## SOLUTIONS PRE-AP PRECAL MATH TEAM

## **SOLUTIONS: Practice Quiz #1 Summer Reading Packet**

1. Using the formulas from the packet, find the sum of the first 20 squares.

$$\frac{h(n+1)(2n+1)}{6} = \frac{20 \cdot 21 \cdot 41}{6} = 2870$$

Using the formulas from the packet, find the sum of  $1^4 + 2^4 + 3^4 + ... + 15^4$ . 2.

$$\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{15(15+1)(2\cdot15+1)(3\cdot15^2+3\cdot15-1)}{30} = \frac{178,312}{178,312}$$

3. What kind of conic is represented by this equation?

$$3x^{2}-4xy+5y^{2}+6x-7y+8=0$$

$$B^{2}-4AC = (-4)^{2}-4(3)(5) = 16-60 = -44 \le 0$$

$$ELLIPSE * if a conic!$$

Simplify the following trig identities using the identities from your packet

4. 
$$\cos^2\theta + \sin^2\theta = 1$$

5. 
$$\cos \alpha \cos \beta - \sin \alpha \sin \beta = (\omega + \beta)$$

6. 
$$1 + \left[\cot^2\theta\right] = \csc^2\theta$$

7. 
$$\sin \alpha \cos \beta + \cos \alpha \sin \beta = \sin(\alpha + \beta)$$

8. 
$$\cos^2\theta - \sin^2\theta = \bigcirc 2\theta$$

$$\cos^2 \theta - \sin^2 \theta =$$
 9.  $\frac{1 - \cos \theta}{\sin \theta} = \frac{\theta}{2}$ 

10. 
$$\cos \frac{\theta}{2} = \pm \sqrt{1 + \cos \theta}$$

11. 
$$\sin\frac{\theta}{2} = \frac{1}{2}\sqrt{1-\cos\theta}$$
 12. 
$$2\cos^2\theta - 1 = \cos^2\theta$$

$$12. \quad 2\cos^2\theta - 1 = \cos 2\theta$$

13. 
$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\tan\theta}{1-\tan^2\theta}$$

14. 
$$\sin 2\theta = 2 \sin \theta \cos \theta$$

15. 
$$\tan(\alpha - \beta) = \frac{1 + \tan \alpha - \tan \beta}{1 + \tan \alpha + \cos \beta}$$

16. 
$$\frac{\sin \theta}{1 - \cos \theta} = \cot \frac{\theta}{2}$$

$$b/c \int ccc procal$$

17. What is the equation of the Law of Sines?

18. What is the equation of the Law of Cosines?

19. Factor:

20. Simplify:

$$8x^3 - 27y^3$$
  $(2x - 3y)(4x^2 + 6xy + 9y^2)$ 

$$|-4+5i| = \sqrt{16+25} = \sqrt{41}$$

21. Solve using the Quadratic Formula.

$$3x^{2}-5x+7=0$$

$$X = \frac{-(-s) \pm \sqrt{(-s)^{2}-4(3)(7)}}{2(3)} = \frac{5 \pm \sqrt{-59}}{6} = \frac{5}{6} \pm \frac{\sqrt{59}i}{6}i$$

22. Determine what values of k will give the following equation real roots:

$$2x^{2}-5x-4k=0$$

$$b^{2}-4ac \ge 0$$

$$(-5)^{2}-4(2)(4k) \ge 0$$

$$25 + 32k \ge 0$$

- 23. The equation, f(-x) = f(x), is true for \_\_\_\_\_\_ functions.
- 24. The equation, f(-x) = -f(x), is true for <u>odd</u> functions.
- 25. The equation, f(g(x)) = g(f(x)) = x, is true for \_\_\_\_\_\_\_ functions.
- 26. What is a formula used to find the n<sup>th</sup> term of an Arithmetic Sequence?

