t-test for means

Homework

- Read Sec 8-4
- Discussion Question p425
- Ex 8-4 p425 3-5, 7, 11, 12, 14, 16, 18

Objective

Perform hypothesis test for means of samples using the t statistic.

t-test for mean

Testing a hypothesis about population means

Compares the means of two **normally** distributed groups to determine if a **difference** exists relative to some value.

Objective: Students will perform hypothesis tests for means of samples using the t statistic.

Statistics 8-4

t-test for mean

For small samples and **when the population standard deviation is unknown**, the statistic used to test the null hypothesis, **H_0**, is the **Student's t-statistic**.

Like the z score, the **t-statistic** is found by comparing an **observed** value (through research) to an **expected** value from a population **assuming the null hypothesis is true.**

Remember: The t-distribution resembles the z distribution with normal shape and mean = 0 but the t distribution has variance > 1, and is a family of curves **based on degrees of freedom**.

Objective: Students will perform hypothesis tests for means of samples using the t statistic.

Statistics 8-4

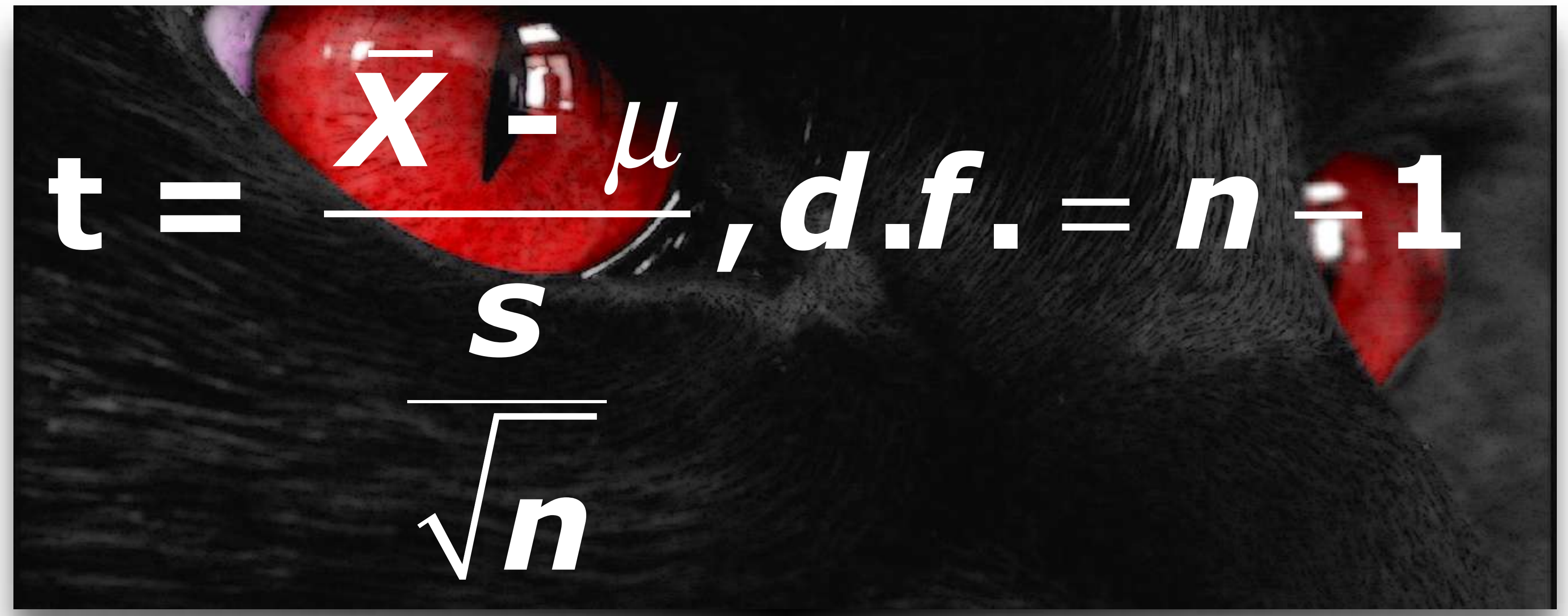
t-test for mean

When deciding to choose between a z statistic or a t statistic; your book and I differ slightly in opinion. Your book suggests that for large samples you can use the z statistic. My decision is simpler. I suggest you use the **t-statistic** any time **the population standard deviation is unknown**.

t-test for mean

$$\text{Test Statistic} = \frac{\text{Observed Value - Expected Value}}{\textit{sample}\ \text{standard error}}$$

$$t = \frac{\text{Sample mean - Population mean}}{\text{adjusted standard deviation}}$$



Look familiar?

