Name SOLUTIONS PRE-AP PRECAL MATH TEAM

SOLUTIONS: Practice Quiz #1 Summer Reading Packet

1. Using the formulas from the packet, find the sum of the first 20 squares.

$$\frac{n(n+1)(2n+1)}{6} = \frac{20\cdot 21\cdot 41}{6} = \boxed{2870}$$

2. Using the formulas from the packet, find the sum of $1^4+2^4+3^4+\ldots+15^4$.

$$\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{15(15+1)(2\cdot 15+1)(3\cdot 15^2+3\cdot 15-1)}{30} = \boxed{178{,}312}$$

3. What kind of conic is represented by this equation?

$\underset{A}{3}x^2 - \underset{B}{4}xy + \underset{C}{5}y^2 + 6x - 7y + 8 = 0$

$$B^2-4AC = (-4)^2-4(3)(5) = 16-60 = -44 < 0$$

$\therefore$ $\boxed{\text{ELLIPSE}}$ *if a conic!

Simplify the following trig identities using the identities from your packet.

4. $\cos^2\theta + \underline{\sin^2\theta} = 1$

5. $\cos\alpha\cos\beta - \sin\alpha\sin\beta = \cos(\alpha+\beta)$

6. $1 + \boxed{\cot^2\theta} = \csc^2\theta$

7. $\sin\alpha\cos\beta + \cos\alpha\sin\beta = \sin(\alpha+\beta)$

8. $\cos^2\theta - \sin^2\theta = \cos 2\theta$

9. $\dfrac{1-\cos\theta}{\sin\theta} = \tan\dfrac{\theta}{2}$

10. $\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

11. $\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$

12. $2\cos^2\theta - 1 = \cos 2\theta$

13. $\dfrac{2\tan\theta}{1-\tan^2\theta} = \tan 2\theta$

14. $\sin 2\theta = 2\sin\theta\cos\theta$

15. $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

16. $\dfrac{\sin\theta}{1-\cos\theta} = \cot\dfrac{\theta}{2}$

b/c ↑ reciprocal of $\tan\frac{\theta}{2}$

17. What is the equation of the Law of Sines?

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

18. What is the equation of the Law of Cosines?

$c^2 = a^2 + b^2 - 2ab\cos C$

19. Factor:

$8x^3 - 27y^3$

$(2x - 3y)(4x^2 + 6xy + 9y^2)$

20. Simplify:

$|-4+5i|$ $= \sqrt{16+25} = \sqrt{41}$

21. Solve using the Quadratic Formula.

$3x^2 - 5x + 7 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(7)}}{2(3)} = \frac{5 \pm \sqrt{-59}}{6} = \boxed{\frac{5}{6} \pm \frac{\sqrt{59}}{6}i}$$

22. Determine what values of k will give the following equation real roots:

$2x^2 - 5x - 4k = 0$

$b^2 - 4ac \geq 0$

$(-5)^2 - 4(2)(-4k) \geq 0$

$25 + 32k \geq 0$

$k \geq \frac{-25}{32}$

23. The equation, $f(-x) = f(x)$, is true for even functions.

24. The equation, $f(-x) = -f(x)$, is true for odd functions.

25. The equation, $f(g(x)) = g(f(x)) = x$, is true for inverse functions.

26. What is a formula used to find the nth term of an Arithmetic Sequence?

$t_n = t_1 + (n-1)d$

27. What is a formula used to find the nth term of an Geometric Sequence?

$t_n = t_1 \cdot r^{n-1}$

28. What is a formula for the sum of an Arithmetic Sequence?

$S_n = \frac{n}{2}(t_1 + t_n)$

29. What are the two formulas used to find the sum of a finite Geometric Sequence?

$S_n = \frac{t_1(1-r^n)}{1-r}$; $S_n = \frac{t_1 - t_n \cdot r}{1-r}$

30. What is the formula used to find the sum of an infinite Geometric Sequence?

$S_\infty = \frac{t_1}{1-r}$ where $-1 < r < 1$

31. Find the 8^{th} term in the expansion of $\left(2x^3-3y^2\right)^{10}$ $k=7$

$$\frac{10!}{3!7!}\left(2x^3\right)^3\left(-3y^2\right)^7 = \frac{10\cdot 9\cdot 8}{3\cdot 2}\cdot\left(8x^9\right)\left(-2187y^{14}\right)$$

$$= 2099520x^9y^{14}$$

32. Find the area of a sector if the radius of the sector is 10 units, and the angle is 120°

$120° = \frac{2\pi}{3}$ radians

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(10)^2\left(\frac{2\pi}{3}\right) = \boxed{\frac{100\pi}{3}\text{ units}^2}$$

33. Simplify.

$_{11}P_3 = \frac{11!}{8!} = 11\cdot 10\cdot 9 = \boxed{990}$

34. $_{11}C_3 = \frac{11!}{8!3!} = \frac{11\cdot 10\cdot 9}{3\cdot 2} = \boxed{165}$

34. For any two events A and B in a sample space,

$P(A)+P(B)-P(A\cap B) =$ $P(A\cup B)$

35. the harmonic mean of 10 and 30 is . . .

$$\frac{2(10)(30)}{10+30} = \frac{2\cdot 10\cdot 3\cancel{0}}{4\cancel{0}} = \frac{60}{4} = \boxed{15}$$

36. What is the length of the latus rectum of the following parabola:

$y = -4x^2+2x+3$?

