7-3 Perpendicular and Angle Bisectors

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I CAN… use perpendicular and angle bisectors to solve problems.

VOCABULARY

- equidistant

MODEL & DISCUSS

A new high school will be built for Brighton and Springfield. The location of the school must be the same distance from each middle school. The distance between the two middle schools is 18 miles.

A. Trace the points for the schools on a piece of paper. Locate a new point that is 12 mi from each school. Compare your point with other students. Is there more than one location for the new high school? Explain.

B. **Reason** Can you find locations for the new high school that are the same distance from each middle school no matter what the given distance? Explain.

ESSENTIAL QUESTION

What is the relationship between a segment and the points on its perpendicular bisector? Between an angle and the points on its bisector?

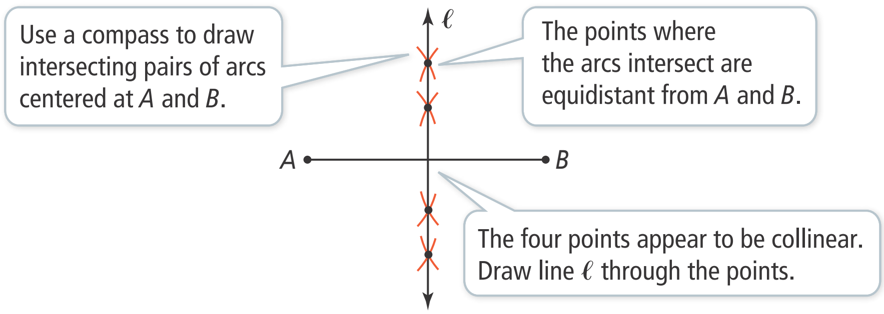
CONCEPTUAL UNDERSTANDING

EXAMPLE 1 Find Equidistant Points

How can you find points that are equidistant from the endpoints of $\overline{AB}$? What do you notice about these points and their relationship with $\overline{AB}$?

A point that is the same distance from two points is **equidistant** from the points.

COMMON ERROR
Be sure not to change the compass setting when drawing each pair of intersecting arcs from each endpoint.



The points that are equidistant from A and B appear to lie on line ℓ. Line ℓ appears to bisect and be perpendicular to $\overline{AB}$. You can use a ruler and a protractor to support this hypothesis.

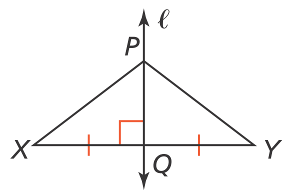
Try It! 1. Draw a pair of points, and find points that are equidistant from the two points. Draw a line through the set of points. Repeat this process for several pairs of points. What conjecture can you make about points that are the same distance from a given pair of points?

THEOREM 7-10 Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

PROOF: SEE EXAMPLE 2.

If...



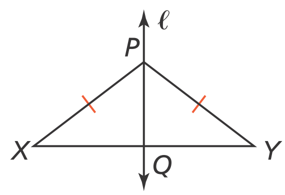
Then... $PX = PY$

THEOREM 7-11 Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

PROOF: SEE EXAMPLE 2 TRY IT.

If...



Then... $XQ = YQ$ and $\overleftrightarrow{PQ} \perp \overline{XY}$

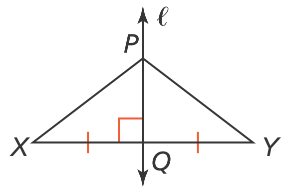
PROOF

EXAMPLE 2 Prove the Perpendicular Bisector Theorem

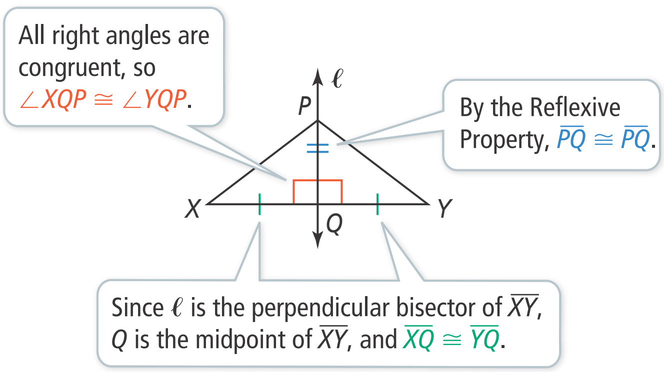
Prove the Perpendicular Bisector Theorem.

