PreCalc Name: ______________________________

Proving Closure Practice

1. Is the set of irrational numbers closed under multiplication? Prove or disprove it.

Pf: The set of irrational numbers is <u>not</u> closed under multiplication as the following counter example proves.

$\sqrt{2} \cdot \sqrt{2} = 2$, 2 is not irrational $\square$

2. Is the set of multiples of 5 closed under addition? Prove or disprove it.

Pf: Let x, y be any integer. Therefore $5x$ and $5y$ are any multiples of 5. $5x + 5y = 5(x+y)$. Since integers are closed under addition, $x+y$ is an integer. Therefore, $5(x+y)$ is a multiple of 5. Multiples of 5 are closed under addition. $\square$

3. If ☺ is a made up operation so that a ☺ b = a^b, is the set of integers closed under the operation ☺ ? Prove or disprove it.

Pf: The set of integers is <u>not</u> closed under the operation ☺ as the following counter example proves.

2 ☺ $-2 = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$\frac{1}{4}$ is not an integer $\square$

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Proving Closure Practice

4. Is the set { -1 , 0 , 1 } closed under multiplication? Prove or disprove it.

Pf: The set $\{-1, 0, 1\}$ is closed under multiplication because

$-1 \cdot -1 = 1$ ✓ $\quad 0 \cdot -1 = 0$ ✓ $\quad 1 \cdot -1 = -1$ ✓

$-1 \cdot 0 = 0$ ✓ $\quad 0 \cdot 0 = 0$ ✓ $\quad 1 \cdot 0 = 0$ ✓

$-1 \cdot 1 = -1$ ✓ $\quad 0 \cdot 1 = 0$ ✓ $\quad 1 \cdot 1 = 1$ ✓

□

* This is an example of proof by exhaustion. *

5. Is the set { -1 , 0 , 1 } closed under division? Prove or disprove it.

Pf: The set $\{-1, 0, 1\}$ is <u>not</u> closed under division as the following counterexample proves.

$1 \div 0$ does not exist

□

6. Is the set of fractions with a numerator of 1 (ex.{ … , -⅓, -½, ½, ⅓ , … } closed under multiplication? Is it closed under division? Prove or disprove it.

Pf: Let x, y be integers. So $\frac{1}{x}, \frac{1}{y}$ are in the set of fractions with a numerator of 1.

$\frac{1}{x} \cdot \frac{1}{y} = \frac{1 \cdot 1}{x \cdot y} = \frac{1}{xy}$ Since integers are closed under multiplication, xy is an integer. Therefore, the set of fractions with a numerator of 1 is closed under multiplication.

□

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Proving Closure Practice

7. Is the set of rational numbers closed under addition? Prove or disprove it.

Pf: Let x, y, a, b be integers. So $\frac{x}{y}$, $\frac{a}{b}$ are rational numbers.

$\frac{x}{y} + \frac{a}{b} = \frac{xb}{yb} + \frac{ay}{yb} = \frac{xb+ay}{yb}$. Since integers are closed under multiplication and addition, $(xb+ay)$ and (yb) are integers. So $\frac{xb+ay}{yb}$ is a rational number. Therefore, rational numbers are closed under addition. $\square$

8. If ☀ is a made up operation so that a ☀ $b = a^2 - b^2$, is the set of negative integers closed under the operation ☀? Prove or disprove it.

Pf: The set of negative integers is <u>not</u> closed under the operation ☀ as proves the counter example:

-4 ☀ -2 $= (-4)^2 - (-2)^2 = 16 - 4 = 12$

12 is not a negative integer. $\square$

9. Is the set of even integers closed under addition? Prove or disprove it.

Pf: Let x, y be integers. So 2x, 2y are even integers.

$2x + 2y = 2(x+y)$ Since integers are closed under addition, x+y is an integer. Therefore 2(x+y) is an even integer. Even integers are closed under addition. $\square$

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Proving Closure Practice

10. Is the set of odd integers closed under the following operations? Prove or disprove it for each operation.

a. Addition

Pf: The set of odd integers is <u>not</u> closed under addition, as proves the following counter example:

$$3+5=8 \qquad \square$$

b. Subtraction

Pf: The set of odd integers is <u>not</u> closed under subtraction, as proves the following counter example:

$$9-3=6 \qquad \square$$

c. Multiplication

Pf: Let x, y be integers so that $2x+1$ and $2y+1$ are odd integers.

	$2x$	1
$2y$	$4xy$	$2y$
1	$2x$	1

$4xy+2x+2y+1 = 2(2xy+x+y)+1$. Since $2xy+x+y$ is an integer, $2(2xy+x+y)+1$ is an odd integer.

Therefore the set of odd integers is closed under multiplication. $\square$

d. Division

Pf: The set of odd integers is <u>not</u> closed under division, as proves the following counter example:

$$3 \div 5 = .6 \qquad \square$$