Precalc takeaways **Name:**

Algebra 2 Unit 4: Polynomial Functions

Standards

A2 4.1 I can determine the degree of a polynomial based on its formula, graph, or table of values.
A2 4.2 I can find all the roots of a polynomial and rewrite it in factored form.
A2 4.3 I can prove that functions are even, odd, or neither.
A2 4.4 I can describe the y-intercept and end behavior of a polynomial.

degrees of polynomial sequences

linear sequences (degree 1) have a constant difference

3, 5, 7, 9, 11
+2, +2, +2, +2

quadratic sequences (degree 2) have a constant second difference

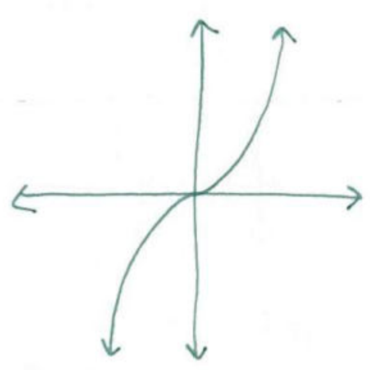
3, 8, 15, 24, 35
+5, +7, +9, +11
+2, +2, +2

cubic sequences (degree 3) have a constant third difference

3, 11, 26, 50, 85, 133
+8, +15, +24, +35, +48
+7, +9, +11, +13
+2, +2, +2

degree 4 sequences have a constant fourth difference
degree 5 sequences have a constant fifth difference
and so on...

Features of $\boxed{f(x) = x^3}$



domain: all real numbers
↳ (all the x-values you can plug in)
range: all real numbers
↳ (all the y-values you get)
(0,0) is the y-intercept
(0,0) is the only x-intercept
the function increases over its entire domain (except maybe at $x=0$?)

Fundamental Theorem of Algebra

A polynomial of degree n has n roots.

→ BUT we have to count multiplicity of roots (for example, double roots count as 2)

→ AND ALSO we have to count roots that are complex numbers

degree — the highest exponent on x (if you write the polynomial in standard form)

root / zero / x-intercept — a value of x that makes the polynomial 0.

factors — $(x-\#)(x-\#)\ldots$ where the #s are the roots

complex roots — roots that are complex numbers

note: non-real complex roots always come in pairs.

if $a+bi$ is a root, then $a-bi$ is also a root.

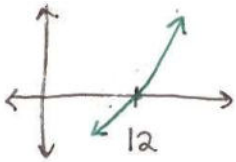
Multiplicity of roots

<u>single root</u>

example: root = 12

factor: $(x-12)$

graph looks like:



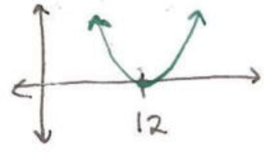
<u>double root</u>

root = 12

factors: $(x-12)^2$

or: $(x-12)(x-12)$

graph looks like:



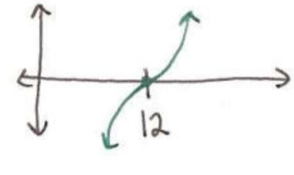
<u>triple root</u>

root = 12

factors: $(x-12)^3$

or: $(x-12)(x-12)(x-12)$

graph looks like:



<u>quadruple root</u>

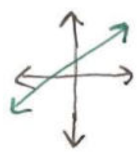
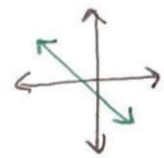
looks like double root but flatter/boxier

<u>quintuple root</u>

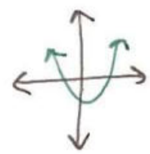
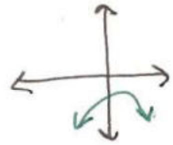
looks like triple root but flatter/boxier

Graphs of polynomials of different degrees

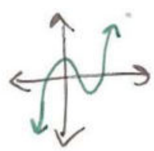
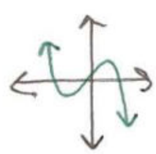
degree 1 (linear) : graph is a line

