10-1 Designing a Study

Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

1. **SCHOOL** A group of high school students is randomly selected and asked to complete the form shown.

SOLUTION:

This is a survey because data are collected from responses to the question.
sample: the students in the study; population: the student body

CCSS ARGUMENTS Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.

3. **LITERACY** A literacy group wants to determine whether high school students that participated in a recent national reading program had higher standardized test scores than high school students that did not participate in the program.

SOLUTION:

Observation study; sample answer: The scores of the participants are observed and compared without them being affected by the study.

Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.

5. Which student council candidate's platform do you support?

SOLUTION:

The question is not confusing, does not cause a strong reaction, does not cause a certain response, and addresses only one issue. So, it is unbiased.

7. **HYBRIDS** A car manufacturer wants to determine what the demand in the U.S. is for hybrid vehicles. State the objective of the survey, suggest a population, and write two unbiased survey questions.

SOLUTION:

objective: to determine how many people in the U.S. are interested in purchasing a hybrid vehicle
population: the people surveyed

sample survey questions:
Do you currently own a hybrid vehicle?
Are you planning on purchasing a hybrid vehicle?

The survey questions are unbiased because they are not confusing, do not cause a strong reaction, do not cause a certain response, and address only one issue.

9. **SPORTS** A research company wants to conduct an experiment to test the claim of the protein shake shown. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.

SOLUTION:

objective: to determine whether the protein shake helps athletes recover from exercise; population: all athletes; experiment group: athletes given the protein shake; control group: athletes given a placebo; sample procedure: The researchers could randomly divide the athletes into two groups: an experimental group given the protein shake and a control group given the placebo. Next, they could have the athletes exercise and then drink the protein shake or placebo. Later, the researchers could interview the athletes to see how they feel.

Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

11. **GRADES** A research group randomly selects 80 college students, half of whom took a physics course in high school, and compares their grades in a college physics course.

SOLUTION:

This is an observational study because the study group is going to observe the students' performance without directly affecting the students. The sample is the 80 physics students because they are the ones being observed. Both halves of the selected students are included because one half is going to be compared to the other half. The population is all college students that take a physics course.

13. **TELEVISION** A television network mails a questionnaire to randomly selected people across the country to determine whether they prefer watching sitcoms or dramas.

SOLUTION:

This is a survey because data are collected from the responses to the questionnaire.

sample: people that receive the questionnaire; population: all viewers

Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.

15. **TRAVEL** A travel agency randomly calls 250 U.S. citizens and asks them what their favorite vacation destination is.

SOLUTION:

Survey; sample answer: The data will be obtained from opinions given by members of the sample population.

17. **ENGINEERING** An engineer is planning to test 50 metal samples to determine whether a new titanium alloy has a higher strength than a different alloy.

SOLUTION:

Experiment; sample answer: Metal samples will need to be tested, which means that the members of the sample will be affected by the study.

Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.

19. Which is your favorite football team, the Dallas Cowboys or the Pittsburgh Steelers?

SOLUTION:

Biased; sample answer: The question only gives two options, and thus encourages a certain response.

21. Don't you agree that students should carpool to school?

SOLUTION:

Biased; sample answer: The question encourages a certain response. The phrase "don't you agree" suggests that the people surveyed should agree.

23. Identify any flaws in the experiment design, and describe how they could be corrected.

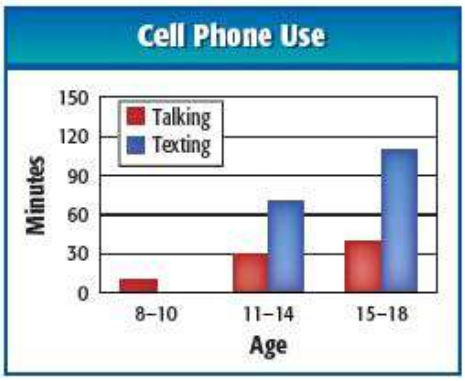
Experiment: A supermarket chain wants to determine whether shoppers are more likely to buy sunscreen if it is located near the checkout line. The experimental group consists of a group of stores in the midwest in which the sunscreen was moved next to the checkout line, and the control group consists of stores in Arizona in which the sunscreen was not moved.

