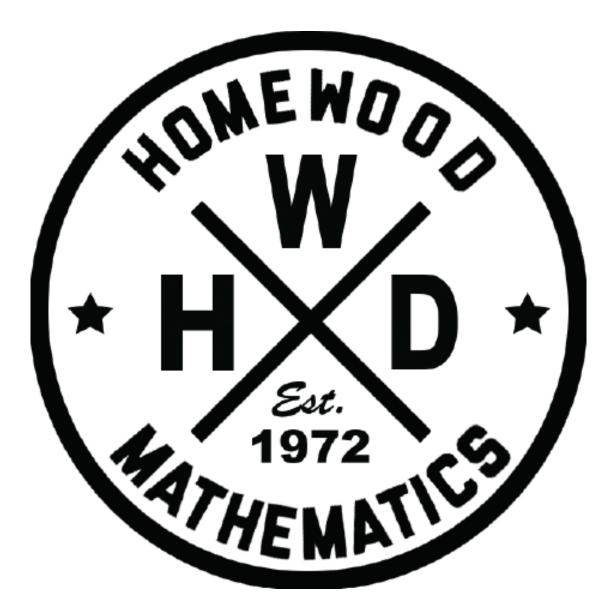
Pre-AP Algebra II Math Team Summer Reading for 2021-2022



Just a little love from Mr. Hurry to you.

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FORMULAS and DEFINITIONS for MATH TEAM

Powers Quizzes

Please memorize all of the values listed directly below. You will need to know these by heart without using a pencil to calculate them. You will be given timed tests over these values beginning the first week of school. If you are really fast, you can actually get bonus points (you may need them!). BTW, *Flash cards are a great idea*. If you are able to complete a quiz like below in under 35 seconds you will be labeled for all eternity as "The Daddy" and will go down in math lore with high honors. If you are able to complete the quiz in under 30 seconds you will be mentioned in the same sentence as "The Great One," Hamilton Simpson, who not only helped put Homewood on the map as a great math team school, but who also holds the record at 27 seconds.

$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$
$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$
$2^4 = 16$	$3^4 = 81$	$4^4 = 256$	$5^4 = 625$
$2^5 = 32$	$3^5 = 243$	$4^5 = 1024$	$5^5 = 3125$
$2^6 = 64$	$3^6 = 729$	$4^6 = 4096$	$5^6 = 15625$
$2^7 = 128$	$3^7 = 2187$		
$2^8 = 256$	$3^8 = 6561$		
$2^9 = 512$			
$2^{10} = 1024$			
$2^{11} = 2048$			
$2^{12} = 4096$			

Here is what a sample "powers quiz" will look like. I would practice these this summer. If you can correctly finish a quiz like this in under 2 minutes you'll be doing A or better work on the quiz.

Name_					WERS QUI	Z	Period	<u> </u>	
1.	2 ¹²	2.	4 ⁵	3.	2 ⁸	4.	5 ⁴	5.	3 ⁸
6.	4 ⁶	7.	27	8.	2 ⁶	9.	55	10.	5 ⁶
11.	35	12.	3 ⁴	13.	2 ¹⁰	14.	5 ³	15.	44
16.	2 ¹¹	17.	3 ⁶	18.	3 ⁷	19.	2°	20.	2 ⁵