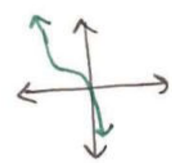
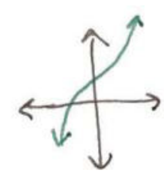
degree 2 (quadratic): graph is a parabola

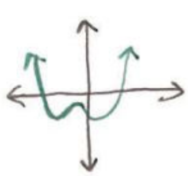
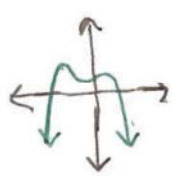
degree 3 (cubic):

could also be less bumpy like:

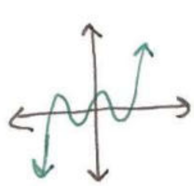
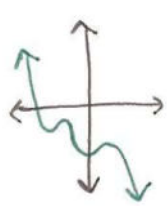
 

degree 4 (quartic):

could also be less bumpy and look like a flatter parabola

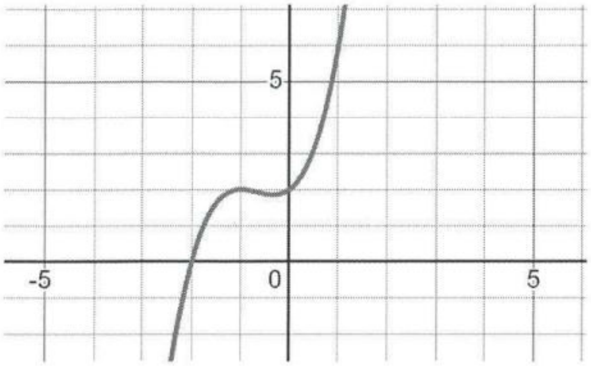
degree 5 (quintic):

could also be less bumpy and look like a cubic

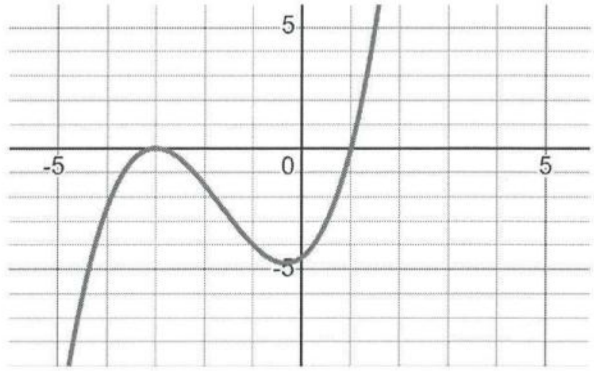
the higher the degree, the bumpier the graph can be.

Examples: 4.2 I can find all the roots of a polynomial and rewrite it in factored form. (Algebra)



How many non-real roots?

looks like a degree 3 polynomial, so 3 total roots. one root is 2, which is real (see x-intercept (-2,0)) the other two roots are non-real (complex)



Leading coefficient is ½

x-intercepts are 1 and -3. -3 is a double root because it "bounces off" the x-axis.

factored form:

$\frac{1}{2}(x+3)^2(x-1)$

or $\frac{1}{2}(x+3)(x+3)(x-1)$

Example: write $f(x)=3x^3-3$ in fully factored form.

Factor out leading coefficient, a.k.a. a: $f(x)=3(x^3-1)$

Graph x^3-1 on Desmos. There is an x-intercept at $x=1$, so 1 is a root and $(x-1)$ is a factor.

