

AP* Calculus Free-response Question Type Analysis and Notes

Revised to include the 2013 Exam

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General note: AP Questions often test several diverse ideas or concepts in the same question. The type names are meant only as a guide and may refer to the form of the question, what it looks like at a glance. Within each type various ideas and concepts may and often are tested.

Many (all) of these ideas may also be tested on multiple-choice questions

Tables listing type by years and question number follow Topic 9 (page 7)

Topic 1: Area – Volume Question

Given equations that define a region in the plane students are asked to find its area and the volume of the solid formed when the region is revolved around a line or used as a base of a solid with regular cross-sections. This standard application of the integral has appeared every year since 1969 on the AB exam and all but one year on the BC exam.

If this appears on the calculator active section: It is expected that the definite integrals will be evaluated on a calculator. Students should write the definite integral with limits on their paper and put its value after it. It is *not* required to give the antiderivative and if students give an incorrect antiderivative they will lose credit even if the final answer is (somehow) correct.

There is a calculator program available that will give the set-up and not just the answer so recently this question has been on the no calculator section. (The good news is that the integrals will be easy or they will be set-up but do not integrate questions.)

What students should know how to do:

- Find the intersection(s) of the graphs and use them as limits of integration (calculator equation solving). Write the equation followed by the solution; showing work is not required. Usually no credit until the solution is used in context.
- Find the area of the region between the graph and the x -axis or between two graphs.
- Find the volume when the region is revolved around a line, not necessarily an axis, by the disk/washer method. (Shell method is *never necessary* but is eligible for full credit if properly used).
- Find the volume of a solid with regular cross-sections whose base is the region between the curves. But see 2009 AB 4(b)
- Find the equation of a vertical line that divides the region in half (area or volume). This involves setting up and solving an integral equation where the limit is the variable for which the equation is solved.

- For BC only – find the area of a region bounded by polar curves.

$$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} (r(\theta))^2 d\theta$$

Topic 2: Particle moving on a line

These questions may give the position equation, the velocity equation or the acceleration equation along with an initial condition. Students may be asked about the motion of the particle: its direction, when it changes direction, its maximum position in one direction (farthest left or right) etc. Speed, the absolute value of velocity, is also a common topic.

The particle may be a “particle,” a person, car, etc. The position, velocity or acceleration may be given as an equation, a graph or a table. There are a lot of different things students may be asked to find. While particles don’t really move in this way, the question is a versatile way to test a variety of calculus concepts.

What students should know how to do (AB and BC):

- Initial value differential equation problems: given the velocity or acceleration with initial condition(s) find the position or velocity.
- Distinguish between position at some time (displacement) and the total distance traveled during the time.
 - The total distance traveled is the definite integral of the absolute value of the rate of change (velocity): $\int_a^b |v(t)| dt$
 - The net distance (displacement) is the definite integral of the rate of change (velocity): $\int_a^b v(t) dt$
 - The final position is the initial position plus the definite integral of the rate of change from $x = a$ to $x = t$: $s(t) = s(a) + \int_a^t v(x) dx$ Notice that this is an accumulation function equation.
- Find the speed at a particular time. The speed is the absolute value of the velocity.
- Find average speed, velocity, or acceleration
- Determine if the speed is increasing or decreasing.
 - If at some time, the velocity and acceleration have the *same* sign then the speed is increasing.
 - If they have *different* signs the speed is decreasing.
 - If the velocity graph is moving away from (towards) the t -axis the speed is increasing (decreasing).
- Use a difference quotient to approximate derivative
- Riemann sum approximations
- Units of measure
- Interpret meaning of a definite integral in context of the problem

Topic 3: Particle moving on a plane for BC (parametric/vector question).

On the BC exam particles often move in the plane. Their position is defined by two parametric equations $x = x(t)$ and $y = y(t)$ or the equivalent vector $\langle x(t), y(t) \rangle$. The velocity is the vector $\langle x'(t), y'(t) \rangle$ and the acceleration is the vector $\langle x''(t), y''(t) \rangle$. Any of these three may be given with initial condition(s) and student may be asked to find the others.

What students should know how to do:

- Initial value differential equation problems – given the velocity or acceleration with initial conditions, find the position and/or velocity.
- Find the speed at time t : $\text{speed} = \sqrt{(x'(t))^2 + (y'(t))^2}$
- Use the definite integral for arc length to find the distance traveled. $\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} dt$
- Vectors are given in ordered pair form; answers may be in ordered pairs form or \vec{i} - \vec{j} form using parentheses () or pointed brackets $\langle \rangle$.