Steps for Hypothesis Testing

1. Formulate all hypotheses: H_0, H_a

2. Determine the **test statistic** and the **critical value of the test statistic** (based on α). (Currently t)

2a. Draw, label, and appropriately shade the curve representing H_0

3. Find value(s) of the **test statistic** based on **the data** and the **p-value** for that statistic.

4. Make a **decision** to **Reject or Fail to reject H_0**.

5. Write your **conclusion** in a complete sentence.

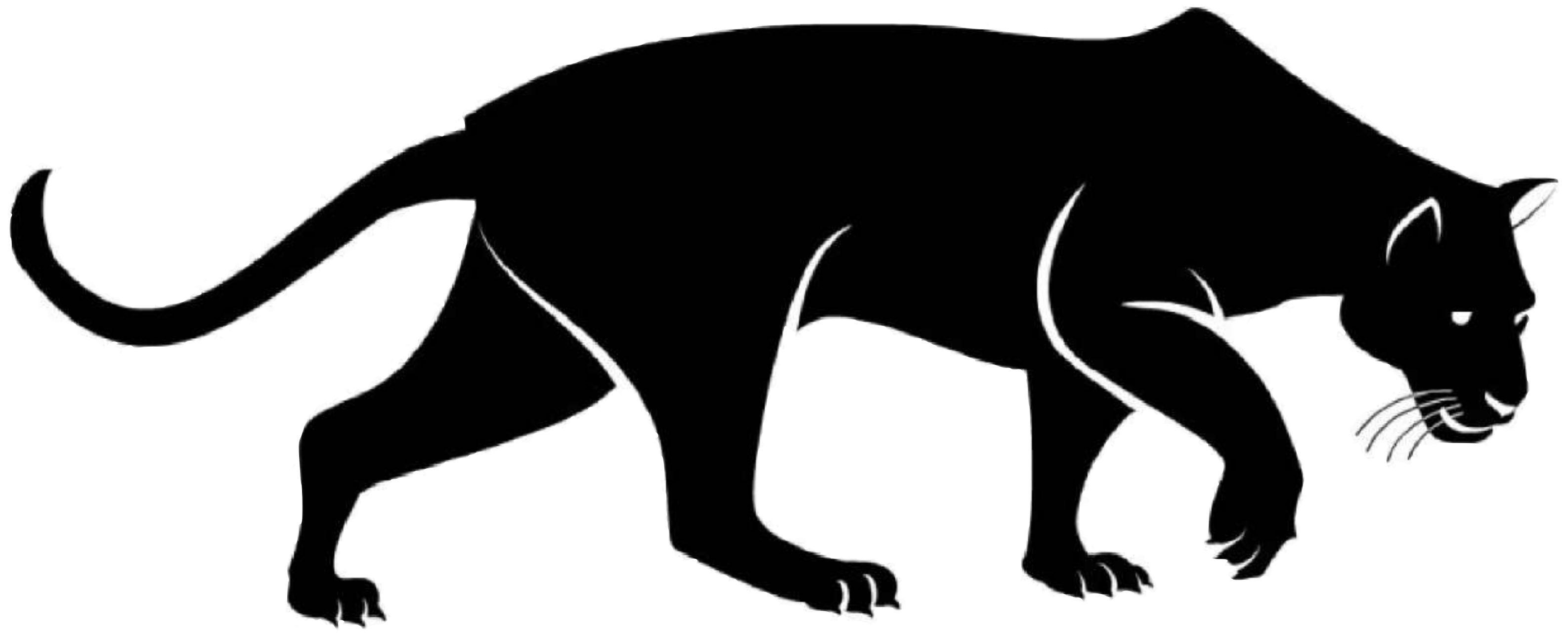
H_0: Null Hypothesis

The null hypothesis, **H_0**, (written as an equation containing the population parameter) states that there is **no** real difference between the population parameter and the sample statistic. Any difference we observe is simply the result of sampling variability (sampling error).

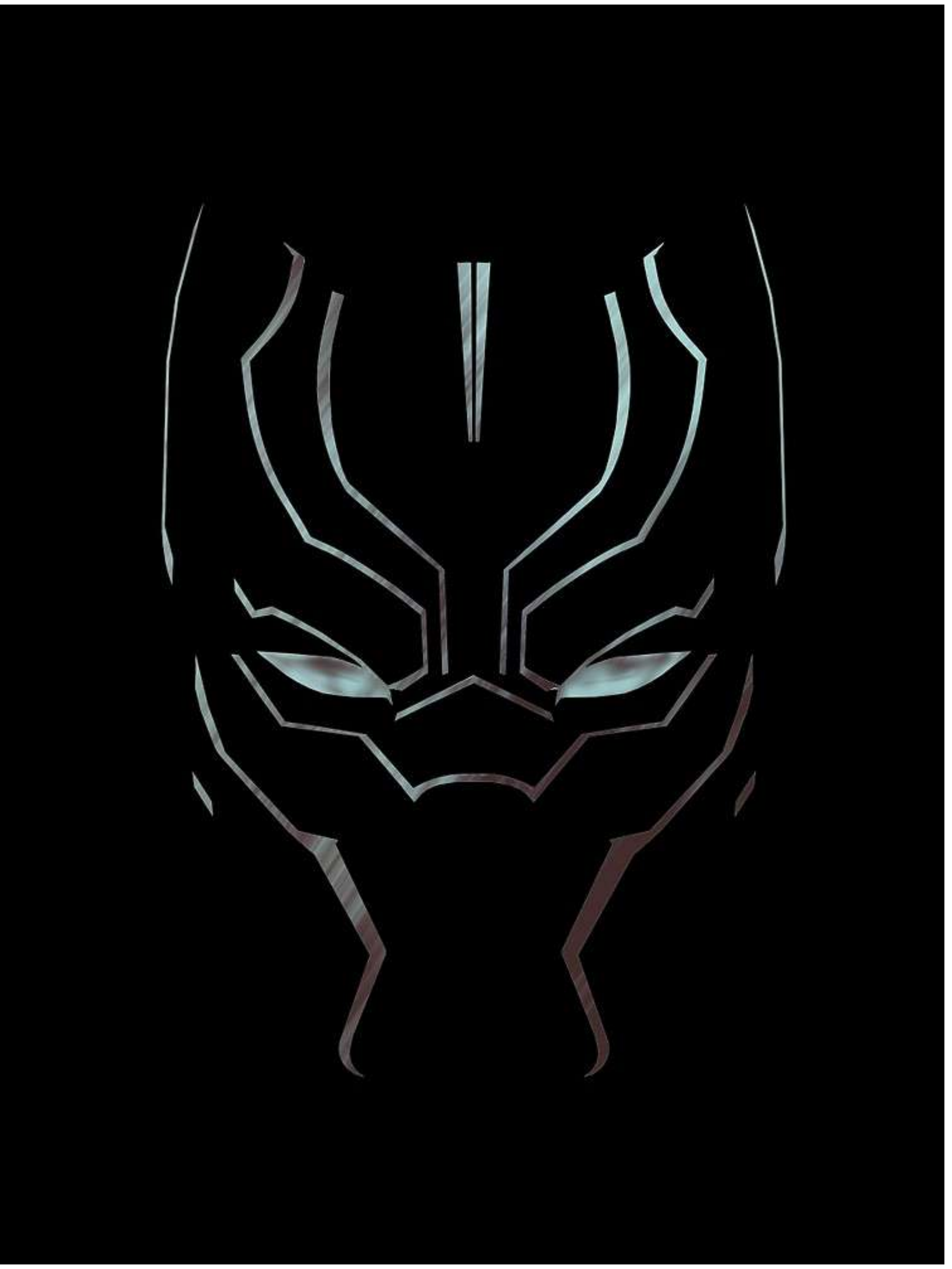
We expect the sample statistic to be different from the population value. The question becomes: when is the difference different enough to reject **H_0**?

H_a: Alternate Hypothesis

The alternative hypotheses, **H_a**, states that there **is** a statistically significant difference between the population parameter and the sample statistic. The difference we observe is not simply the result of sampling variability.



Example



A skeptical consumer (Kaffeine "Kaf" Kirk) tested 18 cans of her favorite caffeine delivery drink and found a sample mean of 15.8 ounces with a standard deviation of 0.4 ounces. Kaf is convinced her distributer is deliberately shorting the cans that are labeled 16 ounces. Is Kaf correct?

$n = 18$, $\bar{x} = 15.8$ ounces, $s = 0.4$ ounces. $\mu = 16$ ounces.

1. Hypotheses

n = 18, $\bar{x}$ = 15.8 ounces, s = 0.4 ounces. μ = 16 ounces.