Given: ℓ is the perpendicular bisector of $\overline{XY}$.

Prove: $PX = PY$



Proof:



By SAS, $\triangle XQP \cong \triangle YQP$. Therefore, $\overline{PX} \cong \overline{PY}$ by CPCTC, so $PX = PY$.

STUDY TIP
Remember that if a line is a perpendicular bisector of a segment, you can conclude two things: the line is perpendicular to the segment, and it bisects the segment.

Try It! **2.** Prove the Converse of the Perpendicular Bisector Theorem.

APPLICATION

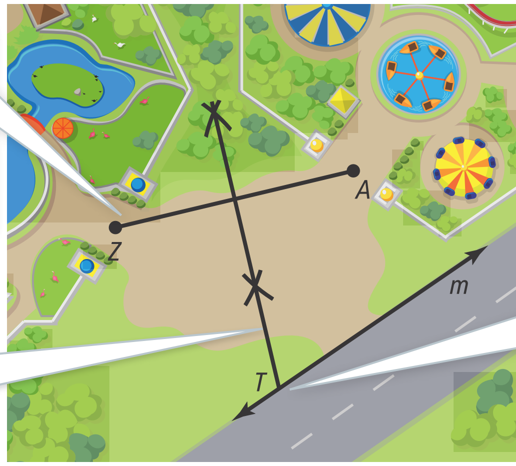
EXAMPLE 3 Use a Perpendicular Bisector

Mr. Lee wants to park his ice cream cart on Main Street so that he is equidistant from the entrances of the amusement park and the zoo. Where should Mr. Lee park? How can he determine where to park?

Mr. Lee can use the perpendicular bisector of the segment that connects the two entrances to find the location.

Step 1 Label the entrances of the amusement park and zoo as points *A* and *Z*, and draw line *m* for Main Street.

Step 2 Draw $\overline{AZ}$, and construct the perpendicular bisector.



Step 3 Mark point *T* where the perpendicular bisector and line *m* intersect.

STUDY TIP
You may need to extend a line to find the point where it intersects with another line.

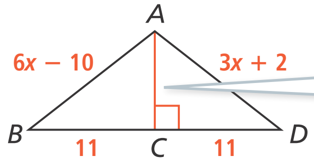
Mr. Lee should park his cart at point *T*, because it is equidistant from both entrances.

Try It! **3.** The entrances are 40 feet apart. Mr. Lee decides to move his cart off Main Street. How can you find where Mr. Lee should park if he must be 30 feet from both entrances?

EXAMPLE 4 Apply the Perpendicular Bisector Theorem

STUDY TIP
Look for relationships in the diagram to help you solve a problem. For example, a right angle marker tells you that two line segments are perpendicular.

What is the value of *AD*?



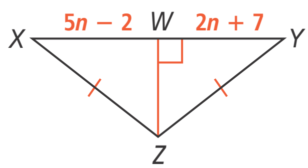
By definition, $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.

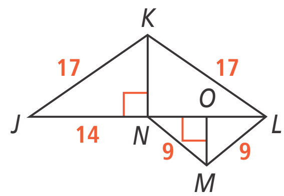
$$AB = AD$$
$$6x - 10 = 3x + 2$$
$$3x = 12$$
$$x = 4$$
$$AD = 3(4) + 2$$
$$AD = 14$$

The lengths are equal by the Perpendicular Bisector Theorem.