Results: The Arizona stores sold more sunscreen than the midwest stores. The company concluded that moving the sunscreen closer to the checkout line did not increase sales.

SOLUTION:

Sample answer: The flaw is that the experimental group consists of stores in the Midwest, and the control group consists of stores in Arizona. On average, the temperature is higher in Arizona than in the Midwest, and people use more sunscreen. Therefore, the sunscreen sales in stores located in those regions would likely be different and should not be compared in an experiment.

25. **REPORTS** The graph shown is from a report on the average number of minutes 8- to 18-year-olds in the U.S. spend on cell phones each day.



a. Describe the sample and suggest a population.
b. What type of sample statistic do you think was calculated for this report?
c. Describe the results of the study for each age group.
d. Who do you think would be interested in this type of report? Explain your reasoning.

SOLUTION:

a. sample: the 8- to 18-year-olds surveyed; population: all 8- to 18-year-olds in the U.S.
b. average time
c. Sample answer: The 8- to 10-year-old group talked for about 10 minutes a day and did not text at all. The 11- to 14-year old group talked for about 30 minutes a day and texted for about 70 minutes a day. The 15- to 18-year-old group talked for about 40 minutes a day and texted for about 110 minutes a day.
d. Sample answer: A cell phone company might use a report like this to determine which age group to target in their ads.

27. **MULTIPLE REPRESENTATIONS** The results of two experiments concluded that Product A is 70% effective and Product B is 80% effective.
a. NUMERICAL To simulate the experiment for Product A, use the random number generator on a graphing calculator to generate 30 integers between 0 and 9. Let 0–6 represent an effective outcome and 7–9 represent an ineffective outcome.

b. TABULAR Copy and complete the frequency table shown using the results. from part **a**. Then use the data to calculate the probability that Product A was effective. Repeat to find the probability for Product B.

Product A	
Number	Frequency
0–6	
7–9	

c. ANALYTICAL Compare the probabilities that you found in part **b**. Do you think that the difference in the effectiveness of each product is significant enough to justify selecting one product over the other? Explain.
d. LOGICAL Suppose Product B costs twice as much as Product A. Do you think the probability of the product's effectiveness justifies the price difference to a consumer? Explain.

SOLUTION:

a. See students' work.

b. Sample answers:

Product A	
Number	Frequency
0–6	𝍸 𝍸 𝍸 ||||
7–9	𝍸 𝍸 |

Product B	
Number	Frequency
0–7	𝍸 𝍸 𝍸 𝍸 |||
8–9	𝍸 ||

The probability that Product A was effective is $\frac{19}{30} \approx$ 63.3%

The probability that Product B was effective is $\frac{23}{30} \approx$ 76.7%

c. 76.7% – 63.3% = 13.4%

The probability that Product B is effective is 13.4% higher than that of Product A. Yes, the difference appears to be significant.

d. Sample answer: It depends on what the product is and how it is being used. For example, if the product is a pencil sharpener, then the lower price may be more important than the effectiveness, and therefore, might not justify the price difference. However, if the product is a life-saving medicine, the effectiveness may be more important than the price, and therefore, might justify the price difference.

REASONING Determine whether each statement is *true* or *false* . If false, explain.

29. Observational studies and experiments can both be used to study cause-and-effect relationships.

SOLUTION:

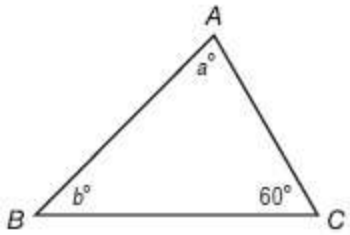
Observation studies are used to study cause-and-effect relationships by observing or measuring the reactions of a group to something that does not affect the group directly. Experiments are used to study cause-and-effect relationships by observing or measuring the reactions of a group to something that *does* affect the group directly. Therefore, the statement is true.