Miscellaneous Algebraic Laws, Rules, Formulas, and Facts

1.	You will also need to have	e memorized t	the following	values		
			$1^2 = 1$	$7^2 = 49$	$13^2 = 169$	$19^2 = 361$
	0!=1		$2^2 = 4$	$8^2 = 64$	$14^2 = 196$	$20^2 = 400$
	1!=1		$3^2 = 9$	$9^2 = 81$	$15^2 = 225$	$21^2 = 441$
	$2! = 1 \cdot 2 = 2$		$4^2 = 16$	$10^2 = 100$	$16^2 = 256$	$22^2 = 484$
	$3! = 1 \cdot 2 \cdot 3 = 6$			$11^2 = 121$		
	$4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$			$12^2 = 144$		
	$5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$		0 = 50	12 -144	10 - 524	$25^2 = 625$
						23 = 023
2.	factorials					
	$n!=n\cdot(n-1)\cdot(n-2)\cdot$	$(n-3)\cdot\ldots\cdot 2\cdot$	1			
2						
3.	double factorial $n!!=n\cdot(n-2)\cdot(n-4)$	(n - 6)	1.2 when n is	s even		
	$n!!=n\cdot(n-2)\cdot(n-4)$					
	For example:	7!!=′	$7 \cdot 5 \cdot 3 \cdot 1 = 105$	5 6!!=	$6 \cdot 4 \cdot 2 = 48$	
4.	Slope definition:	$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slop	be is the " <i>rate of</i>	<i>change</i> " of a l	ine.
		$x_2 x_1$				
		If $Ax + By =$	C then slop	A = -A		
5.	-					
		If $y = mx + b$, then slope =	= <i>m</i>		
6	Point-slope form (of a line)	$y - y_1 = m($	$(\mathbf{r} - \mathbf{r})$		
0.	-			- /		2
	For example: Determine	the equation	of a line goir	g through the p	oint $(2, -3)$ hav	ving a slope of $\frac{2}{3}$.
		$y - \left(-3\right) = \frac{2}{3}\left($	(x-(2))			
		$y + 3 = \frac{2}{3}$	(r-2)			
		$y + 3 = \frac{1}{3}$	(~ _)			

7. Laws of Exponents (multiplication):

$$a^m \cdot a^n = a^{m+n}$$
 $(ab)^m = a^m b^m$ $(a^m)^n = a^{mn}$

Examples:

$$a^{3} \cdot a^{5} = a^{8}$$
 $(ab)^{4} = a^{4}b^{4}$ $(a^{3})^{5} = a^{15}$

8. Laws of Exponents (division):

$$\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}} \qquad \qquad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Examples:

$$\frac{a^{5}}{a^{12}} = a^{5-12} = a^{-7}$$
$$= \frac{1}{a^{12-5}} = \frac{1}{a^{7}}$$
$$\left(\frac{a}{b}\right)^{6} = \frac{a^{6}}{b^{6}}$$

9. Laws of Exponents, continued...

$$a^{0} = 1$$
 $a^{-n} = \frac{1}{a^{n}}$ Examples: $x^{0} = 1$ $x^{-4} = \left|\frac{1}{x^{4}}\right|$ or $\frac{1}{x^{-4}} = \overline{x^{4}}$

10. Know how to use the Quadratic Formula if $ax^2 + bx + c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example:

Solve:
$$7x^2 - 3x - 5 = 0$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(7)(-5)}}{2(7)} = \frac{3 \pm \sqrt{9 + 140}}{14} = \boxed{\frac{3 \pm \sqrt{149}}{14}}$$

11. Relatively prime -

two integers are relatively prime if their greatest common factor is 1.

Example: Even though 8 and 9 are not prime, they are considered relatively prime because their greatest common factor is 1.

a+bi12. Complex Numbers a = real partb = imaginary part $5+2i\sqrt{3}$ 5 - 4ireal part is 5 real part is 5 Ex) Ex) imaginary part is -4imaginary part is $2\sqrt{3}$ $c^2 = a^2 + b^2$ 13. Pythagorean Theorem $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ 14. Distance formula (plane)

15. Midpoint formula (plane)

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

16. Discriminant - the discriminant is the value $b^2 - 4ac$ coming from the equation $ax^2 + bx + c = 0$.

- If $b^2 4ac > 0$ then there are two unequal **real** solutions to the equation $ax^2 + bx + c = 0$ If $b^2 - 4ac = 0$ then there is a **real** double root to the equation $ax^2 + bx + c = 0$
- If $b^2 4ac < 0$ then there are two conjugate **imaginary** roots to the equation $ax^2 + bx + c = 0$

