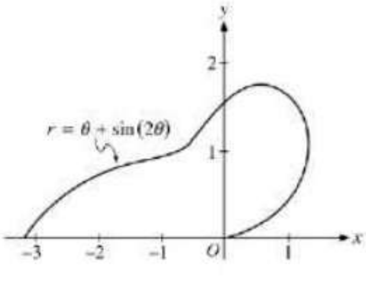



2005 BC2 (Calculator)

The curve above is drawn in xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and $\theta$ is measured in radians. The derivative of r with respect to $\theta$ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

a. Find the slope of the curve at the point $\theta = \frac{\pi}{2}$.

c. Find the angle $\theta$ that corresponds to the point on the curve with x-coordinate -2.

d. For $\frac{\pi}{2} < \theta \le \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

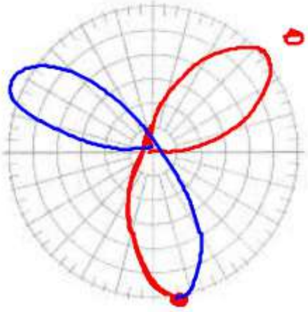
e. Find the value of $\theta$ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2$

? $\cancel{A = \frac{1}{2}\cdot\frac{1}{3}\int_0^{\pi} \sin(3\theta)^2}$

$A = \frac{1}{2}\cdot\frac{1}{3}\int_0^{\pi} \sin(3\theta)^2$

Find the area inside one loop of $\boxed{r = \sin 3\theta}$.



$\theta = \pi/6$

$\theta = \frac{\pi}{2}$

$\frac{1}{2}\int_0^{\pi/2} r^2$

$A = \frac{1}{2}\int_0^{\pi/3} (\sin 3\theta)^2$

$A = \frac{1}{2}(2)\int_0^{\pi/6} (\sin 3\theta)^2$

$0 = \sin 3\theta$

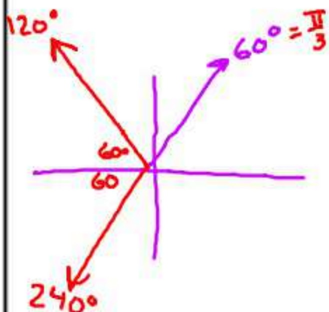
$3\theta = 0, \pi, 2\pi$

$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}$

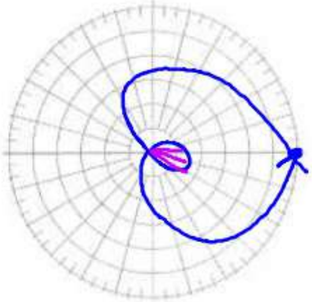
$1 = \sin(3\theta)$

$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}$




Find the area inside the inner loop of $r = 2\cos(\theta) + 1$.



$0 = 2\cos\theta + 1$

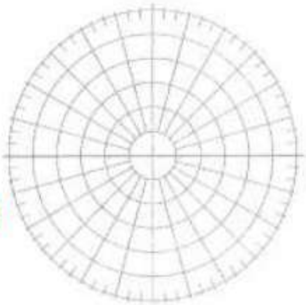
$-1 = 2\cos\theta$

$-\frac{1}{2} = \cos\theta$

$\theta = \frac{2\pi}{3} \quad \theta = \frac{4\pi}{3}$

$A = \frac{1}{2}\int_{2\pi/3}^{4\pi/3} (2\cos\theta + 1)^2$

Find the area inside one loop of $r^2 = 2\sin(2\theta)$.



$r = \pm\sqrt{2\sin(2\theta)}$

$0 = 2\sin(2\theta)$

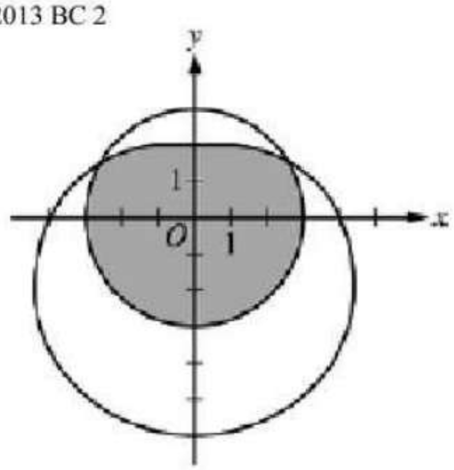
$0 = \sin(2\theta)$

$2\theta = 0, \pi, 2\pi$

$\theta = 0, \frac{\pi}{2}$

$A = \frac{1}{2}\int_0^{\pi/2} 2\sin(2\theta)$

2013 BC 2




The graphs of the polar curves r = 3 and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

a. Let S be the shaded region that is inside the graph of r = 3 and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

b. A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

$x = r\cos\theta$

$x = (4 - 2\sin\theta)\cos\theta$

$-1 = (4 - 2\sin(t^2)\cos(t^2)$

$1.427 = t$

c. For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time t = 1.5.

position $= \langle r\cos t^2, r\sin t^2 \rangle$

$= \langle (4 - 2\sin t^2)\cos t^2, (4 - \sin t^2)\sin(t^2) \rangle$

53. A region R in the xy-plane is bounded below by the x-axis and above by the polar curve defined by $r = \frac{4}{1+\sin\theta}$.

a. Find the area of R by evaluating an integral in polar coordinates.

b. Find the slope of the polar curve when $\theta = \frac{\pi}{2}$