9-6 Right Triangles and the Pythagorean Theorem

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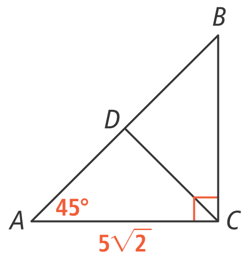
I CAN… prove the Pythagorean Theorem using similarity and establish the relationships in special right triangles.

VOCABULARY

- Pythagorean triple

EXPLORE & REASON

Consider $\triangle ABC$ with altitude $\overline{CD}$ as shown.



A. What is the area of $\triangle ABC$? Of $\triangle ACD$? Explain your answers.

B. Find the lengths of $\overline{AD}$ and $\overline{AB}$.

C. Look for Relationships Divide the length of the hypotenuse of $\triangle ABC$ by the length of one of its sides. Divide the length of the hypotenuse of $\triangle ACD$ by the length of one of its sides. Make a conjecture that explains the results.

ESSENTIAL QUESTION

How are similarity in right triangles and the Pythagorean Theorem related?

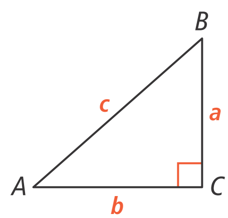
Remember that the Pythagorean Theorem and its converse describe how the side lengths of right triangles are related.

THEOREM 9-8 Pythagorean Theorem

If a triangle is a right triangle, then the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

PROOF: SEE EXAMPLE 1.

If… $\triangle ABC$ is a right triangle.



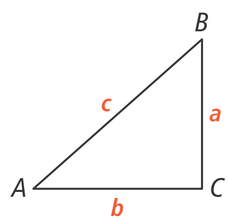
Then… $a^2 + b^2 = c^2$

THEOREM 9-9 Converse of the Pythagorean Theorem

If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.

PROOF: SEE EXERCISE 17.

If… $a^2 + b^2 = c^2$



Then… $\triangle ABC$ is a right triangle.

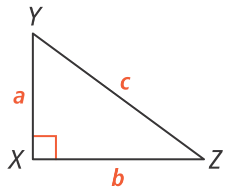
PROOF

EXAMPLE 1 Use Similarity to Prove the Pythagorean Theorem

Use right triangle similarity to write a proof of the Pythagorean Theorem.

Given: $\triangle XYZ$ is a right triangle.

Prove: $a^2 + b^2 = c^2$

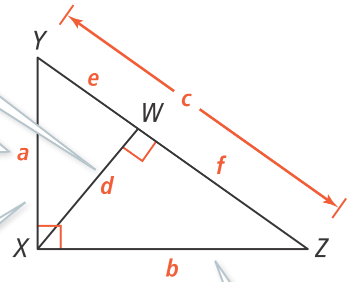


Plan: To prove the Pythagorean Theorem, draw the altitude to the hypotenuse. Then use the relationships in the resulting similar right triangles.

Proof:

Step 1 Draw altitude $\overline{XW}$.

Step 2 $\triangle XYZ \sim \triangle WXZ \sim \triangle WYX$



Step 3 Because $\triangle XYZ \sim \triangle WYX$, $\frac{c}{a} = \frac{a}{e}$. So $a^2 = ce$.

Step 4 Because $\triangle XYZ \sim \triangle WXZ$, $\frac{c}{b} = \frac{b}{f}$. So $b^2 = cf$.

LOOK FOR RELATIONSHIPS
Think about how you can apply properties of similar triangles. What is the relationship between corresponding sides of similar triangles?

Step 5 Write an equation that relates a^2 and b^2 to ce and cf.

$$a^2 + b^2 = ce + cf$$

$$a^2 + b^2 = c(e + f)$$

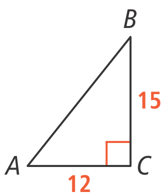
$$a^2 + b^2 = c(c)$$

$$a^2 + b^2 = c^2$$

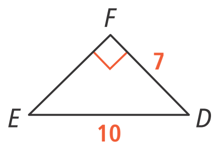
Try It!

1. Find the unknown side length of each right triangle.

a. AB



b. EF



APPLICATION

EXAMPLE 2 Use the Pythagorean Theorem and Its Converse

A. To satisfy safety regulations, the distance from the wall to the base of a ladder should be at least one-fourth the length of the ladder. Did Drew set up the ladder correctly?

