Superposition

Traveling waves can pass through each other. As they do, their displacements add together. This is the **principle of superposition.** Adding waves can be positive or negative.

The surface of the water supports multiple waves. It looks like the waves simply stack on top of each other, which, in fact, they do.

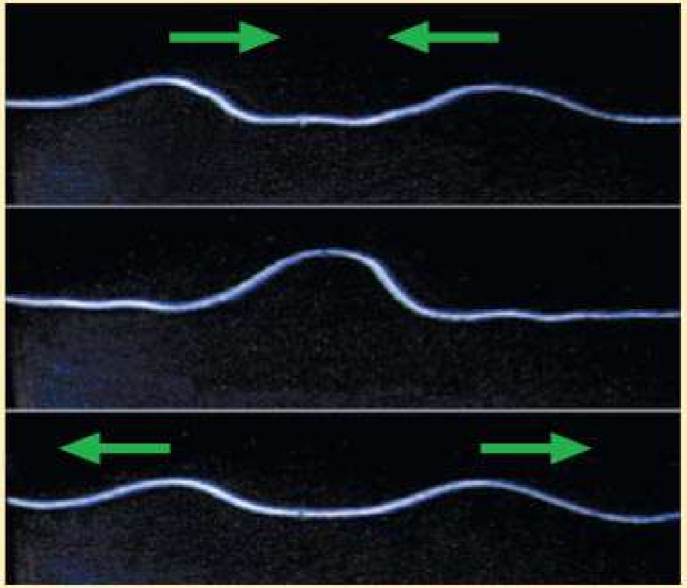
Looking Back

15.2–15.3 The fundamental properties of traveling waves

Superposition

- When two (or more) waves interfere (meet… they're at the same place at the same time) the resultant disturbance is the sum of the disturbances from the individual waves.
- Constructive Interference: meet and amplitudes add. In-phase.
- Destructive Interference: meet and amplitudes destroy momentarily. Out-of-phase by ½λ.
- The waves meet, they interfere—either constructively or destructively—and then the waves continue to pass through each other.
  - http://zonalandeducation.com/mstm/physics/waves/interference/waveInterference2/WaveInterference2.html
  - http://www.cabrillo.edu/~jmccullough/Applets/Applets_by_Topic/Superposition_Interference.html

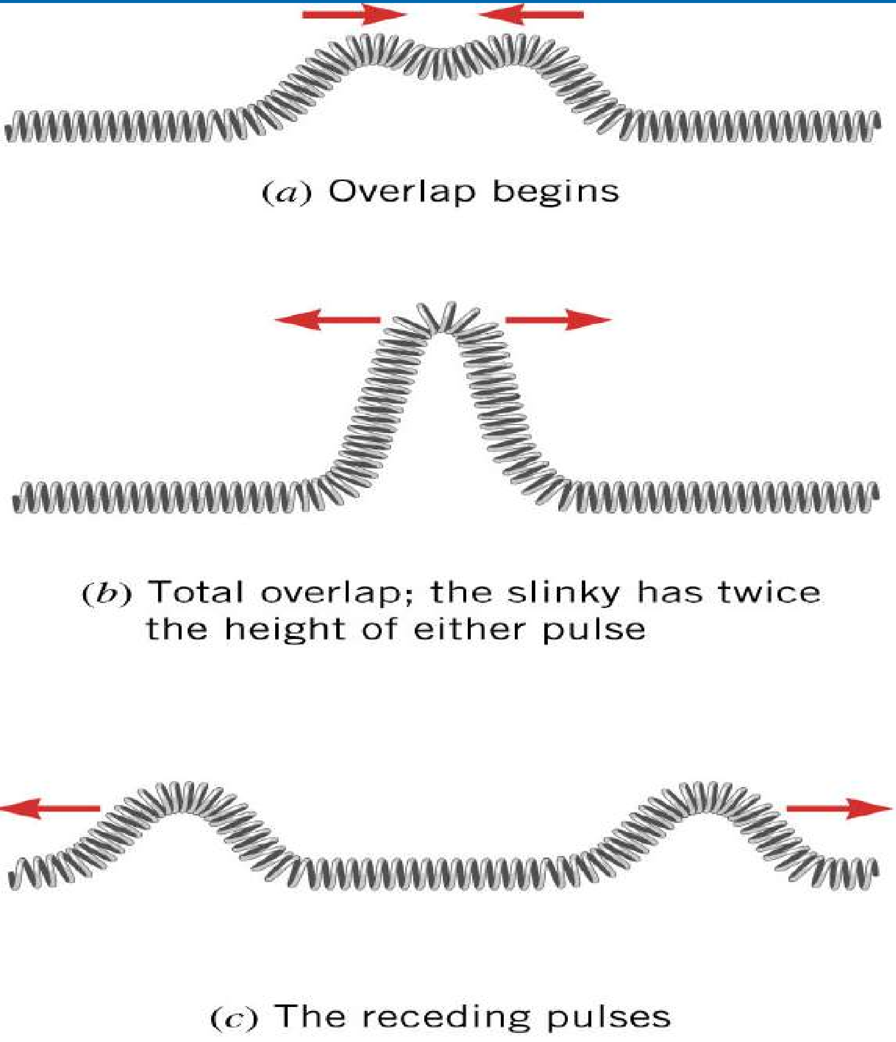
Constructive and Destructive Interference