$x^3-1=(x-1)\cdot(\ ?\ )$

Divide:

	x^2	x	1
x	x^3	x^2	x
-1	$-x^2$	$-x$	-1

so $x^3-1=(x-1)(x^2+x+1)$

Quadratic formula for the quadratic factor:

$$x=\frac{-1\pm\sqrt{1^2-4\cdot1\cdot1}}{2\cdot1}=\frac{-1\pm\sqrt{-3}}{2}$$

$$=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$$

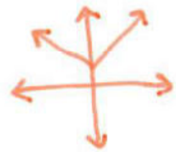
Factored form:

$f(x)=3(x-1)(x+\frac{1}{2}-\frac{\sqrt{3}}{2}i)(x+\frac{1}{2}+\frac{\sqrt{3}}{2}i)$

Even functions

graphs have reflective symmetry over the line $x=0$ (the y-axis)

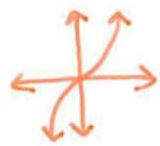
$f(-x)=f(x)$ example: $f(x)=|x|+2$



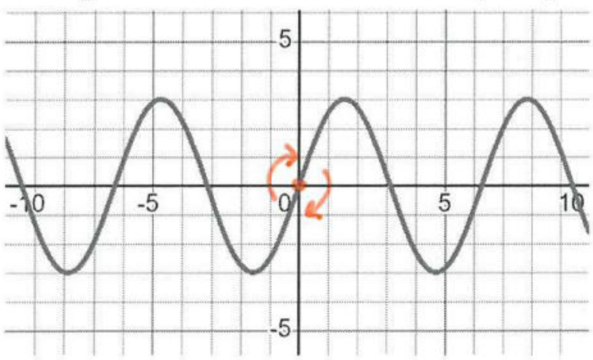
Odd functions

graphs have rotational symmetry $180°$ about the origin (the point $(0,0)$)
(or reflective symmetry across both axes)

$f(-x)=-f(x)$ example: $f(x)=2x^3$



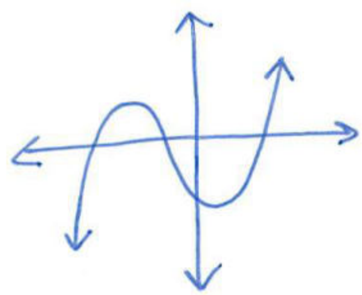
Examples: Are the functions even, odd, or neither?



this function has rotational symmetry $180°$ about the origin, so it is an odd function

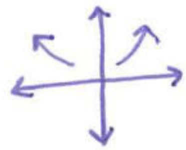
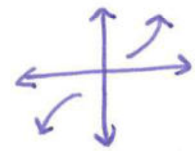
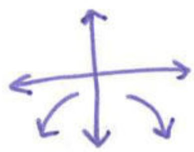
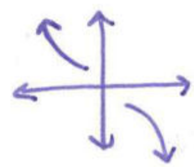
x	$f(x)$
-3	12
-2	-6
-1	3
0	5
1	3
2	-6
3	12

for this function, $f(-x)=f(x)$, so the function is even



this function does not have reflective symmetry over the y-axis, and it does not have rotational symmetry about the origin.
it is neither even nor odd

End behavior of polynomials

	polynomials of even degree	polynomials of odd degree
leading coefficient a is positive	 as $x \to \infty$, $f(x) \to \infty$ as $x \to -\infty$, $f(x) \to \infty$	 as $x \to \infty$, $f(x) \to \infty$ as $x \to -\infty$, $f(x) \to -\infty$
leading coefficient a is negative	 as $x \to \infty$, $f(x) \to -\infty$ as $x \to -\infty$, $f(x) \to -\infty$	 as $x \to \infty$, $f(x) \to -\infty$ as $x \to -\infty$, $f(x) \to \infty$

example: $f(x) = 2(x-1)(x+3)^2$

Find the y-intercept by finding $f(0)$.

$$f(0) = 2(0-1)(0+3)^2 = 2(-1)(3)^2 = -18$$

End behavior:

degree = 3, which is odd. leading coefficient = 2, which is positive.

as $x \to \infty$, $f(x) \to \infty$. as $x \to -\infty$, $f(x) \to -\infty$

example: $f(x) = x^3 - 2x^2 - 7x + 2$

Find the y-intercept $f(0)$: $0^3 - 2 \cdot 0^2 - 7 \cdot 0 + 2 = 2$

End behavior: degree odd, leading coefficient positive

as $x \to \infty$, $f(x) \to \infty$. as $x \to -\infty$, $f(x) \to -\infty$