$\uparrow$ a

L.R. $= \left|\frac{1}{a}\right|$

$\therefore$ LR $= \left|\frac{1}{-4}\right| = \frac{1}{4}$

37. Given $\frac{(x-4)^2}{25}+\frac{(y+5)^2}{36}=1$, what is the area of the ellipse?

$b^2=25$ $a^2=36$

$b=5$ $a=6$

Area $= \pi ab = \pi(6)(5)$

$= \boxed{30\pi}$

38. Solve.

$\log_5 y = 4$

$y = 5^4$

$y = \boxed{625}$

$5^{x-2} = 3$

$\cancel{\log_5} 5^{x-2} = \log_5 3$

$x-2 = \log_5 3$

$x = \boxed{\log_5 3 + 2}$

39. Simplify.

$\log_5 3 + \log_5 7 =$ $\boxed{\log_5 21}$

$\log_5 36 - \log_5 4 =$ $\log_5\left(\frac{36}{4}\right) = \boxed{\log_5 9}$

40. Simplify.

$\log_7 49 = \log_7 7^2 = \boxed{2}$

$\log_{81} 9 = \log_{3^4} 3^2 = \frac{2}{4} = \boxed{\frac{1}{2}}$

41. Simplify.

$\log_{10} 1 = \boxed{0}$

$6^{\log_6 12} = \boxed{12}$

42. Simplify.

$\log_5(-2) = \boxed{\text{DNE}}$

$\log_7 49^x =$

$\cancel{\log_7} \cancel{7}^{2x} = \boxed{2x}$

43-46. Given the following polynomial, $f(x) = 5x^5 - 3x^4 + 2x^3 + 4x^2 - 7x + 12$, find the . . .

odd

Sum of the roots:

$-\frac{b}{a} = \frac{-(-3)}{5} = \boxed{\frac{3}{5}}$

Product of the roots:

$\therefore -\frac{\text{last}}{\text{first}} = -\frac{(12)}{(5)}$

$= \boxed{\frac{-12}{5}}$

Sum of the reciprocals:

$-\frac{\text{2nd to last}}{\text{last}} = \frac{-(-7)}{12}$

$= \boxed{\frac{7}{12}}$

Sum of the squares of the roots:

$\frac{b^2 - 2ac}{a^2} = \frac{(-3)^2 - 2(5)(2)}{(5)^2} = \boxed{\frac{-11}{25}}$

"a" "b" NO "c" term! 2nd to last last

47-50. Given the following polynomial, $f(x) = 2x^8 - 5x^7 + 3x^3 + 7x^2 - 6x + 13$, find the . . .

even

Sum of the roots:

$-\frac{(-5)}{2} = \boxed{\frac{5}{2}}$

Product of the roots:

$+\frac{\text{last}}{\text{first}} = \boxed{\frac{13}{2}}$

Sum of the reciprocals:

$-\frac{(-6)}{13} = \boxed{\frac{6}{13}}$

Sum of the squares of the roots:

$\frac{b^2 - 2ac}{a^2} = \frac{(-5)^2 - \cancel{2(2)(0)}}{(2)^2} = \frac{25 - 0}{4} = \boxed{\frac{25}{4}}$

51. Sum of the measures of interior angles of a 12 sided polygon.

$(12 - 2) \cdot 180$

$= \boxed{1800^\circ}$

52. Number of diagonals of a convex 23 sided polygon.

$\frac{23}{2}(23 - 3) = \boxed{230}$

53. $\lim_{n \to \infty}\left(1 - \frac{3}{n}\right)^n = e^{-3}$

54-57. From the number 3780, find the … $2^2 \cdot 3^3 \cdot 5^1 \cdot 7^1$

…number of positive integral factors.

take the powers… 2 3 1 1

add 1 to each… $(3)(4)(2)(2)$

multiply together… $\boxed{48}$

… number of integral factors.

Same # of pos. as there are neg.

$\therefore\ 2 \times 48 = \boxed{96}$

…sum of the positive integral factors.

