than 1, and is the value we expect to get given the probabilities of each outcome.

27. **ALGEBRA** Paul collects comic books. He has 20 books in his collection, and he adds 3 per month. In how many months will he have a total of 44 books in his collection?

F 5
G 6
H 8
J 15

SOLUTION:

Let x be the number of months required to collect 44 books.

$$\begin{aligned} 20 + 3x &= 44 \\ 3x &= 24 \\ x &= 8 \end{aligned}$$

The correct choice is H.

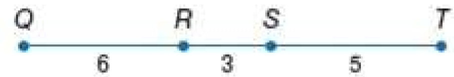
29. **SAT/ACT** If a jar contains 150 peanuts and 60 cashews, what is the approximate probability that a nut selected from the jar at random will be a cashew?

A 0.25
B 0.29
C 0.33
D 0.4
E 0.71

SOLUTION:

There are 60 outcomes in which a cashew can be chosen and 150 total cashews. The probability of selecting a cashew is 60 ÷ 150 = 0.33.

Point X is chosen at random on $\overline{QT}$. Find the probability of each event.

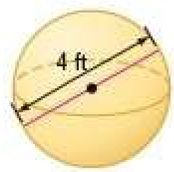


31. $P(X \text{ is on } \overline{RT})$

SOLUTION:

$$\begin{aligned} P(X \text{ is on } \overline{RT}) &= \frac{\text{length of } RT}{\text{length of } QT} \\ &= \frac{3+5}{6+3+5} \\ &= \frac{8}{14} \\ &= \frac{4}{7} \end{aligned}$$

Find the surface area of each figure. Round to the nearest tenth.

33. 

SOLUTION:

The surface area S of a sphere is $S = 4\pi r^2$, where r is the radius. The radius is 2 ft.

$$\begin{aligned} S &= 4\pi(2)^2 \\ &\approx 50.3 \text{ ft}^2 \end{aligned}$$

35. 

SOLUTION:

The surface area S of a sphere is $S = 4\pi r^2$, where r is the radius.

$$\begin{aligned} S &= 4\pi(9)^2 \\ &\approx 1017.9 \text{ in}^2 \end{aligned}$$