The floor, the wall, and the ladder form a right triangle.

Step 1 Find the length of the ladder.

$$a^2 + b^2 = c^2$$

$$2.5^2 + 9^2 = c^2$$

$$87.25 = c^2$$

$$9.34 \approx c$$

Use the Pythagorean Theorem with $a = 2.5$ and $b = 9$.

Step 2 Find $\frac{1}{4}$ the length of the ladder.

$$\frac{1}{4}c \approx \frac{1}{4}(9.34)$$

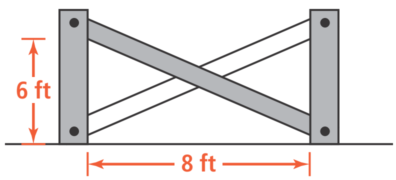
$$\approx 2.335$$

The length of the ladder is 9.34 ft.

Since $2.5 > 2.335$, Drew set up the ladder correctly.

B. The length of each crosspiece of the fence is 10 ft. Why would a rancher build this fence with the measurements shown?

The numbers 6, 8, and 10 form a *Pythagorean triple*. A **Pythagorean triple** is a set of three nonzero whole numbers that satisfy the equation $a^2 + b^2 = c^2$.

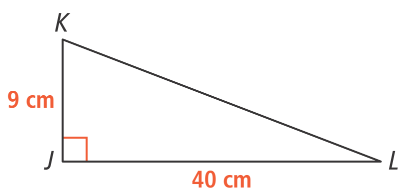


Since $6^2 + 8^2 = 10^2$, the posts, the ground, and the crosspieces form right triangles.

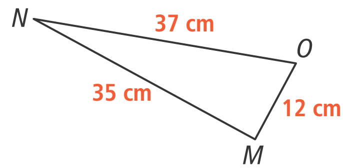
By using those measurements, the rancher knows that the fence posts are perpendicular to the ground, which stabilizes the fence.

STUDY TIP
Learn and recognize common Pythagorean triples such as 3, 4, and 5; and 5, 12, and 13 to speed calculations.

Try It! **2. a.** What is *KL*?



b. Is $\triangle MNO$ a right triangle? Explain.

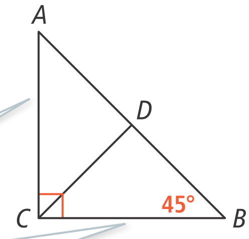


CONCEPTUAL UNDERSTANDING

EXAMPLE 3 Investigate Side Lengths in 45°-45°-90° Triangles

REASON
Think about the properties of a triangle with two congruent angles. How do the properties of the triangle help you relate the side lengths?

Is there a relationship between the lengths of $\overline{AB}$ and $\overline{AC}$ in $\triangle ABC$? Explain.



Draw altitude $\overline{CD}$ to form similar right triangles $\triangle ABC$, $\triangle ACD$, and $\triangle CBD$.

Notice that $\triangle ABC$ is a 45°-45°-90° triangle, and that $AC = BC$.

Use right-triangle similarity to write an equation.

$$\frac{AB}{AC} = \frac{AC}{AD}$$

Since $\triangle ABC \sim \triangle ACD$, AC is the geometric mean of AB and AD.

$$\frac{AB}{AC} = \frac{AC}{\frac{1}{2}AB}$$

Because $\triangle ABC$ is isosceles, $\overline{CD}$ bisects $\overline{AB}$.

$$\frac{1}{2}AB^2 = AC^2$$

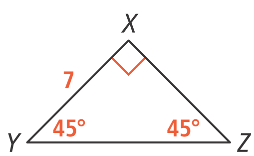
$$AB^2 = 2AC^2$$

$$AB = \sqrt{2} \cdot AC$$

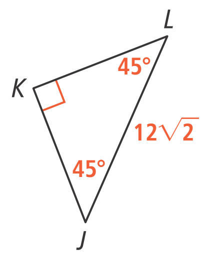
The length of $\overline{AB}$ is $\sqrt{2}$ times the length of $\overline{AC}$.

Try It! **3.** Find the side lengths of each 45°-45°-90° triangle.

a. What are XZ and YZ?



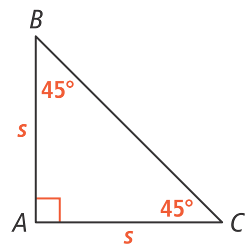
b. What are JK and LK?