Ex) <u>Question</u>: Determine what values of k will give the following equation <u>imaginary</u> roots: $3x^2 - 4x - 2k = 0$

> <u>Ans:</u> Based on the equation above, a = 3, b = -4, and c = -2k. Therefore, since the equation is supposed to give imaginary roots...

$$b^{2} - 4ac < 0$$

$$(-4)^{2} - 4(3)(-2k) < 0$$

$$16 + 24k < 0$$

$$24k < -16$$

$$k < -\frac{2}{3}$$

17. Harmonic mean: a number, m_h , is the harmonic mean of x and y if

 $\frac{1}{m_h}$ is the average of $\frac{1}{x}$ and $\frac{1}{y}$, that is $\frac{2}{m_h} = \frac{1}{x} + \frac{1}{y}$

An easier way to think about the harmonic mean of two numbers x and y can be found by

$$m_h = \frac{2xy}{x+y}$$
 or $t_2 = \frac{2t_1t_3}{t_1+t_3}$

This is essentially saying ... "two times the product divided by the sum,"

ex.) the harmonic mean of 6 and 12 is . . .

$$\frac{2(6 \cdot 12)}{6+12} = \frac{2 \cdot 6 \cdot 12}{18} = 8$$

18. Formula for the Sum of the first *n* numbers

$$1+2+3+\ldots+n = \boxed{\frac{n(n+1)}{2}}$$

Ex) Find the sum of 1+2+3+4+...+50 or

Find the sum of the first 50 terms.

Solution:
$$\frac{n(n+1)}{2} = \frac{50(51)}{2} = 1275$$

19. Formula for the Sum of the first *n* squares

$$1^{2} + 2^{2} + 3^{2} + \ldots + n^{2} = \frac{n(n+1)(2n+1)}{6}$$

Ex) Find the sum of $1^2 + 2^2 + 3^2 + 4^2 + ... + 50^2$

Solution:
$$\frac{n(n+1)(2n+1)}{6} = \frac{50(51)(101)}{6} = 42925$$

20. Formula for the Sum of the first *n* cubes

$$1^{3} + 2^{3} + 3^{3} + \ldots + n^{3} = \boxed{\frac{n^{2} (n+1)^{2}}{4}}$$

Ex) Find the sum of $1^3 + 2^3 + 3^3 + 4^3 + ... + 50^3$

Find the sum of the first 50 cubes.

Solution:
$$\frac{n^2(n+1)^2}{4} = \frac{50^2(51)^2}{4} = 1625625$$

Factoring

Factoring out a greatest common monomial

Ex.)
$$3xy - 12x^2$$
 $4x^3 - 6xy + 10x$

Factoring a difference of two squares

Ex.)
$$x^2 - 25$$
 $4x^2 - 81y^2$ $2a^3 - 72ab^2$

or

or

Factoring a perfect square trinomials

Ex.)
$$x^2 - 10x + 25$$
 $x^2 + 6x + 9$ $4x^2 - 12x + 9$

Factoring a trinomial with a lead coefficient of "1"

Ex.)
$$x^2 - 9x + 14$$
 $x^2 + 4x - 12$ $x^2 - 8x - 9$

Trigonometric Identities and Laws

Law of Sines & Cosines

Law of sines	Law of cosines
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	$a2 = b2 + c2 - 2bc \cos A$ $b2 = a2 + c2 - 2ac \cos B$ $c2 = a2 + b2 - 2ab \cos C$