The null hypothesis is that the mean volume for the caffeine delivery is 16 oz. The alternate hypothesis is the mean volume is less than 16 oz. (Remember, Kaf thinks she is being shorted.)

H_0: $\mu \geq$ 16oz and H_a: $\mu <$ 16oz

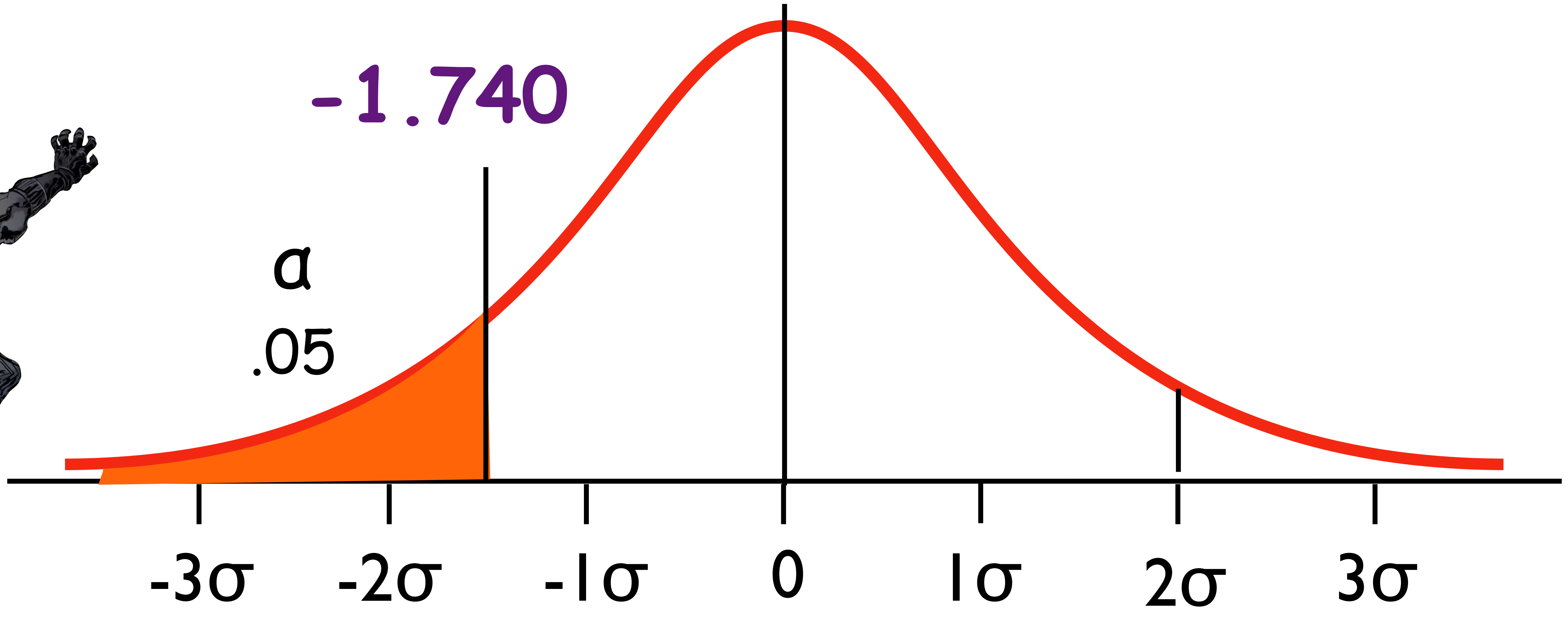
2. Critical Value

$n = 18$, $\bar{x} = 15.8$ ounces, $s = 0.4$ ounces. $\mu = 16$ ounces.

We are not given the level of significance so we default to .05. A one tailed test means all the critical area is below the mean.

The t-statistic corresponding to $\alpha = .05$ and d.f. = 17 is -1.740.

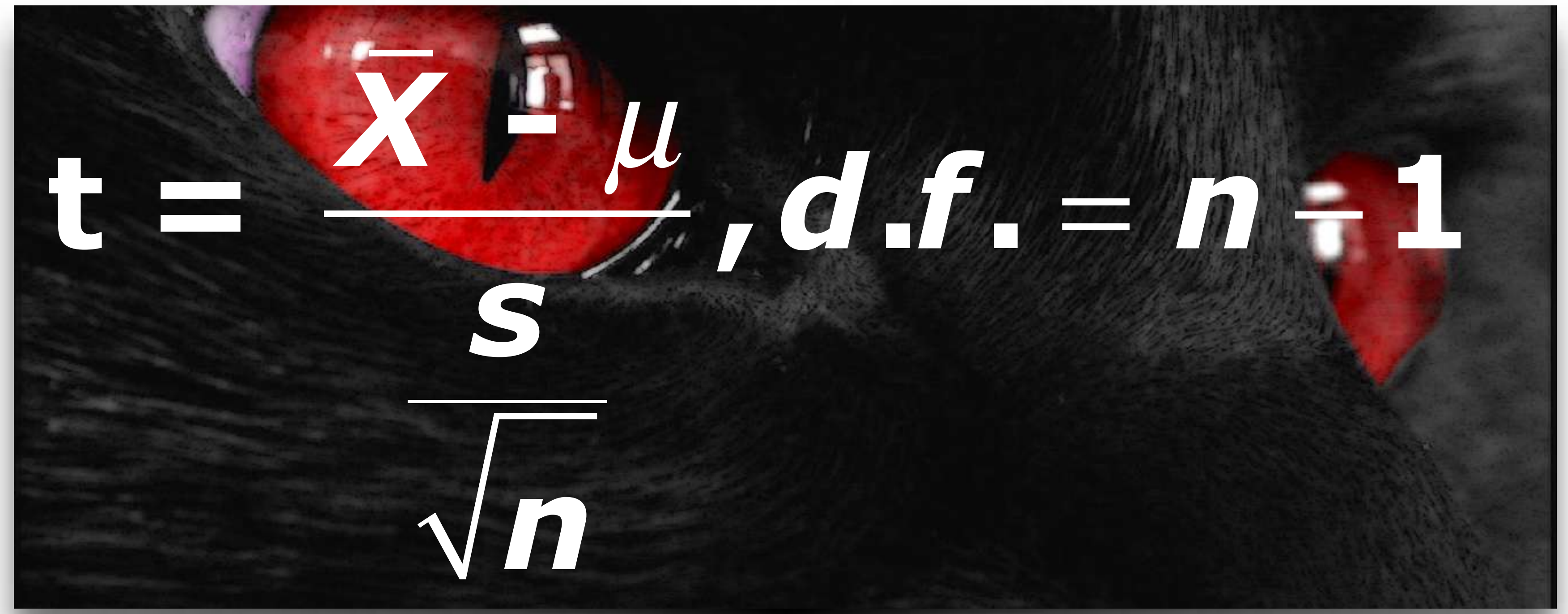
$t_{\alpha/2} = \text{Invt}(.05, 17) = -1.740$



Objective: Students will perform hypothesis tests for means of samples using the t statistic.