THEOREM 9-10 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

If...

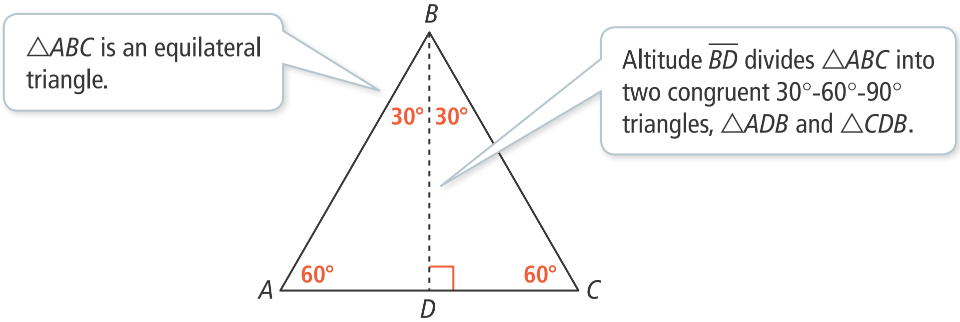


Then... $BC = s\sqrt{2}$

PROOF: SEE EXERCISE 18.

EXAMPLE 4 Explore the Side Lengths of a 30°-60°-90° Triangle

Using an equilateral triangle, show how the lengths of the short leg, the long leg, and the hypotenuse of a 30°-60°-90° triangle are related.



> **STUDY TIP**
> Recall that an altitude of a triangle is perpendicular to a side. Think about what properties of the triangle result in the altitude also being a segment bisector.

Look at △*ADB*. Let the length of the short leg $\overline{AD}$ be *s*.

Find the relationship between *AD* and *AB*.

$AD = CD = s$ — $\overline{BD}$ bisects $\overline{AC}$.

$AC = AD + CD$

$AC = 2s$

$AB = 2s$ — △*ABC* is equilateral, so $AB = AC = 2s$.

Find the relationship between *AD* and *BD*.

$AD^2 + BD^2 = AB^2$ — Use the Pythagorean Theorem.

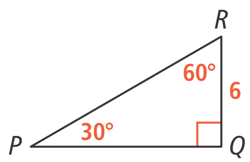
$s^2 + BD^2 = (2s)^2$

$BD^2 = 3s^2$

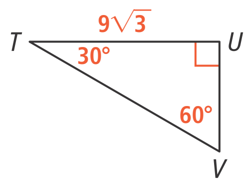
$BD = s\sqrt{3}$

In △*ADB*, the length of hypotenuse $\overline{AB}$ is twice the length of the short leg $\overline{AD}$. The length of the long leg $\overline{BD}$ is $\sqrt{3}$ times the length of the short leg.

Try It! **4. a.** What are *PQ* and *PR*?



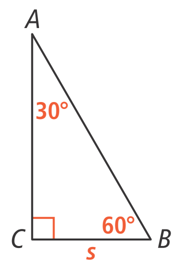
b. What are *UV* and *TV*?



THEOREM 9-11 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the short leg. The length of the long leg is $\sqrt{3}$ times the length of the short leg.

If...



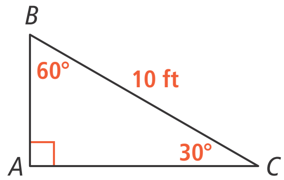
Then... $AC = s\sqrt{3}$, $AB = 2s$

PROOF: SEE EXERCISE 19.

Activity Assess

EXAMPLE 5 Apply Special Right Triangle Relationships

A. Alejandro needs to make both the horizontal and vertical supports, $\overline{AC}$ and $\overline{AB}$, for the ramp. Is one 12-foot board long enough for both supports? Explain.



The ramp and supports form a 30°-60°-90° triangle.

$BC = 2AB$ $\quad$ $AC = AB\sqrt{3}$

$10 = 2AB$ $\quad$ $AC = 5\sqrt{3}$ ft

$AB = 5$ ft

Find the total length of the supports.

$AB + AC = 5 + 5\sqrt{3}$

≈ 13.66 ft

Since $13.66 > 12$, the 12-foot board will not be long enough for Alejandro to make both supports.

COMMON ERROR
Be careful not to mix up the relationship of the shorter and longer legs. Remember that the longer leg is $\sqrt{3}$ times as long as the shorter leg, so the longer leg is between $1\frac{1}{2}$ and 2 times as long as the short leg.

