

# Pre-Calculus H Unit 5: Functions and Graphs

<b>Unit #:</b>	APSDO-00019256	<b>Duration:</b>	7.0 Week(s)	<b>Date(s):</b>	
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**Grades:**  
10, 11, 12

**Subjects:**  
Mathematics

## Unit Focus

In this unit, students identify, graph, and represent simple transformations of functions. They investigate the composition of more than one function, the inverse of a function, and the use functions to model real world data. Students also graph and identify key features of polynomial and rational functions. Summative assessments may include projects, labs and test. Primary instructional materials for this unit include Pre-Calculus with Limits, Larson, Hostetler, and Edwards, 2008.

## Stage 1: Desired Results - Key Understandings

Established Goals	Transfer	
<p><b>Common Core</b> <i>Mathematics: 11</i></p> <ul style="list-style-type: none"> <li>• Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. <i>CCSS.MATH.CONTENT.HSA.CED.A.1</i></li> <li>• Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</li> </ul>	<p><b>T1</b> (T50) Based on an understanding of any problem, initiate a plan, execute it and evaluate the reasonableness of the solution.</p> <p><b>T2</b> (T53) Articulate how mathematical concepts relate to one another in the context of a problem or in the theoretical sense.</p> <p><b>T3</b> (T51) Examine alternate methods to accurately and efficiently solve problems.</p> <p><b>T4</b> (T52) Use appropriate tools strategically to deepen understanding of mathematical concepts.</p> <p><b>T5</b> (T23) Use functions or equations to model relationships among quantities.</p> <p><b>T6</b> (T24) Classify, interpret, and compare functions or equations.</p>	
	Meaning	
	Understandings	Essential Questions
	<p><b>U1</b> (U530) Every problem belongs to a</p>	<p><b>Q1</b> (Q532) Which model best represents this</p>

<p><i>CCSS.MATH.CONTENT.HSS.ID.B.6.A</i></p> <ul style="list-style-type: none"> <li>Graph linear and quadratic functions and show intercepts, maxima, and minima.</li> </ul> <p><i>CCSS.MATH.CONTENT.HSF.IF.C.7.A</i></p> <ul style="list-style-type: none"> <li>Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</li> </ul> <p><i>CCSS.MATH.CONTENT.HSA.APR.A.1</i></p> <ul style="list-style-type: none"> <li>Combine standard function types using arithmetic operations.</li> </ul> <p><i>CCSS.MATH.CONTENT.HSF.BF.A.1.B</i></p> <ul style="list-style-type: none"> <li>Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</li> </ul>	<p>category of problems that has a similar structure and set of characteristics; which means it can be solved using a similar model.</p> <p><b>U2</b> (U560) Patterns and structures are characterized by consistent relationships.</p> <p><b>U3</b> (U206) A function can represent how quantities in the real world relate to one another.</p> <p><b>U4</b> (U207) Recognition of predictable mathematical patterns supports the analysis of functional relationships and the prediction of data.</p>	<p>problem?</p> <p><b>Q2</b> (Q533) How do I use the model to solve other problems?</p> <p><b>Q3</b> (Q560) What is the pattern/structure in this problem?</p> <p><b>Q4</b> (Q561) How does understanding the pattern/structure help me solve the problem?</p> <p><b>Q5</b> (Q562) How do values and/or concrete models relate to each other?</p> <p><b>Q6</b> (Q205) How can I represent this relationship as a function or equation? (Gr. 6-12)</p> <p><b>Q7</b> (Q207) How do I classify, interpret, and compare functions or equations? (Gr. 8-12)</p> <p><b>Q8</b> (Q208) What function best models the data? How do its characteristics help me make predictions? (Gr. 8-12)</p>
<b>Acquisition of Knowledge and Skill</b>		
<b>Knowledge</b>		<b>Skills</b>
<p><i>CCSS.MATH.CONTENT.HSF.IF.C.7.B</i></p> <ul style="list-style-type: none"> <li>Know and apply the Remainder Theorem: For a polynomial <math>p(x)</math> and a number <math>a</math>, the remainder on division by <math>x - a</math> is <math>p(a)</math>, so <math>p(a) = 0</math> if and only if <math>(x - a)</math> is a factor of <math>p(x)</math>.</li> </ul> <p><i>CCSS.MATH.CONTENT.HSA.APR.B.2</i></p> <ul style="list-style-type: none"> <li>Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</li> </ul> <p><i>CCSS.MATH.CONTENT.HSF.IF.A.2</i></p> <ul style="list-style-type: none"> <li>Verify by composition that one function is the inverse of another.</li> </ul> <p><i>CCSS.MATH.CONTENT.HSF.BF.B.4.B</i></p> <ul style="list-style-type: none"> <li>Compose functions.</li> </ul> <p><i>CCSS.MATH.CONTENT.HSF.BF.A.1.C</i></p> <ul style="list-style-type: none"> <li>Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</li> </ul> <p><i>CCSS.MATH.CONTENT.HSF.IF.C.7.C</i></p> <ul style="list-style-type: none"> <li>Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both</li> </ul>		<p><b>S1</b></p> <p>Use a graphical device to create a model from a given set of data (linear, quadratic)</p> <p><b>S2</b></p> <p>Use transformations to graph various functions (linear, absolute value, quadratic, square root, cubic, cube root, trigonometric)</p> <p><b>S3</b></p> <p>Determine whether a function is even or odd (algebraically and graphically)</p> <p><b>S4</b></p> <p>Perform operations with functions</p> <p><b>S5</b></p>