	Fundamental Iden	itities	
Trigonometric Functions	Reciprocal Identities	Quotient identities	
$\sin \theta = \frac{y}{r} = \frac{opp}{hyp}$ $\cos \theta = \frac{x}{r} = \frac{adj}{hyp}$	$\sin \theta = \frac{1}{\csc \theta}$ $\cos \theta = \frac{1}{\sec \theta}$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	
$\tan \theta = \frac{y}{x} = \frac{opp}{adj}$ $\cot \theta = \frac{x}{adj} = \frac{adj}{adj}$	$\tan \theta = \frac{1}{\cot \theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	
$\cot \theta = \frac{r}{y} = \frac{ay}{opp}$ $\sec \theta = \frac{r}{x} = \frac{hyp}{adj}$	$\cot \theta = \frac{1}{\tan \theta}$ $\sec \theta = \frac{1}{\cos \theta}$		
$\csc \theta = \frac{r}{y} = \frac{hyp}{opp}$	$\csc\theta = \frac{1}{\sin\theta}$		

1тЈ . ----1

Math Team related FORMULAS and DEFINITIONS

1. Sum of the roots of a polynomial . . $ax^n + bx^{n-1} + cx^{n-2} + \dots + yx + z = 0$

$$r_1 + r_2 + \dots = -\frac{b}{a}$$
 or $-\frac{2nd \ coefficient}{1st \ coefficient}$

2. Product of the roots of a polynomial... $ax^n + bx^{n-1} + cx^{n-2} + \dots + yx + z = 0$

$$r_1 \cdot r_2 \cdot \ldots = \begin{cases} \frac{z}{a} \text{ or } & \frac{\text{constant}}{\text{first coeff.}}, & \text{if highest power is even} \\ -\frac{z}{a} \text{ or } & -\frac{\text{constant}}{\text{firstcoeff}}, & \text{if highest power is odd} \end{cases}$$

3. Sum of the squares of the roots of . . $ax^n + bx^{n-1} + cx^{n-2} + \dots + yx + z = 0$

$$r_1^2 + r_2^2 + r_3^2 + \dots = \frac{b^2 - 2ac}{a^2}$$

★Example for #1-3:

For the following polynomial, $f(x) = 3x^5 - 7x^4 + 5x^3 + 12x^2 - 6x + 9$, find the ...

Sum of the roots:

 $-\frac{b}{a} = -\frac{2nd \ coefficient}{1st \ coefficient} = -\frac{(-7)}{3} = \boxed{\frac{7}{3}}$

Product of the roots:

odd power, so . . .
$$-\frac{\text{constant}}{\text{first coefficient}} = -\frac{9}{3} = \boxed{-3}$$

Sum of the squares of the roots: If $ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + \dots = 0$ Then $\frac{b^2 - 2ac}{a^2} = \frac{(-7)^2 - 2(3)(5)}{(3)^2} = \boxed{\frac{19}{9}}$

4. Sum of the measures of interior angles of polygon in degrees:

$$S = (n-2)180$$

Example: Find the sum of the measures of the interior angles of a decagon (10 sides).

$$S = (10 - 2)180$$

= 8.180
= 1440°

5. To "Find the number of positive integral factors of a number" ...

Rather than actually write out all of the factors like this: 1,2,3,4,5,6,8,9,10,12,...,120,360,720 which could be extremely difficult and at the very least significantly time consuming...

1. factor a number into a product of prime factors.

$$(ex. \quad 720 \rightarrow 2^4 \cdot 3^2 \cdot 5^1)$$

2. Ignoring the bases, add the "1" to each of the exponents.

 $(ex. 4, 2, 1 \rightarrow 5, 3, 2)$

3. multiply these "new" values together.

(ex. $5 \cdot 3 \cdot 2 = 30$ Therefore, there are 30 <u>positive</u> integral factors of 720)

*note: There are 30 positive integral factors in this case. If the problem asks for the integral factors, then you must multiply by 2 to account for positive and negative factors, giving us 60.

6. To find the "Sum of the *positive* integral factors of a number"... Rather than actually write out all the factors and then add them, ... 1. factor a number into a product of prime factors.