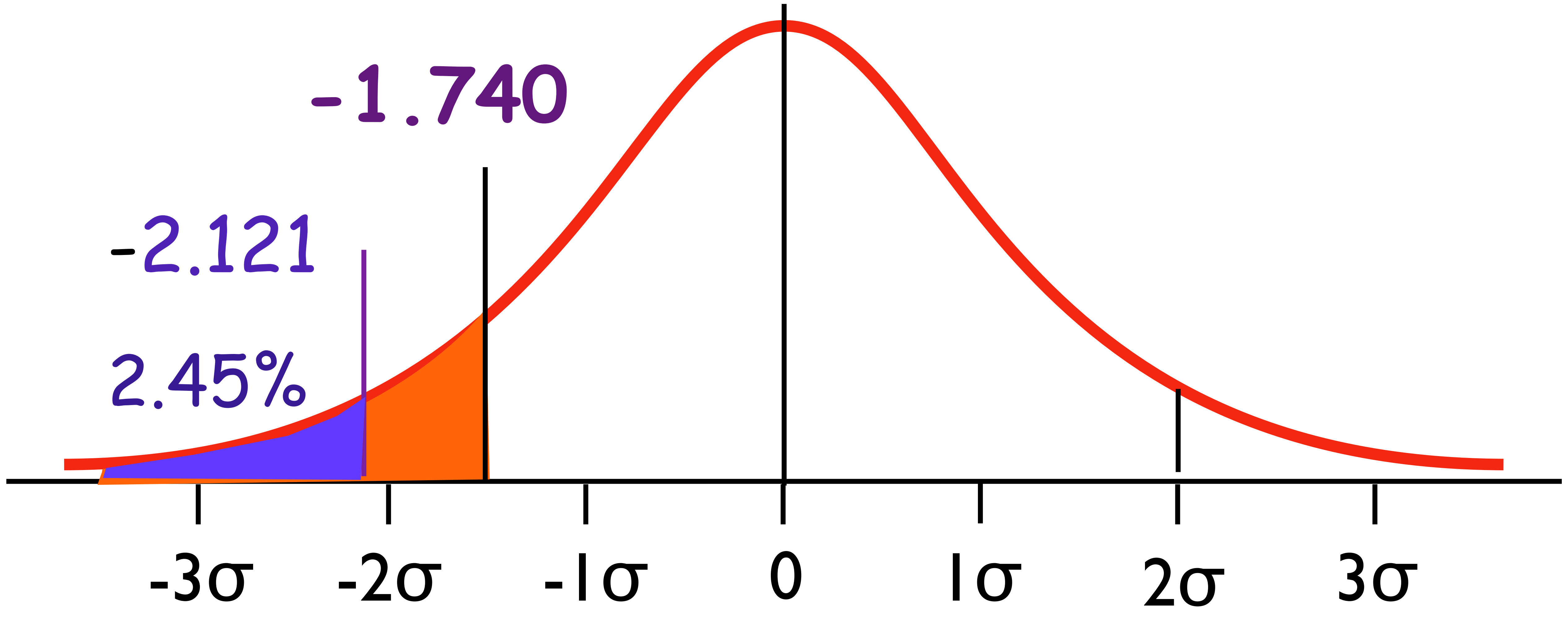
3. Calculate Value & p-value

n = 18, $\bar{x}$ = 15.8 ounces, s = 0.4 ounces. μ = 16 ounces.



$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}, d.f. = n - 1$$

$$t = \frac{15.8 - 16}{\frac{.4}{\sqrt{18}}} = -2.121$$



To run the t-test on the TI-84

TI-84

$n = 18$, $\bar{x} = 15.8$ ounces, $s = 0.4$ ounces. $\mu = 16$ ounces.

STAT ➢ TESTS ⋎ 1:z-Test

Inpt: Data **Stats**
μ_0: **16**
$\bar{x}$: **15.8**
s_x: **.4**
n: **18**
μ: $\neq\mu_0$ **$<\mu_0$** $>\mu_0$
Calculate

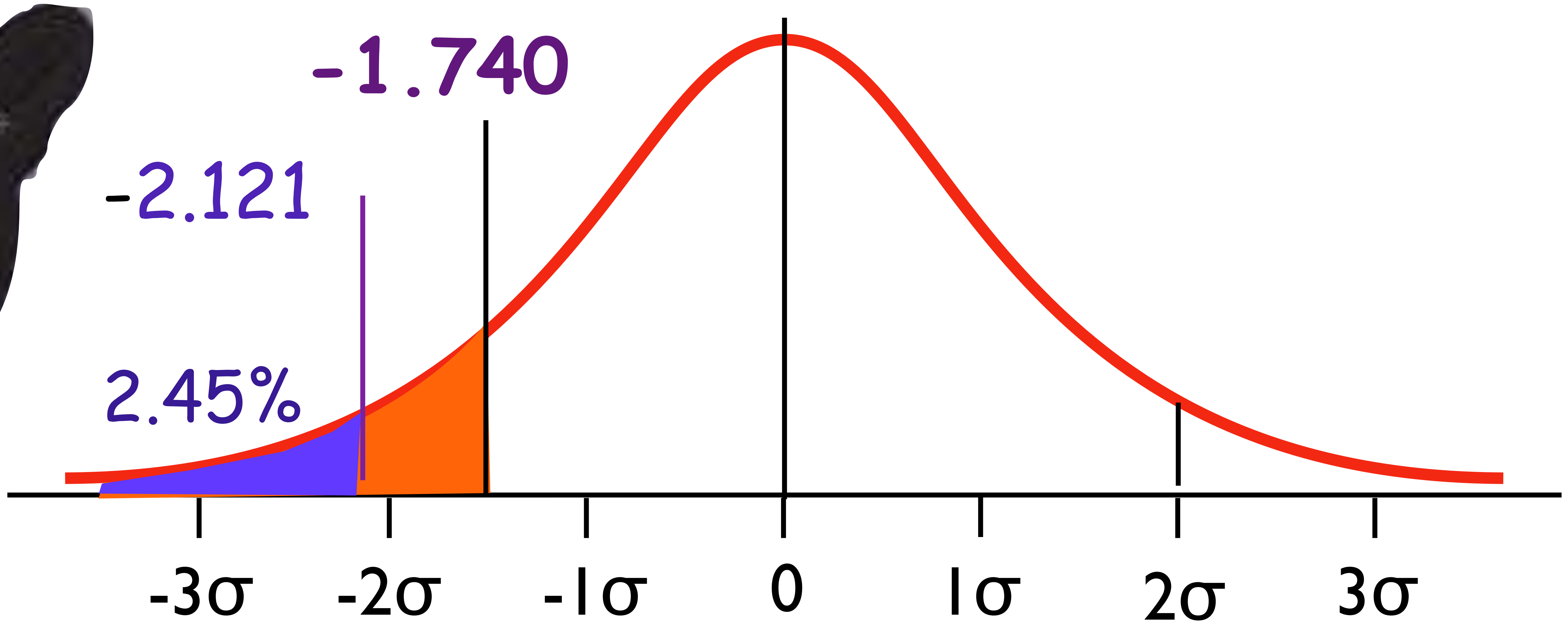
This is **H_a** ⟶ (μ: $\neq\mu_0$ $<\mu_0$ $>\mu_0$)