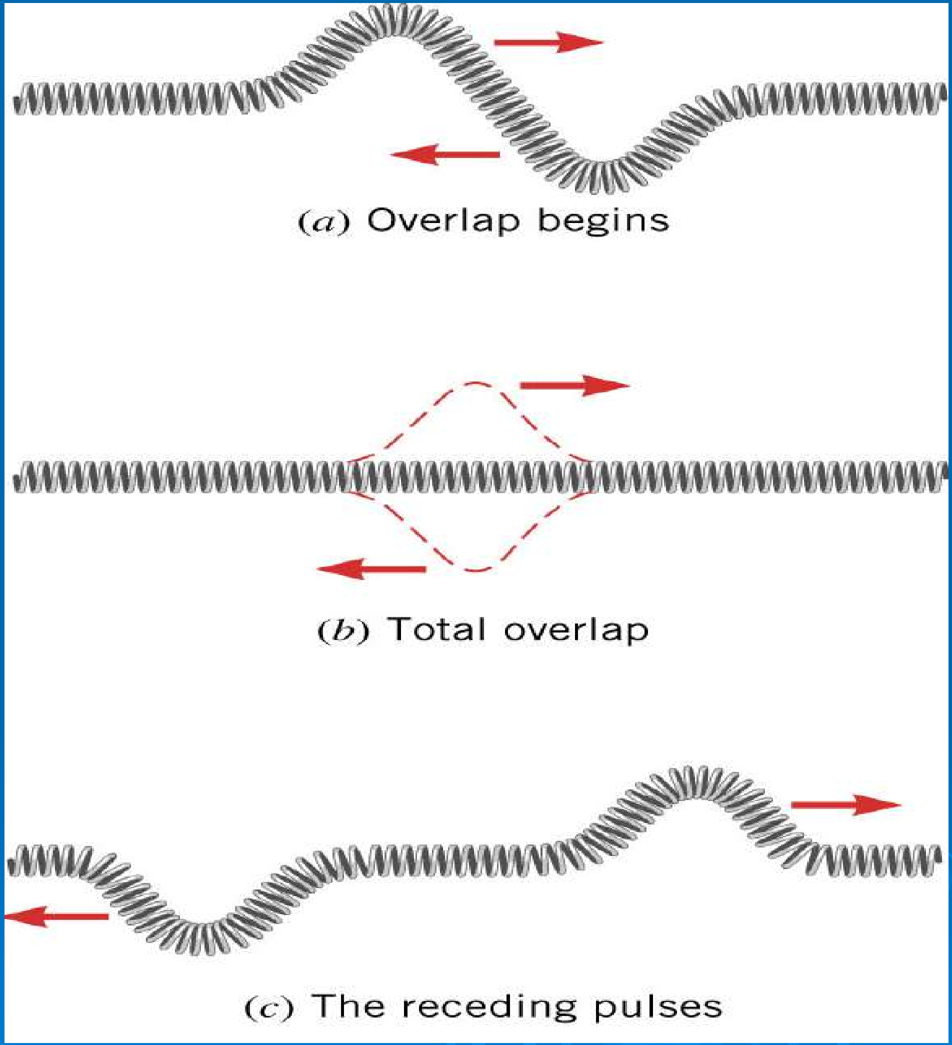


Two waves on a string each displace the string upward. Where the two waves overlap, the displacement is twice that of the individual waves. This is **constructive interference.**