Topic 4: Interpreting Graphs

There are a variety of question types here. Students may be given an equation of the function or its derivative and asked for the location of extreme values, intervals where the function is increasing or decreasing, concavity, etc. Students may be given the graph of the derivative and asked the same kinds of things. They may be asked to find the value of the integral given the graph but no equation.

This may be a particle motion problem where the velocity is given as a graph.

What students should know how to do:

- Reading information about the function from the graph of the derivative. This may be approached as a derivative techniques or antiderivative techniques.
- Find and justify extreme values (1st DT, 2nd DT, Closed interval test (aka. Candidates' test)).
- Find and justify points of inflection.
- Write an equation of tangent line
- Evaluate Riemann sums from graphs only.
- FTC: Evaluate integral from area of regions on the graph.
- FTC: Realize that if $g(x) = g(a) + \int_a^x f(t) dt$, then $g'(x) = f(x)$
- "Family of functions": functions with a parameter;
- Functions defined by other functions.

Topic 5: Accumulation & Rates

The integral of a rate of change gives the amount of change (FTC). The general form of the equation is $f(x) = f(x_0) + \int_{x_0}^x f'(t) dt$, $x = x_0$ is the initial time, and $f(x_0)$ is the initial value. Since this is one of the main interpretations of the definite integral the concept may come up in a variety of situations.

What students should know how to do?

- Understand the question. It is often not necessary to do as much computation as it seems at first.
- The FTC may help differentiating F .
- Often these problems contain a lot of writing; be ready to read and apply; recognize that rate = derivative.
- Recognize a rate from the units given without the words “rate” or “derivative.”
- Explain the meaning of a derivative or definite integral or its value in terms of the context of the problem.
- In-out problems: 2 rates of change work together but in opposite directions.
- Max/min and inc/dec analysis.
- Explain the meaning of a definite integral in context. The explanation should include (1) what the integral gives, (2) the units and (3) an accounting of the limits of integration.

Topic 6: Problems from information given in tables

Tables may be used to test a variety of ideas in calculus including analysis of functions, accumulation, position-velocity-acceleration, *et al.*

What students should be able to do:

- Approximate the derivative using a difference quotient.
- Use Riemann sums (left, right, midpoint) or a trapezoidal approximation to approximate the value of a definite integral using values in the table (typically with uneven subintervals). (Trapezoidal Rule, *per se*, is not required.)
- Average value and the MVT may appear
- Questions about the Rolle’s theorem, MVT, IVT, etc.

Do not: Use a calculator to find a regression equation and then use that to answer parts of the question. (While finding them is perfectly good mathematics, regression equations are not one of the four things students may do with their calculator and give only an approximation of our function.)

Topic 7: Differential Equation Questions

Differential equations are tested every year. The actual solving of the differential equation is usually the main part of the problem, but it is accompanied by a question about its slope field or a tangent line approximation of some sort or something related. BC students may also be asked to approximate using Euler's Method. Large parts of the BC questions are often suitable for AB students and contribute to the AB subscore of the BC exam.

What students should be able to do:

- Find the *general solution* of a differential equation using the method of separation of variables (this is the *only* method tested).
- Find a *particular solution* using the initial condition to evaluate the constant of integration – initial value problem (IVP)
- Understand that proposed solution of a differential equation is a function (not a number) and if it and its derivative are substituted into the given differential equation the resulting equation is true. This may be part of doing the problem even if solving the differential equation is not required (see 2002 BC 5(c))
- Growth-decay problems.
- Draw a slope field by hand.
- Sketch a *particular solution* on a (given) slope field.
- Interpret a slope field.
- For BC only: Use Euler's Method to approximate a solution.
- For BC only: use the method of partial fractions to find the antiderivative after separating the variables.
- For BC only: understand the logistic growth model, its asymptotes, meaning, etc. The exams have never asked students to actually solve a logistic equation IVP.

Topic 8: Power Series (BC only)

Since some graphing calculator can produce Taylor Polynomials, this question appears on the no calculator allowed section. (Questions from 1995 – 1999 before the FR sections was split do not have anything a calculator *could* do. They are interesting and clever and worth looking at.)