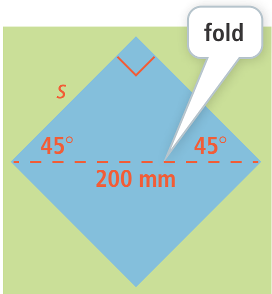
B. Olivia starts an origami paper crane by making the 200-mm diagonal fold. What are the side length and area of the paper square?

Step 1 Find the length of one side of the paper.

$s\sqrt{2} = 200$

$s = \frac{200}{\sqrt{2}}$

$s \approx 141.4$ mm



Step 2 Find the area of the paper square.

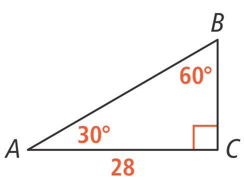
$A = s^2$

$A = (100\sqrt{2})^2$

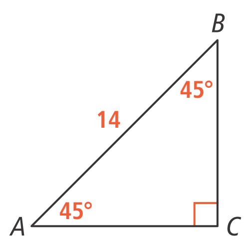
$A = 20{,}000 \text{ mm}^2$

The paper square has side length 141.4 mm and area 20,000 mm^2.

Try It! 5. a. What are AB and BC?



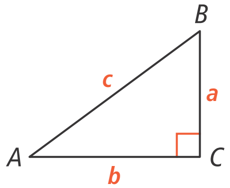
b. What are AC and BC?



CONCEPT SUMMARY The Pythagorean Theorem and Special Right Triangles

THEOREM 9-8 Pythagorean Theorem

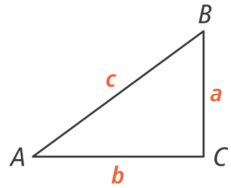
If... $\triangle ABC$ is a right triangle



Then... $a^2 + b^2 = c^2$

THEOREM 9-9 Converse of the Pythagorean Theorem

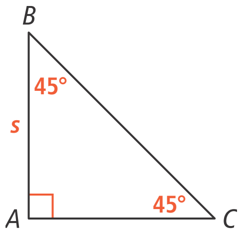
If... $a^2 + b^2 = c^2$



Then... $\triangle ABC$ is a right triangle.

THEOREM 9-10 45°-45°-90° Triangle Theorem

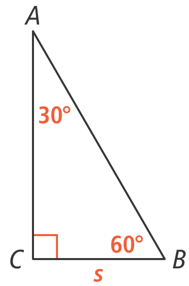
If...



Then... $BC = s\sqrt{2}$

THEOREM 9-11 30°-60°-90° Triangle Theorem

If...

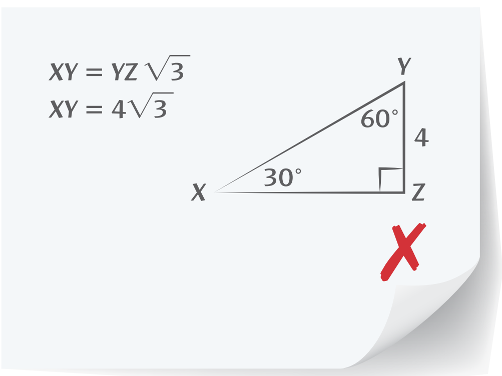


Then... $AC = s\sqrt{3}$, $AB = 2s$

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How are similarity in right triangles and the Pythagorean Theorem related?

2. **Error Analysis** Casey was asked to find XY. What is Casey's error?

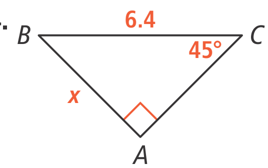


3. **Reason** A right triangle has leg lengths 4.5 and $4.5\sqrt{3}$. What are the measures of the angles? Explain.

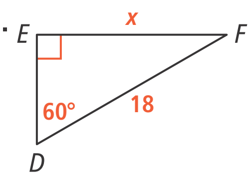
Do You KNOW HOW?

For Exercises 4 and 5, find the value of *x*.

4.



5.



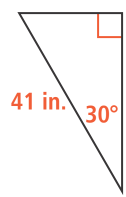
For Exercises 6–8, is $\triangle RST$ a right triangle? Explain.