Evaluate the expression for *AD*.

Try It! **4. a.** What is the value of *WY*? **b.** What is the value of *OL*?





EXAMPLE 5 Find Equidistant Points from the Sides of an Angle

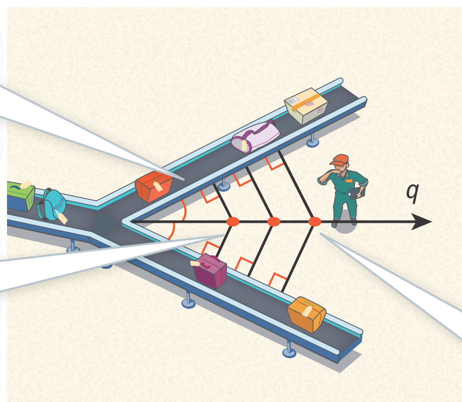
An airport baggage inspector needs to stand equidistant from two conveyor belts. How can the inspector determine where he should stand?

Use pairs of corresponding points on each conveyor belt that are the same distance away from the vertex of the angle. To be equidistant from the conveyor belts, a point must have the same distance from corresponding points.

USE APPROPRIATE TOOLS
Think about the tools you can use to make sure that segments are perpendicular. What tool would you use?

Draw the lines perpendicular from each pair of corresponding points.

The distance between a point and a line is the length of the segment perpendicular from the line to the point.



The points of intersection are equidistant from each belt and appear to be collinear.

Ray q appears to be the angle bisector. You can use a protractor to support this.

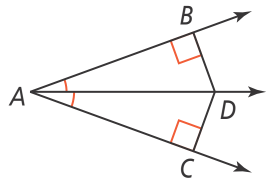
The inspector can determine where to stand by choosing a point on the angle bisector.

Try It! **5.** Consider two triangles that result from drawing perpendicular segments from where the inspector stands to the conveyor belts. How are the triangles related? Explain.

THEOREM 7-12 Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

If...



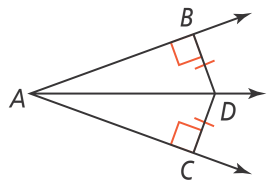
Then... $BD = CD$

PROOF: SEE EXERCISE 9.

THEOREM 7-13 Converse of the Angle Bisector Theorem

If a point is equidistant from two sides of an angle, then it is on the angle bisector.

If...



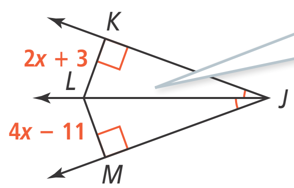
Then... $m\angle BAD = m\angle CAD$

PROOF: SEE EXERCISE 10.

EXAMPLE 6 Apply the Angle Bisector Theorem

STUDY TIP
To apply the Angle Bisector Theorem, be sure a diagram reflects the necessary conditions—angles are marked as congruent and right angles are marked to indicate that segments are perpendicular to the sides.

What is the value of *KL*?



$\overrightarrow{JL}$ is the angle bisector of $\angle KJM$ since $m\angle KJL = m\angle MJL$.

$KL = ML$

$2x + 3 = 4x - 11$

$2x = 14$

$x = 7$

$KL = 2(7) + 3$

$KL = 17$

The lengths are equal by the Angle Bisector Theorem.

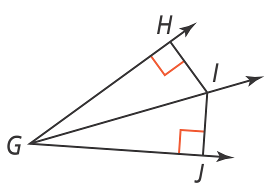
Evaluate the expression for *KL*.

Try It!

6. Use the figure shown.

a. If $HI = 7$, $IJ = 7$, and $m\angle HGI = 25$, what is $m\angle IGJ$?

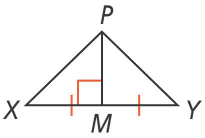
b. If $m\angle HGJ = 57$, $m\angle IGJ = 28.5$, and $HI = 12.2$, what is the value of IJ?