What students should be able to do:

- Find the Taylor (or Maclaurin) polynomial or series for a given function – usually 4 terms and the general term). This may be done by finding the various derivatives, or any other method such as substitution into a known series, long division, the formula for the sum of an infinite geometric series, integration, differentiation, etc.

- Know from memory the Maclaurin series for $\sin(x)$, $\cos(x)$, e^x , and $\frac{1}{1-x}$.
- Find related series by substitution, differentiation, integration or by adapting one of those above.
- Find the radius of convergence (usually by using the Ratio test, or from a geometric series).
- Find the interval of convergence using the radius and checking the endpoints separately.
- Work with geometric series.
- Use the convergence test separately and when checking the endpoints.
- Find a high-order derivative from the coefficient of a term.
- Estimate the error bound of a Taylor or Maclaurin polynomial by using *alternating series error bound* or the *Lagrange error bound*.
- *Do not* claim that a function is equal to (=) its Taylor or Maclaurin polynomial; it is only approximately equal (\approx). This could cost a point.

Topic 9: Other Topics tested occasionally

Implicitly defined relations and implicit differentiation

These questions may ask students to find the first or second derivative of an implicitly defined relation. Often the derivative is given and students are required to show that it is correct. (This is because without the correct derivative the rest of the question cannot be done.)

What students should know how to do?

- Know how to find the first derivative of an implicit relation using the product rule, quotient rule, the chain rule, etc.
- Know how to find the second derivative, including substituting for the first derivative.
- Know how to evaluate the first and second derivative by substituting both coordinates of the point. (Note: the substitution is often easier if done before solving for dy/dx .)
- Analyze the derivative to determine where the relation has horizontal and/or vertical tangents.
- Work with lines tangent to the relation.

Related Rates

Derivatives are rates and when more than one variable is involved the relationships among the rates can be found by differentiating with respect to time. The time variable may not appear in the equations.

What students should know how to do:

- Know how to differentiate using the product, quotient and chain rules.
- Set up and solve related rate problems.
- Interpret the answer.

A Guide to the Free-response Questions

- GCA = Graphing calculator allowed – yes or no; GCA indicates that the question is in the graphing calculator allowed section of the exam; it does *not* indicate that a graphing calculator should or must be used.
- Comments are highlights, not a complete description
- Since often several topics are tested in the same question, some questions are listed under more than one topic.

Topic 1: Area, volume AB & BC		
Year & Question	GCA	Comments
1998 AB1	Yes	Line divides area or volume into equal parts
1998 BC1	Yes	Line divides volume into equal parts. Suitable for AB
1998 AB2-BC2	Yes	(c) find other lane with same volume
2000 AB1-BC1	Yes	Find intersection, Regular cross-section
2001 AB1	Yes	Find intersection; 2 integrals needed.
2002 AB1- BC1	Yes	Includes max/min
2002 AB1 B	Yes	Volume of revolution and regular cross section
2002 BC3 B	Yes	Curve length Parts a and b suitable for AB
2003 AB1-BC1	Yes	Regular cross-section
2003 BC3	Yes	Area and polar graphs, polar area
2003 AB1 B	Yes	Also write tangent line equation
2003 BC2 B	Yes	Same area with dy and dx ; also polar area
2004 AB2-BC2	Yes	Regular cross-section
2004 AB1 B	Yes	Volume of revolution horizontal and vertical lines
2004 AB6-BC6 B	No	Area in terms of parameter.

2004 BC5 B	No	Average value (x2), Improper integral
2005 AB1-BC1	Yes	Curves define 2 regions.
2005 AB1 B	Yes	Regular cross-section (semi-circles)
2005 BC6 B	No	Improper integrals
2006 AB1-BC1	Yes	Rotations horizontal and vertical lines
2006 AB1-BC1 B	Yes	Area, volume rotation, area.
2007 AB1-BC1	Yes	Area, volume rotation, no graph given
2007 AB1-BC1 B	Yes	Area, volume rotation
2008 AB1-BC1	Yes	Area, volume, regular cross section, (d) variable height
2008 AB1 B	Yes	Area, volume, regular cross-section
2008 AB3-BC3 B	Yes	From table: Area, Trap rule, flow volume.
2008 BC 4 B	No	Area, volume perimeter
2009 AB 4	No	Area, cross section square and area given
2009 AB 4 B	No*	Area, square cross section, revolve
2009 BC 1 B	Yes	Area, semi-circle cross section perimeter
2010 AB 4 – BC 4	No	Area, revolve, square cross section
2010 AB 1–BC 1 B	Yes	Area, volume of revolution, square cross section
2010 BC 4 B	No	Equation stem, analyze graph, max/min, asymptote not mentioned but needs to be considered. Area with improper integral.
2011 AB 3	No	Tangent line, area, volume of rotation
2011 BC 3	No	Perimeter, volume of revolution, related rate
2011 AB 3 BC 3 B	No	Area, rectangular cross-section w/ variable height, tangent line perpendicular to other graph.
2012 AB 2	Yes	Horizontal rectangles, square cross-section, divide area in half with horizontal line.
2013 AB 4	No	Area, volume from rotation. Volume square cross-section