$\mu < 16$
t=-2.121320344
p=.0244479997
$\bar{x}$: 15.8
S_x: .4
n: 18

$p(t < -2.121) = \text{tcdf}(-10\text{^}99, -2.121, 17) = 0.0245$

$p(x < 15.8) = \text{tcdf}(-10\text{^}99, 15.8, 17) = 0.0245$

4. Decision



-2.121 < -1.740 (more extreme) and .0245 < .05 so our z statistic falls within the rejection region. The probability of getting that value is less than 5%, thus…

We reject the null

5. Conclusion

Since we reject H_0 …

There is sufficient evidence to suggest the mean volume of the caffeine delivery system is less than 16 oz.

It is plausible to believe the dealer is deliberately short-changing our consumer Kaffeine Kirk.

Kaffeine Kirk may very well be correct.

Objective: Students will perform hypothesis tests for means of samples using the t statistic.

Statistics 8-4

z or t?

Remember:

Z test if sample is large, $n \geq 30$
Z test if σ is known
t test if sample is small and σ is not known

Or:

z test if σ is known
t test if σ is not known

Of course, this assumes (requires) the population is at least approximately unimodal and symmetric. If we cannot be certain the population is sufficiently normally distributed we must ensure our sample is large, i.e. $n \geq 30$.

TI-84

Let us say that in previous years the average temperature for this time of year is 67°F. Students are complaining that this year the temperatures are different. To find out the students record the temps at noon for a two week period. For this example we will assume σ = 5.

59 62 62 70 74 75 69 63 61 68 61 62 64 66

Test the student's conjecture at a significance level of .05

Let us say that in previous years the average temperature for this time of year is 67°F. Students are complaining that this year the temperatures are different. To find out the students record the temps at noon for a two week period. For this example we will assume $\sigma = 5$.

59 62 62 70 74 75 69 63 61 68 61 62 64 66

1. Hypotheses

$H_0: \mu = 67°$ and $H_1: \mu \neq 67°$

Why did we choose a 2-tailed test?

2. Critical Value (z)

Since the purpose at the moment is to learn to use the calculator, we will be finding both z and t.

Enter the following data into L_1:

59 62 62 70 74 75 69 63 61 68 61 62 64 66

To clear an earlier list - stat - 4: 2nd L_1 - Enter

Objective: Students will perform hypothesis tests for means of samples using the t statistic.

Statistics 8-4

2. Critical Value

Finding the critical for both z:

z 2nd VARS (Distr) ⋎ 3:InvNorm(.975) ENTER 1.959963986

& t-values:

t 2nd VARS (Distr) ⋎ 4:Invt(.975, 13) ENTER 2.160368652

Why did we use .975?

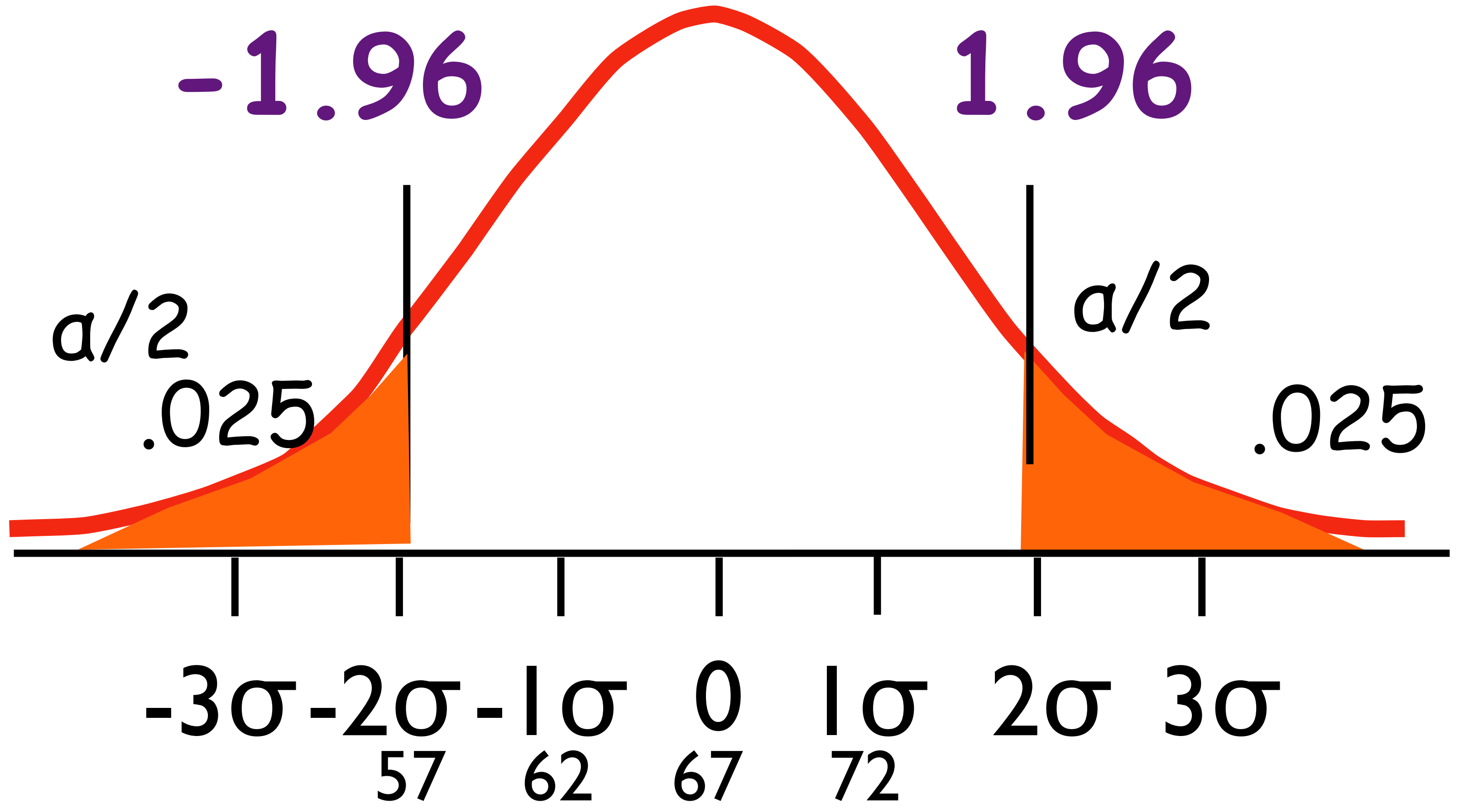
3. Calculate Value (z)