- $(ex. \quad 72 \rightarrow 2^3 \cdot 3^2)$
- 2. for each factorization such as $x^n \cdot y^m \cdot z^p$

find $(x^n + x^{n-1} + \dots + x + 1)(y^m + y^{m-1} + \dots + y + 1)(z^p + z^{p-1} + \dots + z + 1)$

Ex.) Because $72 \rightarrow 2^3 \cdot 3^2$, then $(2^3 + 2^2 + 2 + 1)(3^2 + 3 + 1) = 195$

Therefore, the sum of the positive integral factors of the number 72 is 195

*If you were asked to find the sum of the integral factors (positive & negative), then it would always = 0.

7. Number of diagonals of a convex polygon with *n* vertices (or sides):

number of diagonals = $\frac{n}{2}(n-3)$

Example:

Find the number of diagonals of a regular decagon (10 sides).

Number of diagonals = $\frac{10}{2}(10-3) = 5 \cdot 7 = \boxed{35}$

8. Sum of the coefficients/numbers in the "*n* row" (different from nth row) of Pascal's Triangle? Simply calculate 2ⁿ

(Assume that the top of the "triangle" is the 0 row)

* Remember the 1st row is considered the 0 row, or row 0, because it represents $(a+b)^0$

the 2nd row is considered the 1 row, or row 1, because it represents $(a+b)^1$ etc. . .

Example: Find the sum of the coefficients/numbers in the 12^{th} row of Pascal's Triangle. Solution: Since the 12^{th} row is the "11" row, I will use $2^{11} = \boxed{2048}$ 9. Sum of the coefficients (and constants) in the expansion of $(ax^n + by^m + cz^p + ...)^h$

Simply calculate $(a+b+c+...)^{h}$. We call this rule "Will's Rule" named after Will Gardner, '99.

Example: Find the sum of the coefficients in the expansion of $(2x-3y^2+4)^4$

$$(2x-3y^2+4)^4 \rightarrow (2-3+4)^4 = (3)^4 = 81$$

- 10. **Triangular number:** any of the numbers (as 1,3,6,10,15,21,...) that represent the number of dots in the figures formed starting with one dot and adding rows one after another to form triangles each row of which has one more dot than the one before.
- 11. **Tetrahedral number:** any of the numbers (as $1,4,10,20,35,56,\ldots$).
- 12. **Pentatope number:** any of the numbers (as 1,5,15,35,70,...). See the 5th diagonal of Pascal's triangle.
- 13. A perfect number, *P*, is a number whose factors (other than itself) sum to equal itself. $P = (2^{n-1})(2^{n} - 1)$ where *n* is prime and 2ⁿ - 1 is prime. ex. 6, 28, 496, 8128
 * You will have to memorize the 1st 4 perfect numbers.
- 14. A short cut to finding the fractional equivalent of a repeating decimal (Lewis Lehe ('05) gets the credit from this one, although he said he stole it from some ASFA kid).

Looking at the repeating decimal, let X = the number without the decimal and without the bar over the repeating part and let Y = the number to the left of the repeating part. Find X - Y and divide this value by a value that begins with a "9" for each number under the bar, and ends with a "0" for each number to the right of the decimal but not under the bar (don't forget to simplify the fraction).

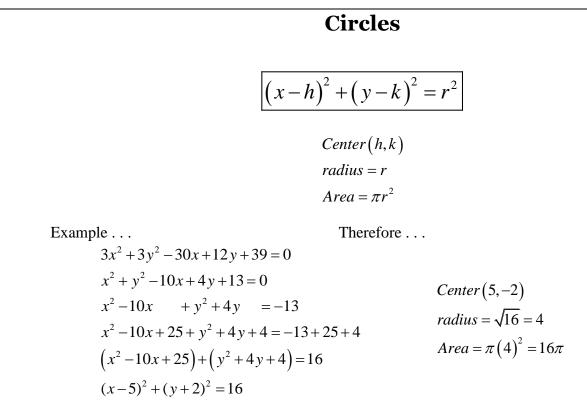
Ex)
$$3.12\overline{43} = \frac{31243 - 312}{9900}$$