Topic 2: Motion on a line for AB and BC		
Year & Question	GCA	Comments
1998 AB3	Yes	Given graph & table: difference quotient, Riemann sum
1999 AB 1	Yes	Given v: analyze motion, find a, s and total distance
2000 AB2-BC2	Yes	Given graph & equation
2001 AB3-BC3	Yes	Given a graph: analyze v, max/min, FTC
2002 AB3	Yes	Given v equation: Speed/velocity, Speed increasing?
2002 AB3 B	Yes	Given v equation: Sketch graph, analyze motion
2003 AB2	Yes	Given v graph: Speed increasing? Analyze motion
2003 AB4 B	No	Given v graph: Speed increasing? Analyze motion
2004 AB3	Yes	Given v graph: Speed increasing? Analyze motion
2004 AB3-BC3 B	Yes	Given table of v: explain integral, MVT, average v
2005 AB5-BC5	No	Given v graph: distance, a, average rate of change
2005 AB3 B	Yes	Given v equation: analyze motion.
2006 AB4	No	Table & equation, average, Riemann sum,
2006 AB6 B	No	From table, distance, FTC, analyze motion
2007 AB 4	No	Also find and use x''
2007 AB 2 B	Yes	Acceleration, distance position
2008 AB2-BC2 B	Yes	Given "speed": Accumulation, rate of change
2009 AB1 BC 1	Yes	Graph of v, find distance, describe trip
2009 AB 6 B	No	Explain integral, Trap rule, acceleration.
2010 AB-4 BC-4 B	No	Analyze motion, distance, accumulation, graph stem
2010 AB 6 B	No	2 particles, analyze motion, speeding up?
2011 AB 1	Yes	Speed, average velocity, distance, turning point
2011 AB 5 BC 5 B	No	Table of v, find acceleration, interpret integral, left Riemann sum, MVT, related rate
2012 AB 6	No	Direction, total distance, acceleration and speed, final position
2013 AB 2	Yes	Equation stem, speed inc/dec, find position, change direction

Topic 3: Particle motion in the plane; parametric equation, vectors for BC		
Year & Question	GCA	Comments
1998 BC6	Yes	Rectangular and parametric together, position, speed.
1999 BC1	Yes	Given position: graph, max/min, acceleration, speed
2000 BC4	No	Given v: acceleration, position, slope, limit as $t \rightarrow \infty$
2001 BC1	No	Given v: Distance, position by accumulation, speed
2002 BC3	Yes	Slopes, distance, max/min
2002 BC1 B	Yes	Given position: draw graph, speed, distance
2003 BC2	Yes	Given x' but not y' : an atypical problem
2003 BC4 B	No	Given position: when moving horizontally and vertically
2004 BC3	Yes	Given position: tangent lines, speed
2004 BC1 B	Yes	Given velocity vector
2005 BC1 B	Yes	Equation of tangent line; when is particle at rest?
2006 AB4-BC4	No	Table & equation, average, Riemann sum,
2006 BC3	Yes	Parametric velocity, tangent line, limits, improper integral
2006 BC2 B	Yes	Tangent line, acceleration speed, distance traveled, explain.
2007 BC 2 B	Yes	Speed, distance, position, tangent line, acceleration
2008 AB4-BC4	No	From graph: Particle motion, inc/dec, speed
2008 BC 1 B	Yes	Given velocity: acceleration, position, speed distance
2009 BC 3	Yes	Diver, max, total distance,
2010 BC 3	Yes	Speed, distance, interpret slopes, accumulation (x2)
2010 BC 2 B	Yes	Vertical tangent, tangent line, speed, acceleration.
2011 BC 1	Yes	Speed, slope, position, total distance traveled
2012 BC 2	Yes	Slope, position, speed, acceleration, total distance