To find the z stat:

STAT ➢ TESTS ∨ 1:z-Test

Inpt: Data Stats
μ_0: 67
σ: 5
List: L_1
Freq:1
μ: $\neq\mu_0$ $<\mu_0$ $>\mu_0$
Calculate

This is H_a ⟶ μ: $\neq\mu_0$



Results & Decision

Z-Test

$\mu \neq 67$

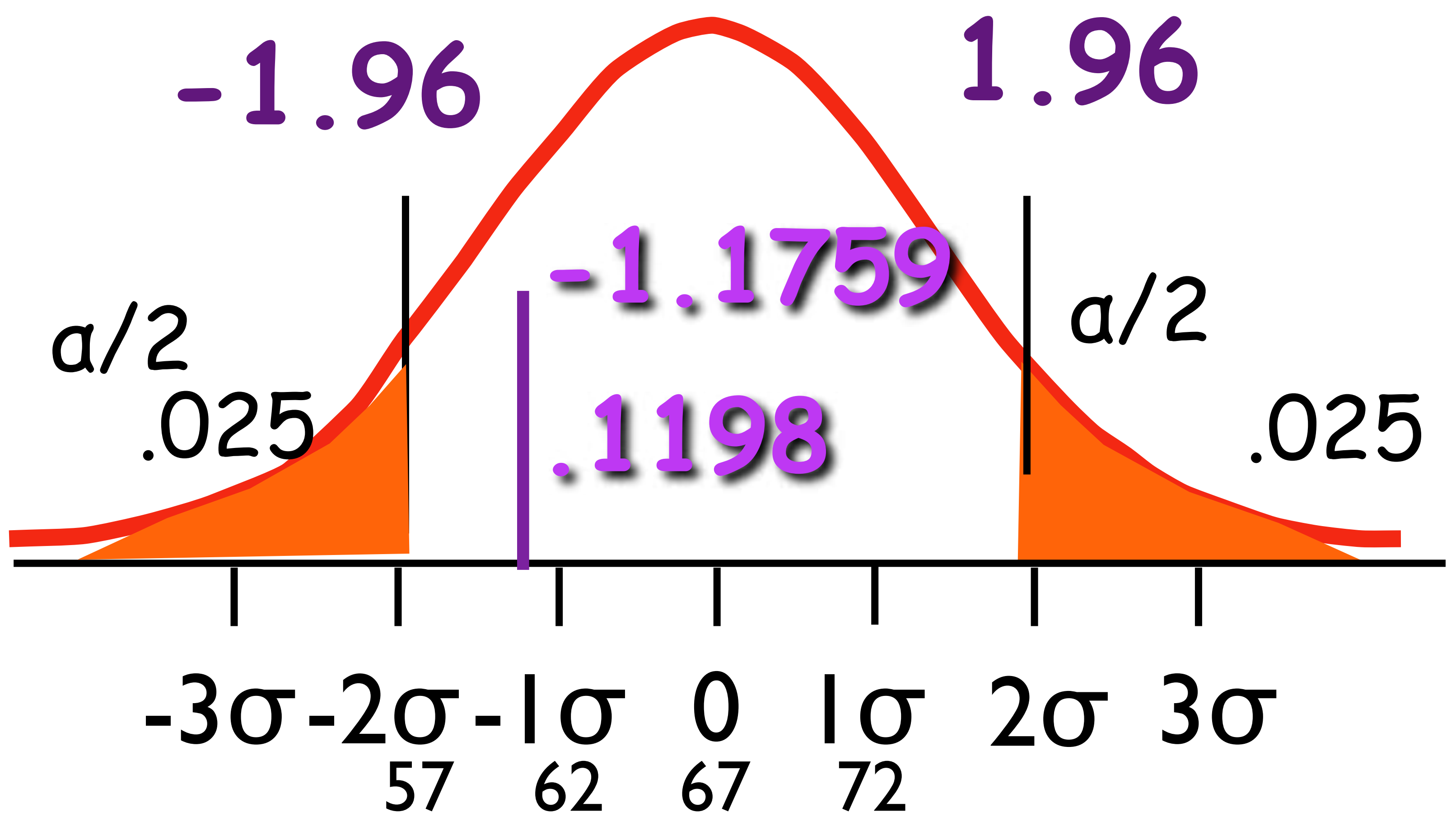
$-1.1759 > -1.96 \longrightarrow z = -1.175949464$

$.2396 > .05 \longrightarrow p = .2396151867$

$\bar{x} = 65.42857143$

$S_x = 5.033950669$

$n = 14$



$$t = \frac{65.4286 - 67}{\frac{5}{\sqrt{14}}} = -1.1759$$

Fail to Reject H_0

Objective: Students will perform hypothesis tests for means of samples using the t statistic.