6. $RS = 20$, $ST = 21$, $RT = 29$

7. $RS = 35$, $ST = 36$, $RT = 71$

8. $RS = 40$, $ST = 41$, $RT = 11$

9. Charles wants to hang the pennant shown vertically between two windows that are 19 inches apart. Will the pennant fit? Explain.



PRACTICE & PROBLEM SOLVING

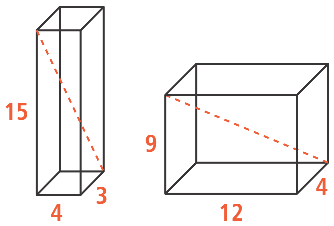
Scan for Multimedia

Practice Tutorial

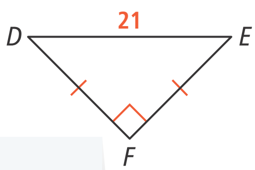
Additional Exercises Available Online

UNDERSTAND

10. **Mathematical Connections** Which rectangular prism has the longer diagonal? Explain.

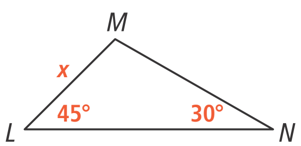


11. **Error Analysis** Dakota is asked to find *EF*. What is her error?

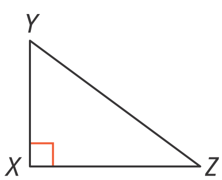


There is not enough information to find *EF* because you need to know either the length of $\overline{DF}$ or one of the other angle measures.

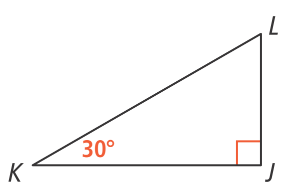
12. **Make Sense and Persevere** What are expressions for *MN* and *LN*? *Hint*: Construct the altitude from *M* to $\overline{LN}$.



13. **Higher Order Thinking** Triangle *XYZ* is a right triangle. For what kind of triangle would $XZ^2 + XY^2 > YZ^2$? For what kind of triangle would $XZ^2 + XY^2 < YZ^2$? Explain.



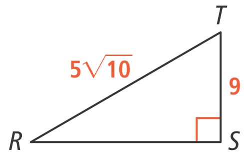
14. **Look for Relationships** Write an equation that represents the relationship between *JK* and *KL*.



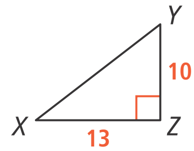
PRACTICE

For Exercises 15 and 16, find the unknown side length of each triangle. SEE EXAMPLE 1

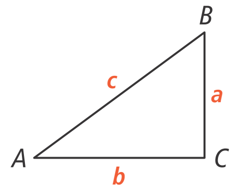
15. *RS*



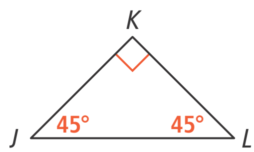
16. *XY*



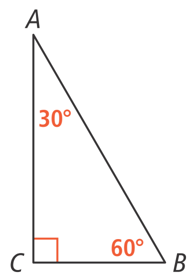
17. Given $\triangle ABC$ with $a^2 + b^2 = c^2$, write a paragraph proof of the Converse of the Pythagorean Theorem. SEE EXAMPLE 2



18. Write a two-column proof of the 45°-45°-90° Triangle Theorem. SEE EXAMPLE 3

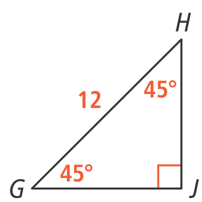


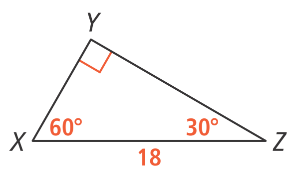
19. Write a paragraph proof of the 30°-60°-90° Triangle Theorem. SEE EXAMPLE 4



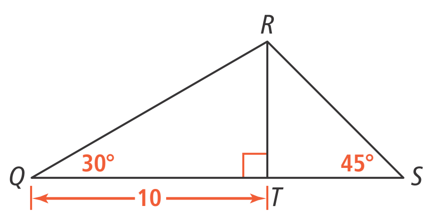
For Exercise 20 and 21, find the side lengths of each triangle. SEE EXAMPLES 3 AND 4