Topic 4: Information from graph or about the graph, family of functions, f, f', f''		
Year & Question	GCA	Comments
1998 AB2- BC2	Yes	Limits, max/min, “family”
1999 AB4	Yes	From equation, 2-DT, function defined by other function
1999 AB5-BC5	Yes	From graph: Max/min, inc/dec, tangent line,
2000 AB3	Yes	From graph: f, f', f'' , inc/dec, POI
2001 AB3-BC3	Yes	Particle motion (q.v.) setting
2001 AB4-BC4	No	From derivative equation: Max/min, tangent line, concavity
2002 AB1 BC1 (c)	Yes	From Equation; Includes max/min area/volume
2002 AB4-BC4	No	Given f' graph: inc/dec, concavity, draw graph f
2002 BC5 (d)	No	Slope field w/ draw solution, Euler, max/min
2002 AB2 B	Yes	Given equation of f' : Max/min, accumulation
2002 AB4-BC4 B	No	Given graph f : values, inc/dec, concavity, Trap rule.
2003 AB3	Yes	Given table and graph: Diff quotient, Riemann sum, max/min, average value, accumulation
2003 AB4-BC4	No	Given graph f' : inc/dec, tangent line, FTC by area.
2003 AB5 B	No	Given graph f : values, average values, MVT, POI
2004 AB5	No	Given graph f' : values, FTC, max/min, POI
2004 AB2 B	Yes	From equation; Accumulation setting, max/min, inc/dec.
2004 AB4-BC4 B	No	Given graph f' : inc/dec, product rule
2005 AB4	No	Tabular information about f, f', f'' , FTC, max/min, draw graph, POI
2005 AB4-BC4 B	No	Given graph f' : values, FTC from graph area, inc/dec.
2006 AB3	Yes	Periodic function, accumulation, tangent line
2006 AB 2 B	Yes	Concavity, max/min, tangent line
2006 AB4-BC4 B	Yes	From graph, derivative, area
2007 AB2-BC2	Yes	Rate in/out from equations and graph
2007 BC4	No	Tangent, concavity
2007 AB 4 B	No	Given graph of f' ; max, concavity, POI, area

2007 AB 6	No	Find f' and f'' , max/min, POI from equation
2008 AB4-BC4	No	From graph: Particle motion, inc/dec, speed
2008 AB 6	NO	Write tangent line, critical points, POI, limit
2008 BC 5	NO	Given derivative: critical points, inc/dec, IVP
2008 AB5-BC5 B	No	POI, max/min, average rate of change, MVT
2009 AB1 BC1	Yes	Particle motion
2009 AB 6	No	Values, point of inflection max/min
2009 AB3 BC 3 B	Yes	Differentiability, roc, average roc, concavity, MVT
2009 AB5 BC 5 B	No	Tangent, max/min, average roc
2010 AB 3	Yes	Accumulation, inc/dec max/min
2010 AB 5	No	$f - f' - f''$, inflection points, max/min
2010 AB 2 B	Yes	$f - f' - f''$, horizontal tangent, concavity, tangent line from equation
2010 AB-4 BC-4 B	No	Analyze motion, distance, accumulation, graph stem
2010 BC 4 B	No	Equation stem, analyze graph, max/min, asymptote not mentioned but needs to be considered. Area with improper integral.
2011 AB 4	No	Max/min, POI, average rate of change, MVT
2011 AB 4 B	No	Max,min, concavity, IVP or accumulation
2011 AB 6	No	Accumulation, critical point, FTC
2011 AB 4 B	No	Given graph of f work with related graphs: Average value, FTC, concavity, inc/dec, arc length
2012 AB 3 - BC 3	No	Graph stem, $f - f' - f''$, extreme values, POI
2013 AB 4 - BC 4	No	Graph stem, max/min, concavity, chain rule