Statistics 8-4

3. Calculate Value (t)

To find the t stat:

STAT ➢ TESTS ∨ 2:T-Test

Inpt: Data Stats
μ_0: **67**
List: **L_1**
Freq: **1**
This is **H_a** ⟶ μ: $\neq\mu_0$ $<\mu_0$ $>\mu_0$
Calculate

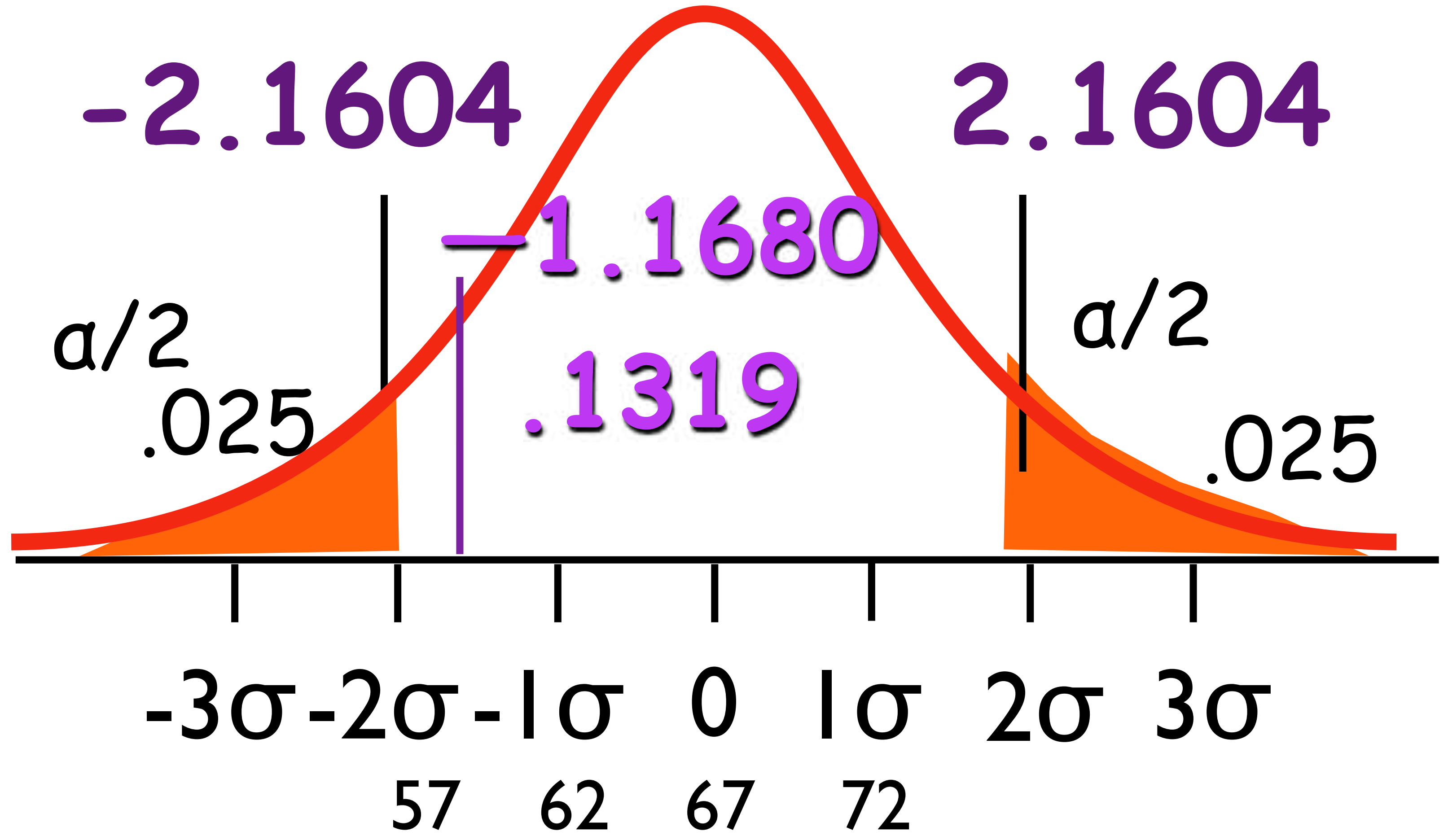
$$t = \frac{65.4286 - 67}{\frac{5.0340}{\sqrt{14}}} = -1.1680$$

Why is this t statistic different from the previous z statistic?

Results & Decision

$\mu \neq 67$
t=-1.1680
p=.2637747346
$\bar{x}$: 65.42857143
S_x: 5.033950669
n: 14

Fail to Reject H_0



Conclusion

In this case, the decision and conclusion are the same for both z- and t-tests.

There is **not sufficient evidence** to conclude the temperature for this time of year is different than the historical average.

Now run the tests using statistics instead of the data.

z-test $\mu = 67$, $\sigma = 5$

t-test $\mu = 67$, $s = 5.034$

Stats

If we know the sample mean we can test the hypotheses using the means and standard deviation.

Suppose a population with a mean of 135 lbs has a standard deviation of 32 lbs. Would a sample of 38 items weighing in at 138 lbs be enough to reject H_0?

1. Hypotheses

H_0: $\mu = 135$ lbs and H_a: $\mu \neq 135$ lbs

Objective: Students will perform hypothesis tests for means of samples using the t statistic.