27. What is a formula used to find the n<sup>th</sup> term of an Geometric Sequence?

28. What is a formula for the sum of an Arithmetic Sequence?

$$S_n = \frac{n}{2} (t_1 + t_n)$$

29. What are the two formulas used to find the sum of a finite Geometric Sequence?

$$S_n = \frac{t_1(1-r^n)}{1-r}$$
 :  $S_n = \frac{t_1-t_n r}{1-r}$ 

30. What is the formula used to find the sum of an infinite Geometric Sequence?

31. Find the 8<sup>th</sup> term in the expansion of 
$$(2x^3 - 3y^2)^{10}$$

$$\frac{10!}{3!7!} (2x^3)^3 (-3y^2)^7 = \frac{10.9.8}{3.2} (8x^9)(-2187y^{14})$$

$$= (2099520x^{9})^{14}$$

Find the area of a sector if the radius of the sector is 10 units, and the angle is 
$$120^{\circ}$$

$$A = \frac{1}{2}r^2\Theta = \frac{1}{2}(10)^2(\frac{2\pi}{3}) = \frac{100\pi}{3} \text{ with}$$

33. Simplify. 34.
$${}_{11}P_3 = \frac{11!}{8!} = 11 \cdot 10 \cdot 9 = \boxed{990} {}_{11}C_3 = \frac{11!}{8!3!} = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2} = \boxed{165}$$

$$P(A) + P(B) - P(A \cap B) = P(A \cup B)$$

$$\frac{2(10)(30)}{10+30} = \frac{2 \cdot 10 \cdot 30}{40} = \frac{60}{4} = \boxed{15}$$

What is the length of the latus rectum of the following parabola:  

$$y = 4x^2 + 2x + 3$$
?

LR =  $\frac{1}{4}$ 

37. Given 
$$\frac{(x-4)^2}{25} + \frac{(y+5)^2}{36} = 1$$
, what is the area of the ellipse?  

$$\frac{7}{36} = 36$$
 Area = Tab = 1

$$6^{2}=25$$
  $a^{2}=36$  Area =  $\pi (6)(5)$   
 $6=5$   $a=6$  =  $30\pi$ 

$$\int_{S}^{\log_{S} y = 4}$$

$$y = 5$$

$$y = 625$$

$$5^{x-2} = 3$$
 $4x^{2} = \log_{5} 3$ 
 $x-2 = \log_{5} 3$ 
 $x = \log_{5} 3 + 2$ 

$$\log_5 3 + \log_5 7 =$$

$$\log_5 36 - \log_5 4 =$$

$$\log_5\left(\frac{36}{4}\right) = \log_59$$

$$\log_{10} l = \bigcirc$$

$$\log_5(-2) = \boxed{DNE}$$

$$\log_{81} 9 = \log_{34} 3^2 = \frac{2}{4} = \boxed{\frac{1}{2}}$$
  $6^{\log_6 12} = \boxed{12}$ 

$$\log_7 49^x = 2x$$

43-46. Given the following polynomial,  $f(x) = 5x^{5} - 3x^{4} + 2x^{3} + 4x^{2} - 7x + 12$ , find the . . .

Sum of the roots:

$$\frac{-b}{a} = \frac{-(-3)}{5} = \boxed{\frac{3}{5}}$$

$$\therefore -\frac{last}{first} = -\frac{(12)}{(5)}$$

Sum of the squares of the roots:  

$$\frac{b^2 - 2ac}{a^2} = \frac{(-3)^2 - 2(5)(2)}{(5)^2} = \boxed{\frac{-11}{25}}$$

 $\frac{b^2 - 2ac}{a^2} = \frac{(-3)^2 - 2(5)(2)}{(5)^2} = \frac{-11}{25}$   $\frac{-12}{5}$ 47-50. Given the following polynomial,  $f(x) = 2x^8 - 5x^7 + 3x^3 + 7x^2 - 6x + 13$ , find the . . .