Topic 5: Accumulation & Rates		
Year & Question	GCA	Comments
1998 AB5-BC5	Yes	Draw graph, average value, accumulation
1999 AB3-BC3	Yes	From table: Riemann sum, Rolle's theorem – MVT, average value
2000 AB4	No	2 solution methods: accumulation, max/min.
2002 AB2-BC2	Yes	"Amusement Park:" In-out, amounts, max/min, values
2002 AB2-BC2 B	Yes	Pollutants: max/min, amounts, accumulation
2003 AB3	Yes	From graph and table: difference quotient, max/min, Riemann sum, interpret integrals.
2003 AB2 B	Yes	Heating oil: In-out, inc/dec, accumulation, max/min
2004 AB1-BC1	Yes	Traffic flow: average value, average rate of change, accumulation, inc/dec.
2004 AB2 B	Yes	Mosquitoes: values, inc/dec, accumulation, max/min
2005 AB2	Yes	"Sandy Beach:" In-out, amounts, max/min, FTC
2005 AB2- BC2 B	Yes	Water tank: In-out, amount, max/min, inc/dec
2006 AB2-BC2	Yes	"Thomasville:" Average value and application
2007 AB2-BC2	Yes	Water Tank; rate in/out
2007 AB3-BC3 B	Yes	Wind chill
2008 AB2-BC2	Yes	"Concert tickets" from table. Derivative, Trap Rule, max/min, accumulation
2008 AB 3	Yes	Related Rate, max/min, accumulation
2008 AB2-BC2 B	Yes	Given "speed": Accumulation, rate of change, Related Rate
2009 AB 2 BC 2	Yes	Rate and extensions
2009 AB 3	Yes	Cost and profit
2009 AB 1 B	Yes	Rates, related rates
2009 AB 2 B	Yes	Distance, interpret derivative,
2010 AB 1 – BC 1	Yes	Accumulation, in-out type, piecewise function
2010 AB 2 – BC 2	Yes	Difference quotient, Trap rule, explain, accumulation, max/min

2010 AB 3	Yes	Accumulation, from graph, graph analysis
2010 BC 3	Yes	Speed, distance, interpret slopes, accumulation (x2)
2011 AB 2 BC 2	Yes	Table, average value, Trap sum, accumulation
2011 AB 1 BC 1 B	Yes	Accumulation, average rate of change, related rate, IVT
2011 AB 2 B	Yes	Continuity, average rate of change, accumulation.
2012 AB 1	Yes	Table stem, average ROC, interpret integrals, left-Riemann sum, over/under estimate?, Accumulation from equation.
2013 AB 1 – BC 1	Yes	Equation stem, rate, amount, extreme value, inc/dec
2013 AB 3 (d)	No	Total amount

Topic 6: Table		
Year & Question	GCA	Comments
1998 AB3	Yes	Given graph & table: difference quotient, Riemann sum
1999 AB3-BC3	Yes	From table: Riemann sum, Rolle – MVT, average value
2001 AB2-BC2	Yes	Difference quotient, Trap rule, explain, average value
2002 AB6	No	Definite integral, tangent line, MVT, interpret, limits
2003 AB3	Yes	From graph and table: difference quotient, max/min, Riemann sum, interpret integrals.
2003 AB3 B	Yes	Average value, Riemann sum, explain, MVT
2004 AB3-BC3 B	Yes	Given table of v : explain integral, MVT, average v
2005 AB3-BC3	Yes	Tabular information, FTC, max/min, draw graph, POI
2005 AB4	No	Tabular information about f, f', f'' , FTC, max/min, draw graph, POI
2006 AB4	No	Table & equation, particle motion: average, Riemann sum,
2006 AB6 B	No	From table, distance, FTC, analyze motion
2007 AB 3	Yes	IVT, MVT, FTC, tangent line to inverse
2007 AB5-BC5	No	Relate rate, Riemann sum, rate of change,
2008 AB2-BC2	Yes	Difference Quotient , Trap Rule, max/min, accumulation
2008 BC 3	Yes	Taylor polynomial, LaGrange error

2008 AB 4 B	No	FTC, Chain rule (integral), tangent line, max/min
2009 AB 5 BC 5	No	Derivative, integral, left Riemann sum, tangent line secant line and error analysis
2009 AB 6 B	No	Particle motion, explain integral, Trap rule, acceleration.
2010 AB 2 - BC 2	Yes	Difference quotient, Trap rule, explain, accumulation, max/min
2010 AB-3 BC-3 B	Yes	Midpoint Riemann sum, accumulation, Related rate
2011 AB 2 BC 2	Yes	Table, average value, Trap sum, accumulation
2011 AB 5/BC 5 B	No	Table of v, find acceleration, interpret integral, left Riemann sum, MVT, related rate
2012 AB 1	Yes	Table stem, average ROC, interpret integrals, left-Riemann sum, over/under estimate?, Accumulation from equation.
2012 BC 4	No	4 Approximations: Linear, midpoint-Riemann sum, Euler's method, Taylor polynomial
2013 AB 3 - BC 3	No	Approx. derivative, MVT, Midpoint sum, accumulation from equation