$(2^2+2^1+1)(3^3+3^2+3^1+1)(5^1+1)(7^1+1)$

$= 7 \cdot 40 \cdot 6 \cdot 8 = \boxed{13440}$

…sum of the integral factors.

add pos. & neg.

$\therefore\ 13440 + (-13440)$

$= \boxed{0}$

58. Find the sum of the coefficients/numbers in the 12 row of Pascal's Triangle (this is technically the 13th row, but since it begins and ends with "1 12 …12 1" we call it the "12 row").

$2^{12} = \boxed{4096}$

59. Find the unit's digit of 2^{2363}?

$2^{2363} \Rightarrow 2^{63} \Rightarrow 2^3 \quad \therefore \quad \boxed{8}$

$$\begin{array}{r} 590 \\ 4\overline{)2363} \\ \underline{20} \\ 36 \\ \underline{36} \\ \boxed{3} \end{array}$$

60. Find the sum of the coefficients (and constants) in the expansion of

$\left(2x^3+4y^5-3z^2+m^3\right)^6$ $\quad (2+4-3+1)^6 = 4^6 = \boxed{4096}$

61. Simplify

$e^{180° i} =$

$\cos 180° + i \sin 180°$

$-1 + 0i = \boxed{-1}$

62. What are the first 4 perfect numbers?

6, 28, 496, 8128

(I'd memorize these…)

63. Determine the number of trailing zeros in 2013!

$\left[\!\left[\frac{2013}{5}\right]\!\right] + \left[\!\left[\frac{402}{5}\right]\!\right] + \left[\!\left[\frac{80}{5}\right]\!\right] + \left[\!\left[\frac{16}{5}\right]\!\right]$

$[\![402.6]\!] + [\![80.4]\!] + [\![16]\!] + [\![3.2]\!]$

$402 + 80 + 16 + 3 = \boxed{501}$

64. Write the repeating decimal as a simplified fraction:

$5.12\overline{36}$ $\quad \dfrac{51236-512}{9900} = \dfrac{50724}{9900} = \boxed{\dfrac{1409}{275}}$

54-57. From the number 3780, find the … $= 2^2 \cdot 3^3 \cdot 5^1 \cdot 7^1$

…number of positive integral factors.

take the powers… 2 3 1 1

add 1 to each… 3 4 2 2

multiply together $3 \cdot 4 \cdot 2 \cdot 2$

$= \boxed{48}$

… number of integral factors.

Same number of positive factors as there are negative

$\therefore\ 2 \times 48 = \boxed{96}$

…sum of the positive integral factors.

$(2^2+2^1+1)(3^3+3^2+3^1+1)(5^1+1)(7^1+1)$

$7 \cdot 40 \cdot 6 \cdot 8$

$= \boxed{13440}$

…sum of the integral factors.

add pos. to neg.

$13440 + (-13440)$

$= 0$

58. Find the sum of the coefficients/numbers in the 12 row of Pascal's Triangle (this is technically the 13th row, but since it begins and ends with " 1 12 …12 1" we call it the "12 row").

$2^{12} = 4096$

59. Find the unit's digit of 2^{2363}?

$2^{2363} \Rightarrow 2^{63} \Rightarrow 2^3 \therefore \boxed{8}$

or $2363 \div 4 = 590$ R 3

$$\begin{array}{r} 590 \\ 4\overline{)2363} \\ \underline{20} \\ 36 \\ \underline{36} \\ 03 \end{array}$$

60. Find the sum of the coefficients (and constants) in the expansion of

$\left(2x^3+4y^5-3z^2+m^3\right)^6$ $\therefore$ $(2+4-3+1)^6 = 4^6 = \boxed{4096}$

61. Simplify

$e^{180^\circ i} = \cos 180^\circ + i \sin 180^\circ$

$= -1 + 0i = \boxed{-1}$

62. What are the first 4 perfect numbers?

6, 28, 496, 8128

(memorize these!)

63. Determine the number of trailing zeros in 2013!

$\left\lfloor\frac{2013}{5}\right\rfloor + \left\lfloor\frac{402}{5}\right\rfloor + \left\lfloor\frac{80}{5}\right\rfloor + \left\lfloor\frac{16}{5}\right\rfloor$

$402 + 80 + 16 + 3$

$= \boxed{501}$

64. Write the repeating decimal as a simplified fraction: $5.12\overline{36}$

$\frac{51236-512}{9900} = \frac{50724}{9900} = \boxed{\frac{1409}{275}}$

65. Simplify.

a) $7!! =$

$7 \cdot 5 \cdot 3 \cdot 1$

$= \boxed{105}$

b) $10!!! =$

$10 \cdot 7 \cdot 4 \cdot 1$

$= \boxed{280}$

66. Simplify.

$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\ldots = \frac{\pi^2}{6}$

memorize these!