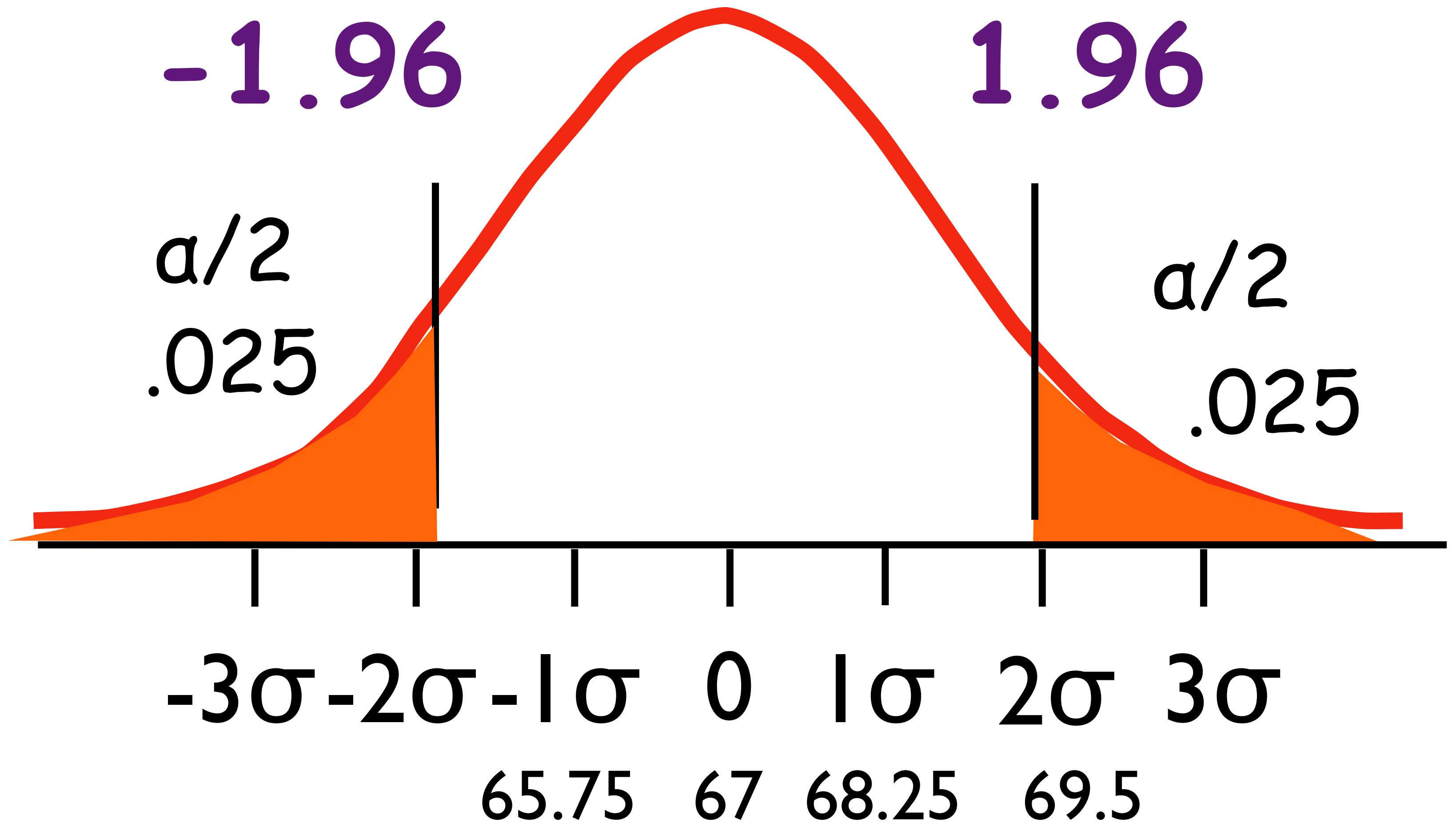
Statistics 8-4

2. Critical Value (z)

Distr

2nd VARS ⋎ 3:InvNorm(.975) ENTER

1.959963986



3. Calculate Value (z)

We do not have any data to enter.

To find the z stat:

STAT ➢ TESTS ⋎ 1:z-Test

Inpt: Data Stats

μ_0: 135

σ: 32

$\bar{x}$: 138

n: 38

This is H_a ⟶ μ: $\neq\mu_0$ $<\mu_0$ $>\mu_0$

Calculate

Objective: Students will perform hypothesis tests for means of samples using the t statistic.

Statistics 8-4

Results

Z-Test

$\mu \neq 135$

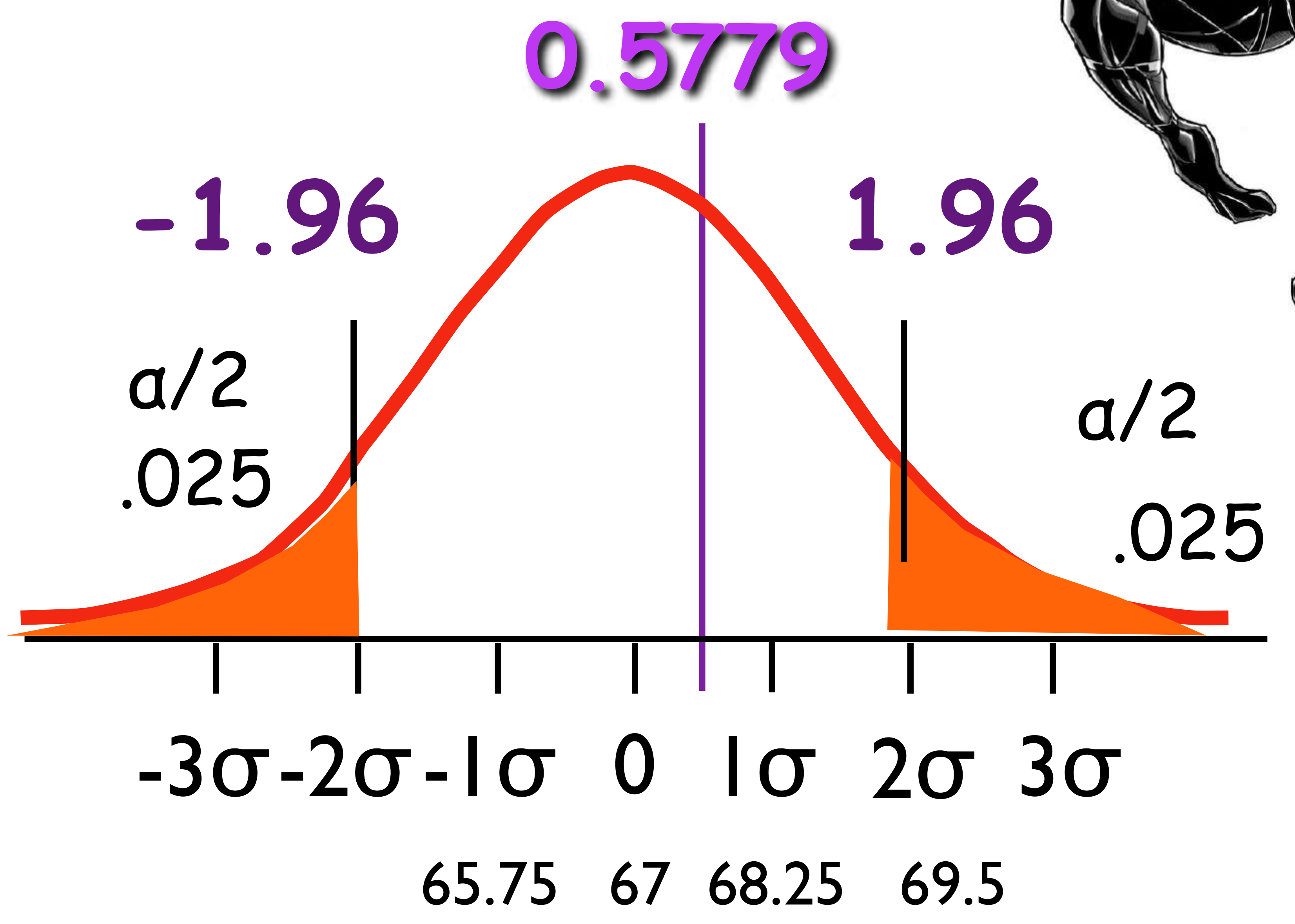
$.5779 < 1.96 \longrightarrow z = .5779138128$

$.563 > .05 \longrightarrow p = .56332222$

$x = 138$

$n = 38$

$$z = \frac{138 - 135}{\frac{32}{\sqrt{38}}} = .5779$$



Fail to reject H_0