CONCEPT SUMMARY Perpendicular and Angle Bisectors

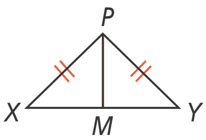
THEOREM 7-10 Perpendicular Bisector Theorem

If...



$XM = YM$ and $\overline{PM} \perp \overline{XY}$

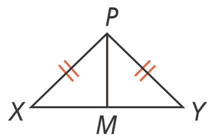
Then...



$PX = PY$

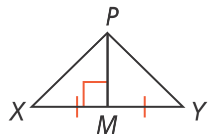
THEOREM 7-11 Converse of Perpendicular Bisector Theorem

If...



$PX = PY$

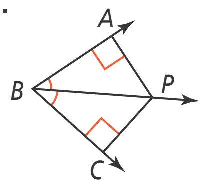
Then...



$XM = YM$ and $\overline{PM} \perp \overline{XY}$

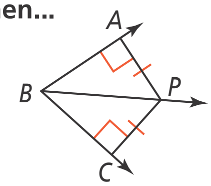
THEOREM 7-12 Angle Bisector Theorem

If...



$\angle ABP \cong \angle CBP$

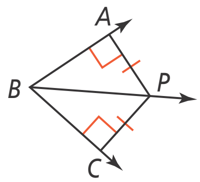
Then...



$AP = CP$

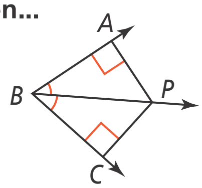
THEOREM 7-13 Converse of Angle Bisector Theorem

If...



$AP = CP$

Then...



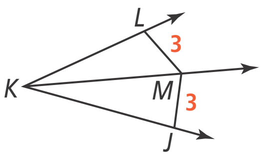
$\angle ABP \cong \angle CBP$

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** What is the relationship between a segment and the points on its perpendicular bisector? Between an angle and the points on its bisector?

2. **Vocabulary** How can you determine if a point is *equidistant* from the sides of an angle?

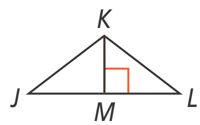
3. **Error Analysis** River says that $\overrightarrow{KM}$ is the bisector of $\angle LKJ$ because $LM = MJ$. Explain the error in River's reasoning.



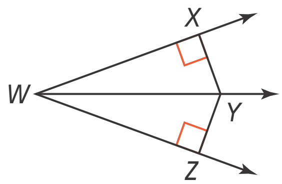
4. **Construct Arguments** You know that $\overline{AB}$ is the perpendicular bisector of $\overline{XY}$, and $\overline{XY}$ is the perpendicular bisector of $\overline{AB}$. What can you conclude about the side lengths of quadrilateral $AXBY$? Explain.

Do You KNOW HOW?

5. If $JL = 14$, $KL = 10$, and $ML = 7$, what is JK?



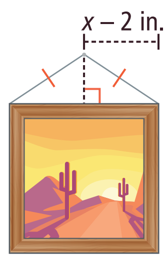
Use the figure shown for Exercises 6 and 7.



6. If $\angle XWY \cong \angle ZWY$ and $XY = 4$, what is YZ?

7. If $XY = ZY$ and $m\angle ZWY = 18$, what is $m\angle XWZ$?

8. What is an algebraic expression for the area of the square picture and frame?



PRACTICE & PROBLEM SOLVING

Scan for Multimedia

Practice Tutorial

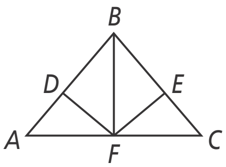
Additional Exercises Available Online

UNDERSTAND

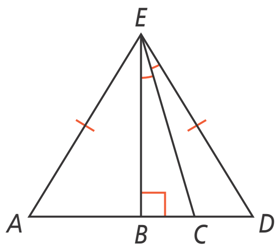
9. **Construct Arguments** Write a two-column proof for the Angle Bisector Theorem.