20. What are *GJ* and *HJ*? **21.** What are *XY* and *YZ*?





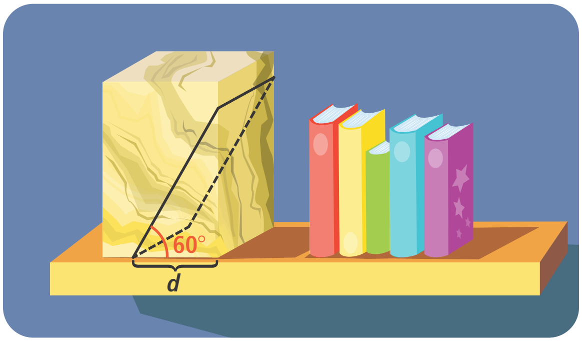
22. What is *QS*? SEE EXAMPLE 5



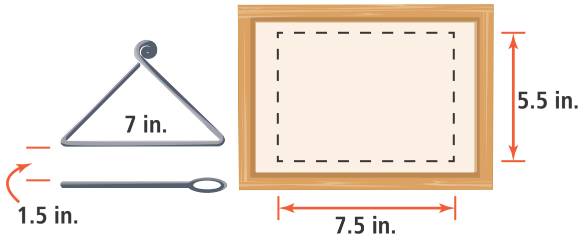
Practice Tutorial
Mixed Review Available Online

APPLY

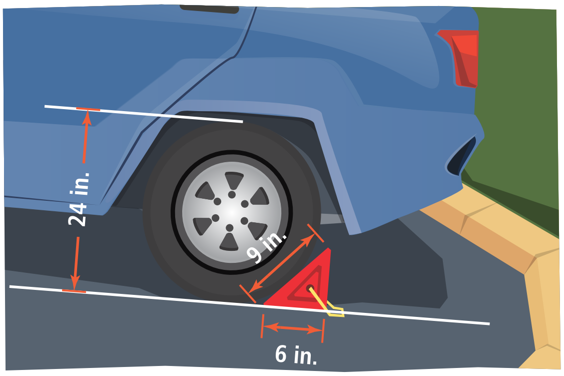
23. Reason Esteban wants marble bookends cut at a 60° angle, as shown. If Esteban wants his bookends to be between 7.5 in. and 8 in. tall, what length *d* should the marble cutter make the base of the bookends? Explain.



24. Communicate Precisely Sarah finds an antique dinner bell that appears to be in the shape of an isosceles right triangle, but the only measurement given is the longest side. Sarah wants to display the bell and wand in a 5.5-in. by 7.5-in. picture frame. Assuming that the bell is an isosceles right triangle, can Sarah display the bell and wand within the frame? Explain.



25. Construct Arguments When Carmen parks on a hill, she places chocks behind the wheels of her car. The height of the chocks must be at least one-fourth of the height of the wheels to hold the car securely in place. The chock shown has the shape of a right triangle. Is it safe for Carmen to use? Explain.

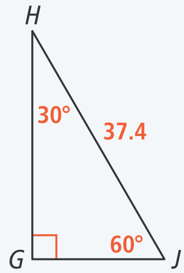


ASSESSMENT PRACTICE

26. Match each set of triangle side lengths with the best description of the triangle.

I. $\sqrt{2}$, $\sqrt{2}$, $\sqrt{3}$	**A.** right triangle
II. 5, $3\sqrt{2}$, $\sqrt{43}$	**B.** 30°-60°-90° triangle
III. 8, 8, $8\sqrt{2}$	**C.** 45°-45°-90° triangle
IV. 11, $11\sqrt{3}$, 22	**D.** not a right triangle

27. SAT/ACT What is *GJ*?



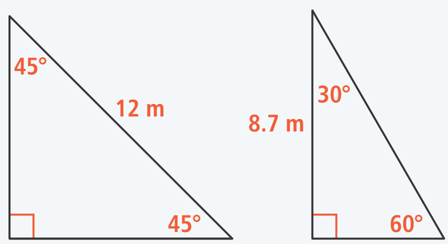
Ⓐ 18.7

Ⓑ $18.7\sqrt{2}$

Ⓒ $18.7\sqrt{3}$

Ⓓ 74.8

28. Performance Task Emma designed two triangular sails for a boat.



Part A What is the area of Sail A?

Part B What is the area of Sail B?

Part C Is it possible for Emma to cut both sails from one square of sailcloth with sides that are 9 meters in length? Draw a diagram to explain.

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