 $= \frac{28469}{9900}$

Ex)
$$1.01\overline{4} = \frac{1014 - 101}{900}$$

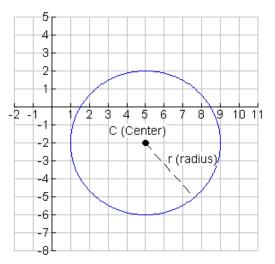
 $= \frac{913}{900}$

Conic Sections (Circles & Parabolas)



How can you tell if an equation is a circle without completing the square?

- Both variables are squared, somewhere in the equation.
- Both x^2 and y^2 have exactly the same coefficient and the same sign.



Parabolas

$$y - k = a(x - h)^2$$
Vertex(h,k) length of lat. rect. = $\left|\frac{1}{a}\right| = |4c|$ $|c| = \frac{1}{4}$ LR

c =dist. from Vertex to Focus or from Vertex to Directrix

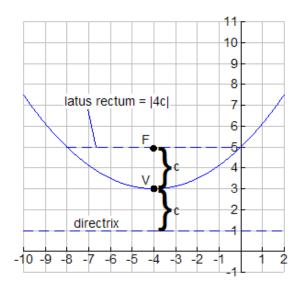
$$\begin{array}{ll} \underline{\text{Completing the Square...}}\\ 2x^{2} + 16x - 16y + 80 = 0\\ -16y + 80 = -2x^{2} - 16x\\ -16y + 80 = -2\left(x^{2} + 8x\right)\\ -16y + 80 - 32 = -2\left(x^{2} + 8x + \underline{16}\right)\\ -16y + 48 = -2\left(x + 4\right)^{2}\\ \left(-\frac{1}{16}\right)\left(-16y + 48\right) = \left(-\frac{1}{16}\right)\left(-2\left(x + 4\right)^{2}\right)\\ y - 3 = \frac{1}{8}\left(x + 4\right)^{2}\end{array}$$

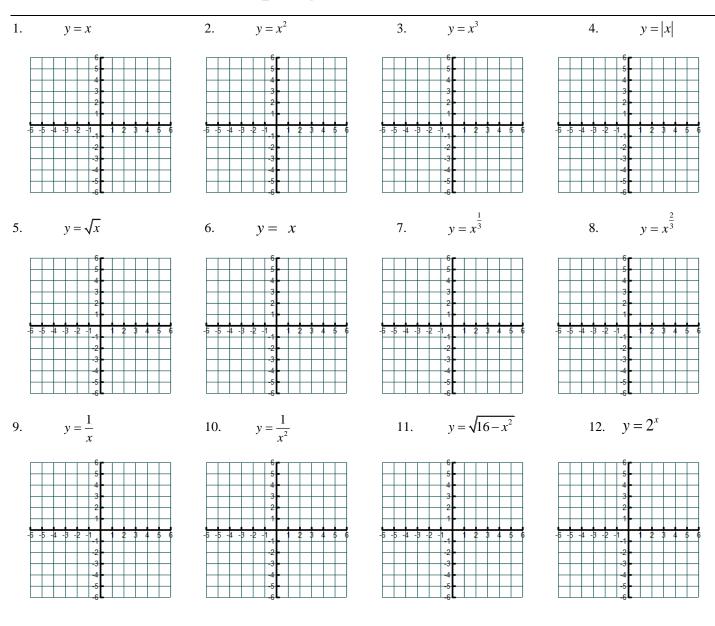
$$\begin{array}{l} \underline{\text{Therefore } \dots}\\ \text{Vertex}(-4,3)\\ a = \frac{1}{8}\\ \text{length of L.R.} = \left|\frac{1}{a}\right| = \left|\frac{1}{\left(\frac{1}{8}\right)}\right| = 8\\ |c| = \frac{1}{4}(L.R.) = \frac{1}{4} \cdot 8 = 2\\ y - 3 = \frac{1}{8}\left(x + 4\right)^{2}\end{array}$$

* This graph of this particular equation is directed upward, because the *x*-term is squared and the *a* is positive

How can you tell if an equation is a parabola without completing the square?