10. **Construct Arguments** Write a paragraph proof for the Converse of the Angle Bisector Theorem.

11. **Reason** In the diagram below, $AB = BC$, $DF = EF$, and $m\angle BDF = m\angle BEF = 90°$. Is $\triangle ADF \cong \triangle CEF$? Justify your answer.



12. **Error Analysis** A student analyzed the diagram and incorrectly concluded that $AB = 2BC$. Explain the student's error.



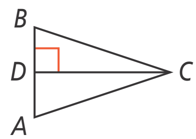
$\overline{EB}$ is the perpendicular bisector of $\overline{AD}$,
so $AB = BD$.
$\angle BEC \cong \angle DEC$, so
$BC = CD$.
$BC + CD = BD = AB$, and
$BC + CD = BC + BC = 2BC$,
so $AB = 2BC$. ✗

13. **Higher Order Thinking** Describe the process of constructing the bisector of an angle. Draw a diagram and explain how this construction can be related to the Angle Bisector Theorem.

PRACTICE

Use the figure shown for Exercises 14 and 15.

SEE EXAMPLES 1–3

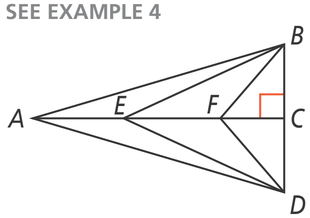


14. If $AD = 3$, $AC = 8$, and $BD = 3$, what is the perimeter of $\triangle ABC$?

15. If $BC = 10$, $AB = 7$, and the perimeter of $\triangle ABC$ is 27, what is the value of BD?

Use the figure shown for Exercises 16 and 17.

SEE EXAMPLE 4

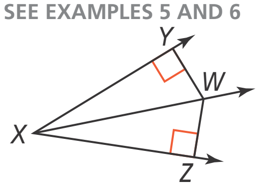


16. If $AD = 21$, $BF = 8$, and $DF = 8$, what is the value of AB?

17. If $EB = 6.2$, $CD = 3.3$, and $ED = 6.2$, what is the value of BD?

Use the figure shown for Exercises 18 and 19.

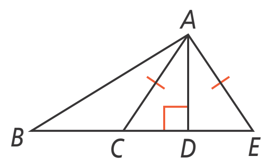
SEE EXAMPLES 5 AND 6



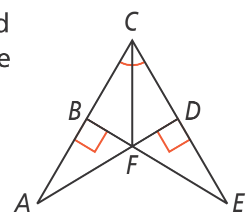
18. If $m\angle YXW = 21$, $YW = 5$, and $WZ = 5$, what is $m\angle ZXY$?

19. If $m\angle YXZ = 38$, $m\angle WXZ = 19$, and $WZ = 8.1$, what is the value of YW?

20. If $CD = 4$ and the perimeter of $\triangle ABC$ is 23, what is the perimeter of $\triangle ABE$?



21. Given that $\angle ACF \cong \angle ECF$ and $m\angle ABF = m\angle EDF = 90$, write a two-column proof to show that $\triangle ABF \cong \triangle EDF$.