31. **CHALLENGE** What factors should be considered when determining whether a given statistical study is reliable?

SOLUTION:

An invalid sampling method and type of sample can produce bias. For example, if a sample is not random, the person conducting the study can influence the results by selecting a specific sample of people. Also, if an experiment is used when an observational study is the more logical type of study to be used, the study can be unreliable. For example, if someone wants to analyze the speeds of vehicles on a specific stretch of highway and decides to place an empty police car on the side of the road, the data will be affected by the police car. The results of this study will show lower speeds than are normally driven on the highway. Biased survey questions and incorrect procedures can affect the reliability of a study as well. A survey question that is poorly written may result in a response that does not accurately reflect the opinion of the participant.

33. **GEOMETRY** In ΔABC, $BC > AB$. Which of the following must be true?



A $AB = BC$
B $AC > AB$
C $a > 60$
D $a = b$

SOLUTION:

Since the side BC > the side AB, $\angle BAC > \angle ACB$. Therefore, $a > 60$
Option C is the correct answer.

35. **SAT/ACT** A pie is divided evenly between 3 boys and a girl. If one boy gives one half of his share to the girl and a second boy keeps two thirds of his share and gives the rest to the girl, what portion will the girl have in all?

F $\frac{5}{24}$
G $\frac{11}{24}$
H $\frac{1}{2}$
J $\frac{13}{24}$
K $\frac{13}{12}$

SOLUTION:

Each of them having $\frac{1}{4}$ portion in all.

The girl getting one half of one boy's share and one third of the second boy's share.

The girl will have $\frac{1}{4}+\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\cdot\frac{1}{3}$ in all.

$$\frac{1}{4}+\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\cdot\frac{1}{3}=\frac{1}{4}+\frac{1}{8}+\frac{1}{12}$$
$$=\frac{6+3+2}{24}$$
$$=\frac{11}{24}$$

Option G is the correct answer.

37. Prove that the statement $9n - 1$ is divisible by 8 is true for all natural numbers.

SOLUTION:

Step 1: $9^1 - 1 = 8$, which is divisible by 8. The statement is true for $n = 1$.

Step 2: Assume that $9^k - 1$ is divisible by 8 for some positive integer k. This means that $9^k - 1 = 8r$ for some whole number r.

Step 3:

$$9^k - 1 = 8r$$
$$9^k = 8r + 1$$
$$9^{k+1} = 72r + 9$$
$$9^{k+1} - 1 = 72r + 8$$
$$9^{k+1} - 1 = 8(9r + 1)$$

Since r is a whole number, $9r + 1$ is a whole number. Thus, $9^k + 1 - 1$ is divisible by 8, so the statement is true for $n = k + 1$. Therefore, $9^n - 1$ is divisible by 8 for all positive integers n.

Solve each system of equations.

39. $y = x + 3$
$y = 2x^2$

SOLUTION:

Substitute $x + 3$ for y and solve for x.

$$2x^2 = x + 3$$
$$2x^2 - x - 3 = 0$$
$$(2x - 3)(x + 1) = 0$$

Therefore, $x = -1$ and $\frac{3}{2}$.

Substitute the values of x in the first equation and evaluate.

$$y = (-1) + 3 \quad \text{and} \quad y = \left(\frac{3}{2}\right) + 3$$
$$= 2 \quad \text{and} \quad y = \frac{9}{2}$$

Therefore, the solutions are $(-1, 2)$ and $\left(\frac{3}{2}, \frac{9}{2}\right)$.