Topic 7: Differential Equations		
Year & Question	GCA	Comments
1998 AB4	Yes	IVP, equation of tangent line
1998 BC4	Yes	IVP field, Euler, solve
1999 BC6	Yes	Tangent line with approximation, Euler, FTC with "impossible integrand" in (c)
2000 AB6	No	IVP, find domain and range of solution.
2000 BC6	No	Slope field with analysis, IVP, range
2001 AB6	No	Second derivative by implicit differentiation of y' , IVP
2001 BC5	No	Improper integral, Euler, IVP
2002 BC5	No	Draw solution on given slope field, Euler, max/min, check solution, 2-DT,
2002 AB5-BC5 B	No	2-DT, IVP
2003 AB5-BC5	No	Related Rate, IVP
2003 BC6 (c)	No	From power series check solution of differential equation

2003 AB 6 B	No	Second derivative by implicit differentiation of y' , IVP
2004 AB6	No	Draw slope field, generalize, IVP
2004 BC5	No	Logistics Diff Eq, limits from y' and y , interpret, IVP
2004 AB5 B	No	Draw slope field, generalize, IVP
2005 AB6	No	Draw slope field, tangent line approximation, IVP
2005 BC4	No	Draw slope field and sketch solution on it; max/min, Euler, implicit differentiation for y'' and analyze.
2005 AB6 B	No	Draw slope field, tangent line, solve
2006 AB 5	No	Draw slope field, solve, domain
2006 BC 5	No	Implicit y'' , write Taylor Poly, Euler
2006 AB 5 B	No	Draw slope field, solve IVP
2006 BC5 B	No	Solve IVP, Analyze logistic DE w/o solving.
(2007 AB 4 part b)	No	Similar to some DE work.
2007 AB5 B	No	Slope field, no solution required, find coefficients,
2007 BC 5 B	No	Find coefficients, Euler,
2008 AB 5	No	Slope field, solve IVP, limit
2008 BC 6	No	Draw solution on slope field, IVP, Euler, Taylor, range
2008 BC 5	NO	Given derivative: critical points, inc/dec, IVP
2009 BC 4	No	Solve, Euler, Taylor polynomial
2010 AB 6	No	Tan line, tan line approximation, concavity, solve DEq
2010 BC 5	No	Euler's method, L'Hôpital's Rule, Solve DEq
2010 AB 5 B	No	Slope field, solve DEq
2010 BC 6 B	No	Ratio test, half-open interval, use to verify DEq solution
2011 AB 5 BC 5	No	Tangent line approx., solve (NB: Median score = 0)
2011 AB 4	No	IVP in part (d)
2012 AB 5	No	Interpret derivative at point, second derivative (implicit) and discuss graph, solve separable IVP
2013 AB 6	No	Tangent line approximation. Solve separable IVP
2013 BC 5	No	L'Hôpital's Rule, Euler's Method, solve separable IVP

Topic 8: Power Series BC only		
Year & Question	GCA	Comments
1998 BC3	Yes	Write Taylor, new series by substitution, and by integration, analyze. CAS no help
1999 BC4	Yes	Write Taylor series, Lagrange error bound, max/min CAS no help
2000 BC3	No	Write series, radius of convergence, Alternating series error bound
2001 BC6	No	Interval of convergence, limit, integrate, Geometric series sum
2002 BC6	No	Interval of convergence, differentiate.
2002 BC6	No	Substitute, series convergence
2003 BC6	No	Max/min 2-DT, Alternating series error; check solution of differential equation
2003 BC6 B	No	Write series, radius of convergence, interval of convergence
2004 BC6	No	Write series, Find high-order coefficient, Lagrange error bound, integrate
2004 BC2 B	Yes	Find derivatives, analyze, Lagrange error bound
2005 BC6	No	Write series, find general coefficient, interval of convergence
2005 BC3 B	Yes	Given $f^{(n)}$, max/min 2-DT, write series, find radius of convergence.
2006 BC 5	No	Implicit y'', write Taylor Poly, Euler
2006 BC 6	No	Interval w/ endpoints, y' and y'' from coefficients max/min
2006 BC6 B	No	Differentiate, integrate, alternating series error bound.
2007 BC 6	No	Write terms, find limit using series, estimate using series, alternating series test
2007 BC 6 B	No	Find terms, integrate, find coefficients, recognize know series.
2008 BC 3	Yes	From Table: Taylor polynomial, LaGrange error
2008 BC 6	No	Draw solution on slope field, IVP, Euler, Taylor, range
2008 BC 6 B	No	Maclaurin series, integrate, error
2009 BC 6	No	Taylor, Ratio test, points of inflection
2009 BC 6 B	No	Geometric, sum, integral, substitute.