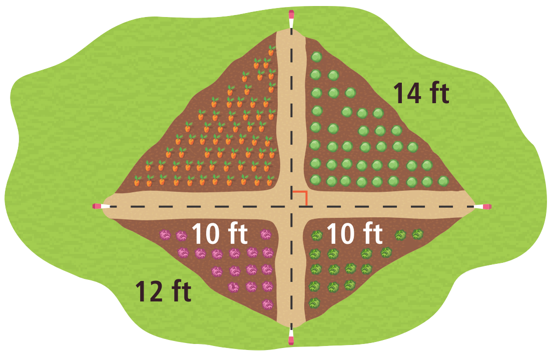


PRACTICE & PROBLEM SOLVING

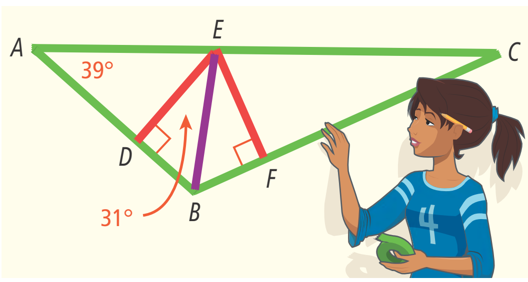
Practice Tutorial
Mixed Review Available Online

APPLY

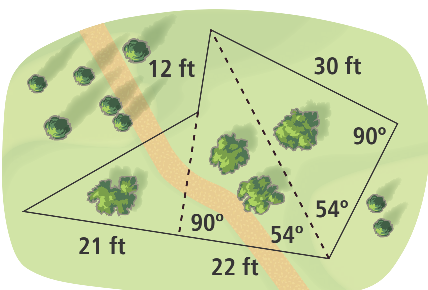
22. Make Sense and Persevere A gardener wants to replace the fence along the perimeter of her garden. How much new fencing will be required?



23. Look for Relationships An artist uses colored tape to divide sections of a mural. She needs to cut a piece of paper to cover $\triangle EFC$ while she works on other sections. What angles should she cut so she only covers the triangle?



24. Mathematical Connections A surveyor took some measurements of a piece of land. The owner needs to know the area of the land to determine the value. What is the area of the piece of land?



ASSESSMENT PRACTICE

25. $\overleftrightarrow{AB}$ is the perpendicular bisector of $\overline{XY}$. Point P is the midpoint of $\overline{XY}$. Is each statement true? Select *Yes* or *No*.

	Yes	No
$AP = XP$	❑	❑
$AB = XY$	❑	❑
$AP = BP$	❑	❑
$XB = YB$	❑	❑
$AY = XB$	❑	❑
$XP = YP$	❑	❑

26. SAT/ACT Points G, J, and K are not collinear, and $GJ = GK$. If P is a point on $\overline{JK}$, which of the following conditions is sufficient to prove that $\overleftrightarrow{GP}$ is the perpendicular bisector of $\overline{JK}$?

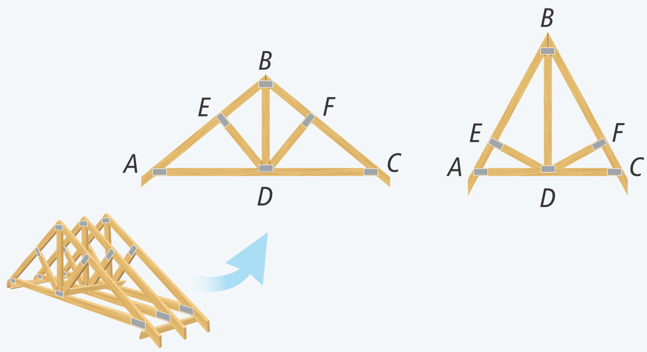
Ⓐ $JG = PG$

Ⓑ $m\angle GPJ = 90$

Ⓒ $\angle GJK \cong \angle GKJ$

Ⓓ $PK = PG$

27. Performance Task A manufacturer makes roofing trusses in a variety of sizes. All of the trusses have the same shape with three supports, as shown.



Part A One builder needs $\angle ABD$ and $\angle CBD$ to be congruent for a project. You need to check that a truss meets the builder's requirement. The only tools you have are a measuring tape and a steel square, which is a carpentry tool for measuring right angles. How can you use these tools to verify the angles are congruent?

Part B In addition to the requirement of the first builder, another builder also needs $\overline{AB}$ and $\overline{BC}$ to be congruent as well as $\overline{AD}$ and $\overline{DC}$. Using the same tools, how can you efficiently verify that all three pairs are congruent? Explain.