<p>positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. <i>CCSS.MATH.CONTENT.HSF.BF.B.3</i></p> <ul style="list-style-type: none"> <li>• Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. <i>CCSS.MATH.CONTENT.HSA.APR.B.3</i></li> <li>• For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>CCSS.MATH.CONTENT.HSF.IF.B.4</i></li> <li>• Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. <i>CCSS.MATH.CONTENT.HSF.IF.C.7.D</i></li> <li>• Produce an invertible function from a non-invertible function by restricting the domain. <i>CCSS.MATH.CONTENT.HSF.BF.B.4.D</i></li> <li>• Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>CCSS.MATH.CONTENT.HSF.IF.B.5</i></li> <li>• Rewrite simple rational expressions in different forms; write <math>a(x)/b(x)</math> in the form <math>q(x) + r(x)/b(x)</math>, where <math>a(x)</math>, <math>b(x)</math>, <math>q(x)</math>, and <math>r(x)</math> are polynomials with the degree of <math>r(x)</math> less than the degree of <math>b(x)</math>, using inspection, long division, or, for the more complicated examples, a computer algebra system.</li> </ul>		<p>Evaluate functions using function notation algebraically and graphically</p> <p><b>S6</b></p> <p>Form a composite function and find its domain</p> <p><b>S7</b></p> <p>Write and graph inverse functions from a graph or equation</p> <p><b>S8</b></p> <p>Restrict the domain of a function to ensure that the inverse is also a function</p> <p><b>S9</b></p> <p>Identify key features of a graph (domain, range, intervals of increasing/decreasing, relative and absolute minima/maxima)</p> <p><b>S10</b></p> <p>Sketch polynomial functions using intercepts, end behavior, and additional coordinates</p> <p><b>S11</b></p> <p>Understand continuity of a function</p> <p><b>S12</b></p> <p>Understand how multiplicity of roots affects the graph</p> <p><b>S13</b></p> <p>Find real zeros of polynomial functions using long division or synthetic division</p> <p><b>S14</b></p> <p>Use the remainder theorem to evaluate</p>
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<p><i>CCSS.MATH.CONTENT.HSA.APR.D.6</i></p> <ul style="list-style-type: none"> <li>Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</li> </ul> <p><i>CCSS.MATH.CONTENT.HSF.TF.B.6</i></p> <ul style="list-style-type: none"> <li>Solve quadratic equations with real coefficients that have complex solutions.</li> </ul> <p><i>CCSS.MATH.CONTENT.HSN.CN.C.7</i></p> <ul style="list-style-type: none"> <li>Extend polynomial identities to the complex numbers. For example, rewrite <math>x^2 + 4</math> as <math>(x + 2i)(x - 2i)</math>.</li> </ul> <p><i>CCSS.MATH.CONTENT.HSN.CN.C.8</i></p> <ul style="list-style-type: none"> <li>Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</li> </ul> <p><i>CCSS.MATH.CONTENT.HSN.CN.C.9</i></p> <ul style="list-style-type: none"> <li>Look for and make use of structure.</li> </ul> <p><i>CCSS.MATH.MP.7</i></p> <ul style="list-style-type: none"> <li>Model with mathematics.</li> </ul> <p><i>CCSS.MATH.MP.4</i></p>		<p>functions</p> <p><b>S15</b></p> <p>Use a given root of a polynomial to find remaining roots</p> <p><b>S16</b></p> <p>Perform operations with and plot complex numbers</p> <p><b>S17</b></p> <p>Graph rational functions and identify key features (domain, limits, asymptotes, intercepts, holes) algebraically or from a graph</p> <p><b>S18</b></p> <p>Solve optimization problems using polynomials or rational functions</p>
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**Stage 2: Assessment Evidence**

**Performance Task(s)**

<b>Coding</b>	<b>Code</b>	<b>Description</b>
<p><b>T/U/Q/K/S</b></p> <ul style="list-style-type: none"> <li>U1</li> <li>U2</li> <li>U3</li> <li>U4</li> <li>S1</li> <li>S2</li> <li>S3</li> <li>S4</li> <li>S5</li> </ul>	<p>PT1</p>	<p><b>Untitled</b></p> <p><b>Performance Task</b></p> <p>Unit #5 Summative Assessment</p>

<ul style="list-style-type: none"> <li>• S6</li> <li>• S7</li> <li>• S8</li> <li>• S9</li> <li>• S10</li> <li>• S11</li> <li>• S12</li> <li>• S13</li> <li>• S14</li> <li>• S15</li> <li>• S16</li> <li>• S17</li> <li>• S18</li> </ul>		
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### Stage 3: Learning Plan

Coding	Code	Description of Learning Activity
<b>T/U/Q/K/S</b> <ul style="list-style-type: none"> <li>• Q3</li> <li>• Q4</li> <li>• Q7</li> <li>• S9</li> </ul>	LA1	<p><b>Learning Activity</b></p> <p><b>Learning Activity</b></p> <p>Lesson #2 Flipped Classroom: Graphs of Functions</p> <p><b>Resources</b></p> <p>RES1 Pre-Reading Section 1.3 <span style="float: right;"><a href="#">Download File</a></span></p> <p>RES4 Post Reading Section 1.3 <span style="float: right;"><a href="#">Download File</a></span></p>