2010 BC 6	NO	Write series (x2) max/min, Alternating series error bound.
2010 BC 6 B	No	Ratio test, half-open interval, use to verify DEq solution
2011 BC 6	NO	Sin(x) and cos(x) series, Lagrange error bound from graph.
2011 BC 6	No	Write general term, interval of convergence given radius, approximation, alternating series error bound.
2012 BC 4	No	Table stem: 4 Approximations: Linear, midpoint-Riemann sum, Euler's method, Taylor polynomial
2012 BC 6	No	Find interval of convergence (ratio test and endpoints), Alternating series error bound, write terms
2013	No	Find derivatives, coefficients, Taylor polynomial from composition.

Topic 9 Miscellaneous		
Year & Question	GCA	Comments
1998 AB6	Yes	Implicit diff, analyze implicit relation, tangent lines
1999 AB6	Yes	Related Rate
2000 AB5-BC5	No	Implicit diff, analyze implicit relation, tangent lines
2001 AB5	No	Unique graph problem, FTC
2002 AB5	No	Related Rate, units of measure
2002 AB6 B	No	Related Rate
2003 AB5-BC5	No	Related Rate w/ solve differential equation IVP
2003 AB6	No	Continuity, average value, parameters
2003 BC3	Yes	Area and polar graphs, polar area
B 2003 BC3	Yes	Includes polar area
2004 AB4-BC4	No	Implicit diff, analyze, second derivative
2004 AB6-BC6 B	No	Use integral with parameter
2005 BC2	Yes	Polar Graph, max/min, interpret $dr/d\theta$
2005 AB5-BC5 B	No	Implicit diff, analyze implicit relation
2006 AB6	No	Differentiation without functions.

2006 BC3	Yes	Parametric velocity, tangent line, limits, improper integral
2006 BC 5	No	Implicit y'' , write Taylor Poly, Euler
2006 AB3-BC3 B	Yes	Graph investigation
2007 BC 3	Yes	Polar Equation, area, interpret derivatives
2007 AB 3	Yes	IVT, MVT, FTC, tangent line to inverse
2007 AB 6 B	No	IVP, MVT, POI from generic function
2008 AB2	Yes	Related Rate, max/min, accumulation
2008 AB5-BC5 B	No	POI, max/min, average rate of change, MVT
2008 AB 2 B	Yes	Accumulation, Related Rate
2008 AB 6 B	No	Implicit differentiation, tangent, analyze
2009 AB 1 B	Yes	Related rate
2009 BC 4 B	No	Polar, area, derivative, tangent line
2010 AB-3 BC-3 B	Yes	Midpoint Riemann sum, accumulation, Related rate
2010 BC 4 B	No	Equation stem, analyze graph, max/min, asymptote not mentioned but needs to be considered. Area with improper integral.
2011 AB 6	No	Continuity, piecewise function, average value.
2011 BC 3	No	Perimeter, volume of revolution, related rate
2011 AB 1 BC 1 B	Yes	Accumulation, average rate of change, related rate, IVT
2011 AB 2 B	Yes	Continuity, average rate of change, accumulation.
2011 AB 5 BC 5 B	No	Table of v , find acceleration, interpret integral, left Riemann sum, MVT, related rate
2011 BC 2 B	Yes	Area, graph, find and interpret dy/dt
2012 AB 4	No	Tangent line, continuity of piecewise function, u-sub integration by hand,
2013 BC 2	Yes	Polar: area, analyze motion, find position and velocity vectors

MVT:1999 AB3, 2002 AB6, 2003B AB3, 2004B AB3, 2005 AB3, 2006B AB6, 2007B AB6, 2007 AB3, 2008 AB2, 2009B AB3/BC3, 2013 AB3/BC3 (b)