- Only x or y is squared, but not both.
- If x is squared, then the graph goes up (a > 0) or down (a < 0) $y k = a(x h)^2$
- If y is squared, then the graph goes right (a > 0) or left (a < 0) $x h = a(y k)^2$





Graphing "12 Basic Functions"

Quiz #1 - Summer Reading Packet Upcoming MTA2W/T classes

1. Find the slope of the line which goes through the points (2, -6) and (-7, -12).

2.	What is the equation of a line, in point-slope form, that goes through the point $(3, -12)$
	and has the slope $-\frac{2}{3}$.

3.	Simplify:			
	$x^4 \cdot x^7 =$	$(3xy)^4 =$		$\left(2x^3\right)^4 =$
4.	Simplify:			
7.	Simpiny.			
	$\frac{7x^6}{14x^9} =$	$\left(\frac{x^3}{2y}\right)^5 =$		
5.	Simplify			
	$\left(4x^0y^3\right)^2 = \frac{x^3}{x^5}$	4 - =	$\frac{x^{12}}{x^{-4}} =$	
6.	Factor			
	$64x^2 - 9y^2$	$x^2 - 6x - 7$		$49x^2 + 28x + 4$
7.	Solve using the Quadratic Formu	la. 8.	Find the distance be	etween the following points:
	$5x^2 - 4x - 3 = 0$		(2,-6) and $(-7,-2)$	12)

9. Find the Midpoint of the following points:(2,-6) and (-7,-12)

10. What is the discriminant of the following equation? $5x^2 - 4x - 3 = 0$

11. Determine the harmonic mean of 12 and 15.

12. Using the formula from the packet, find the sum of the first 100 numbers (1+2+3+...+100)

13. Using the formula from the packet, find the sum of the first 20 squares $(1^2 + 2^2 + 3^2 + \dots + 20^2)$.

14. Using the formula from the packet, find the sum of the first 15 cubes $(1^3 + 2^3 + 3^3 + \dots + 15^3)$.

15. Which version of the law of sines would you use if you knew $\angle A, \angle C$, and side *c*?

16. Which version of the law of cosines would you use if you wanted to solve for $\angle B$?

17.	Simplify:		
	$\frac{x}{r} = \frac{y}{x} = \frac{y}{x} = \frac{r}{y} = \frac{r}{y} = \frac{r}{y}$	$\frac{1}{\csc \theta} = \frac{1}{\cos \theta} = \frac{1}{\sin \theta} = \frac{1}{\sin \theta} = \frac{1}{\sin \theta} = \frac{1}{\cos \theta} = \frac{1}$	$\frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{\sin\theta} = \cos\theta$
18.	Given the following po	lynomial, $f(x) = 5x^5 - 3x^4 + 2x^3$	$+4x^2-7x+12$, find the
<u>Sum</u>	of the roots:	Product of the roots:	Sum of the squares of the roots:
19.	Given the following po	lynomial, $f(x) = 2x^8 - 5x^7 + 3x^3$	$+7x^2-6x+13$, find the
<u>Sum</u>	of the roots:	Product of the roots:	Sum of the squares of the roots:
20.	Find the sum of the me	asures of interior angles of a poly	gon with 12 sides:

...sum of the positive integral factors.

...sum of the integral factors.

23. Determine the sum of the terms in the 12 row of Pascal's triangle.

24. Find the sum of the coefficients (and constants) in the expansion of $(3x^3 - 4y^2 + 5z - 2)^8$

25. What are the first 4 perfect numbers?

26. Write the following repeating decimal as a fraction in simplest terms. $5.12\overline{34}$