Sum of the roots:

$$-\frac{(-s)}{z}=\boxed{\frac{5}{z}}$$

$$+ \frac{\log t}{\text{first}} = \boxed{\frac{13}{2}}$$

$$-\frac{(-6)}{13} = \boxed{\frac{6}{13}}$$

Sum of the squares of the roots:

$$\frac{b^2 - 2ac}{a^2} = \frac{(-5)^2 - 2(2)(0)}{(2)^2} = \frac{25 - 0}{4} = \boxed{\frac{25}{4}}$$

$$\frac{23}{2}\left(\underline{23}-3\right) = \boxed{230}$$

53. 
$$\lim_{n\to\infty} \left( \frac{3}{n} \right)^n = e^{-3}$$

54-57. From the number	378	0, fin	d the	·	22.	3.5	.7
number of pos	itive	integr	al fa	ctors.			nu
take the powers		3	(	1			9
	1 - 1		1 -	1/-1			

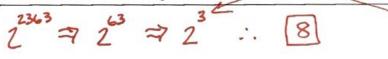
imber of integral factors. same # of pos. as there are neg. -: 2×48 = (96

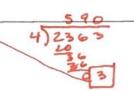
...sum of the positive integral factors.
$$(2^{2}+2^{1}+1)(3^{2}+3^{2}+3^{1}+1)(5^{1}+1)(7^{1}+1)$$
= 7 · 40 · 6 · 8 = [13440]

...sum of the integral factors. add pos. is neg. (-13440)

58. Find the sum of the coefficients/numbers in the 12 row of Pascal's Triangle (this is technically the 13th row, but since it begins and ends with "1 12 ... 12 1" we call it the "12 row").

Find the unit's digit of 2<sup>2363</sup>?





Find the sum of the coefficients (and constants) in the expansion of 60.

$$(2x^3 + 4y^5 - 3z^2 + m^3)^6 \qquad (2 + 4 - 3 + 1)^6 = 4 = 4096$$

61.

62. What are the first 4 perfect numbers? Simplify

(00 180 + isin 180 -1 + Oi

6,28,496,8128 (I'd memoringe these ...)

Determine the number of trailing zeros in 2013! 63.

[402.6] /+ [80.4] /+ [16] +/ [3.2]

Write the repeating decimal as a simplified fraction: 64.

51236-512 5.1236

## 54-57. From the number 3780, find the ... = 22-33-51-7 ...number of positive integral factors. ... number of integral factors. same number of positive factors as there are negative take the powers ... 2 3 1 1 add 1 to each ... 3 4 2 multiply togetum 3-4-2-2 : 2x48 = 96 ...sum of the positive integral factors. ...sum of the integral factors. (22+2+1)(33+3+1)(5+1)(7+1) add poo to neg. 13440 + (-13440) = 13440 58. Find the sum of the coefficients/numbers in the 12 row of Pascal's Triangle (this is technically the 13th row, but since it begins and ends with "1 12...12 1" we call it the "12 row"). 1 = 4096 Find the unit's digit of $2^{2363}$ ? 59. 2363 => 2 => 7 : 8 60. Find the sum of the coefficients (and constants) in the expansion of $(2x^3+4y^5-3z^2+m^3)^6$ : $(2+4-3+1)^6 = 4^6 = 4096$ 62. Simplify What are the first 4 perfect numbers? 61. $e^{180^{\circ}} = 60 180^{\circ} + i \sin(80^{\circ})$ 6, 28, 496, 8128 (memoringe tuese!) 64. Write the repeating decimal as a simplified fraction: 63. Determine the number of trailing 5.1236 zeros in 2013! $\frac{51236-512}{9900} = \frac{50724}{9900}$ 80 1+ 160 + 3 66. 65. Simplify. Simplify. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots = \frac{\pi^2}{4}$ a) 7!! =b) 10!!!=

b) 10!!!=  $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+...=$  7.5.3.1 10.7.4.1 = 280