41. $y^2 + x^2 = 9$
$y = 7 - x$

SOLUTION:

Substitute $7 - x$ for y in the first equation and solve for x.

$$(7 - x)^2 + x^2 = 9$$
$$49 - 14x + x^2 + x^2 = 9$$
$$2x^2 - 14x + 40 = 0$$
$$x^2 - 7x + 20 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(20)}}{2(1)}$$
$$= \frac{7 \pm \sqrt{49 - 80}}{2}$$
$$= \frac{7 \pm \sqrt{-31}}{2}$$

Since the radicand is negative there is no real solution for the system.

43. $x^2 + y^2 = 64$
$x^2 + 64y^2 = 64$

SOLUTION:

Solve the first equation for y^2 and substitute in the second equation.

$$x^2 + y^2 = 64$$
$$y^2 = 64 - x^2$$

$$x^2 + 64y^2 = 64$$
$$x^2 + 64(64 - x^2) = 64$$
$$4096 - 63x^2 = 64$$
$$x^2 = \frac{4096 - 64}{63}$$
$$= 64$$
$$x = \pm 8$$

Substitute the values of x in the first equation and find the corresponding values of y.

$$(\pm 8)^2 + y^2 = 64$$
$$64 + y^2 = 64$$
$$y^2 = 0$$
$$y = 0$$

Therefore, the solutions are $(\pm 8, 0)$.

Find the distance between each pair of points with the given coordinates.

45. (9, –2), (12, –14)

SOLUTION:

Substitute the given points in the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(12 - 9)^2 + ((-14) - (-2))^2}$$
$$= \sqrt{3^2 + (-12)^2}$$
$$= \sqrt{153}$$
$$d = 3\sqrt{17} \text{ units}$$

47. (1, –14), (–6, 10)

SOLUTION:

Substitute the given points in the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-6 - 1)^2 + (10 - (-14))^2}$$
$$= \sqrt{(-7)^2 + (24)^2}$$
$$= \sqrt{625}$$
$$d = 25 \text{ units}$$

49. (2.3, –1.2), (–4.5, 3.7)

SOLUTION:

Substitute the given points in the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-4.5 - 2.3)^2 + (3.7 - (-1.2))^2}$$
$$= \sqrt{(-6.8)^2 + 4.9^2}$$
$$d = \sqrt{70.25} \text{ units}$$

Simplify. Assume that no variable equals 0.

51. $(5cd^2)(-c^4d)$

SOLUTION:

$$(5cd^2)(-c^4d) = -5c^5d^3$$

53. $\frac{a^2n^6}{an^5}$

SOLUTION:

$$\frac{a^2n^6}{an^5} = a^{2-1}n^{6-5}$$
$$= an$$

55. $\frac{-y^5z^7}{y^2z^5}$

SOLUTION:

$$\frac{-y^5z^7}{y^2z^5} = -y^{5-2}z^{7-5}$$
$$= -y^3z^2$$

Write a quadratic equation with the given root (s). Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

57. –3, 9

SOLUTION:

$$(x-p)(x-q)=0$$
$$(x-(-3))(x-9)=0$$
$$(x+3)(x-9)=0$$
$$x^2-9x+3x-27=0$$
$$x^2-6x-27=0$$

59. 4, –5

SOLUTION:

$$(x-p)(x-q)=0$$
$$(x-4)(x-(-5))=0$$
$$(x-4)(x+5)=0$$
$$x^2+5x-4x-20=0$$
$$x^2+x-20=0$$

61. **DRIVING** During a 10-hour trip, Kwan drove 4 hours at 60 miles per hour and 6 hours at 65 miles per hour. What was her average rate, in miles per hour, for the entire trip?

SOLUTION:

The distance traveled in the first four hours is 4×60 or 240 miles.

The distance traveled in the next six hours is 6×65 or 390 miles.

Total distance traveled is 240 + 390 or 630 miles.

Her average rate for the entire trip is $\frac{630}{10}$ or 63 mph.