Noise-canceling headphones create a sound wave that is inverted from the ambient sound. When the waves are added, they cancel, producing a much smaller wave. This is **destructive interference.**

Superposition of Transverse Pulses

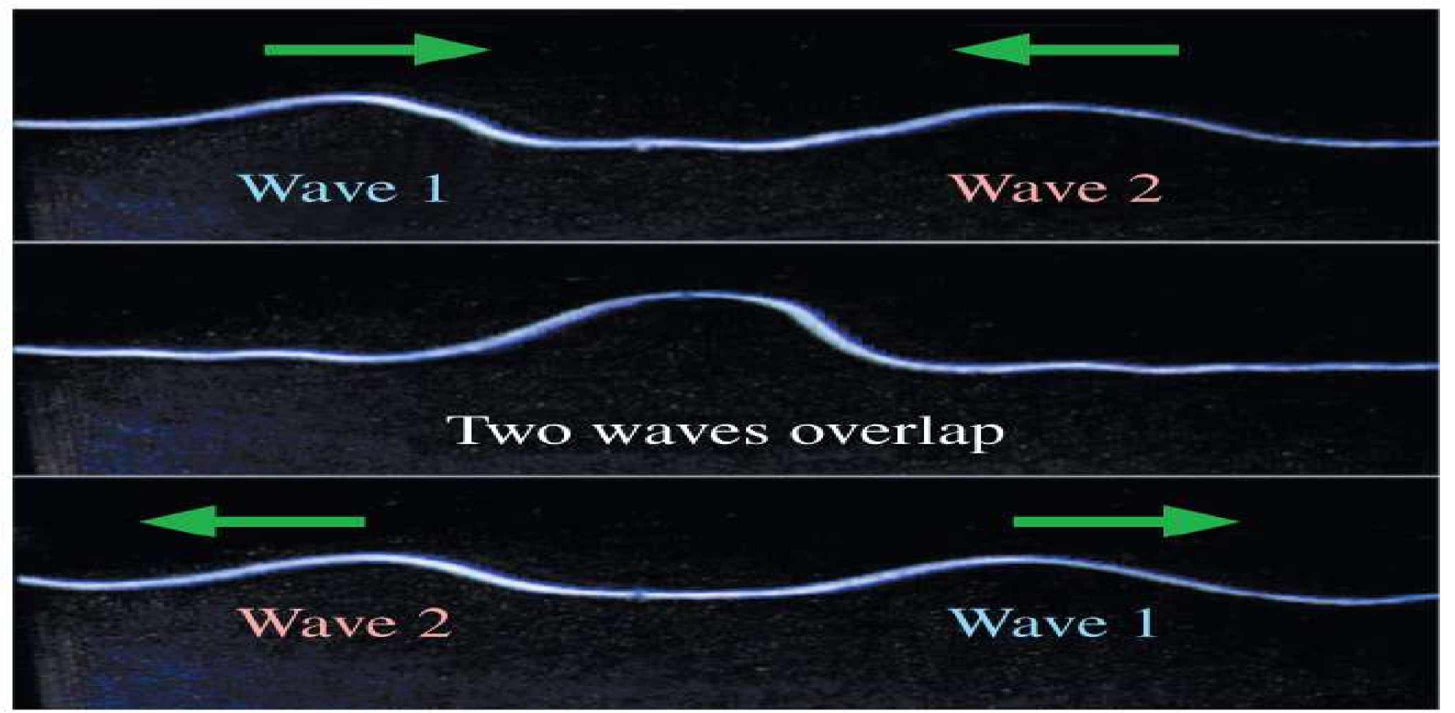




Exactly in-phase: crest meets crest, exhibiting constructive interference.

Exactly out-of-phase: crest meets trough, exhibiting destructive interference.

Principle of Superposition



Principle of superposition When two or more waves are *simultaneously* present at a single point in space, the displacement of the medium at that point is the sum of the displacements due to each individual wave.

Question:

1. When two waves overlap, the displacement of the medium is the sum of the displacements of the two individual waves. This is the principle of ___________.

   A. constructive interference
   B. destructive interference
   C. standing waves
   D. superposition

Answer:

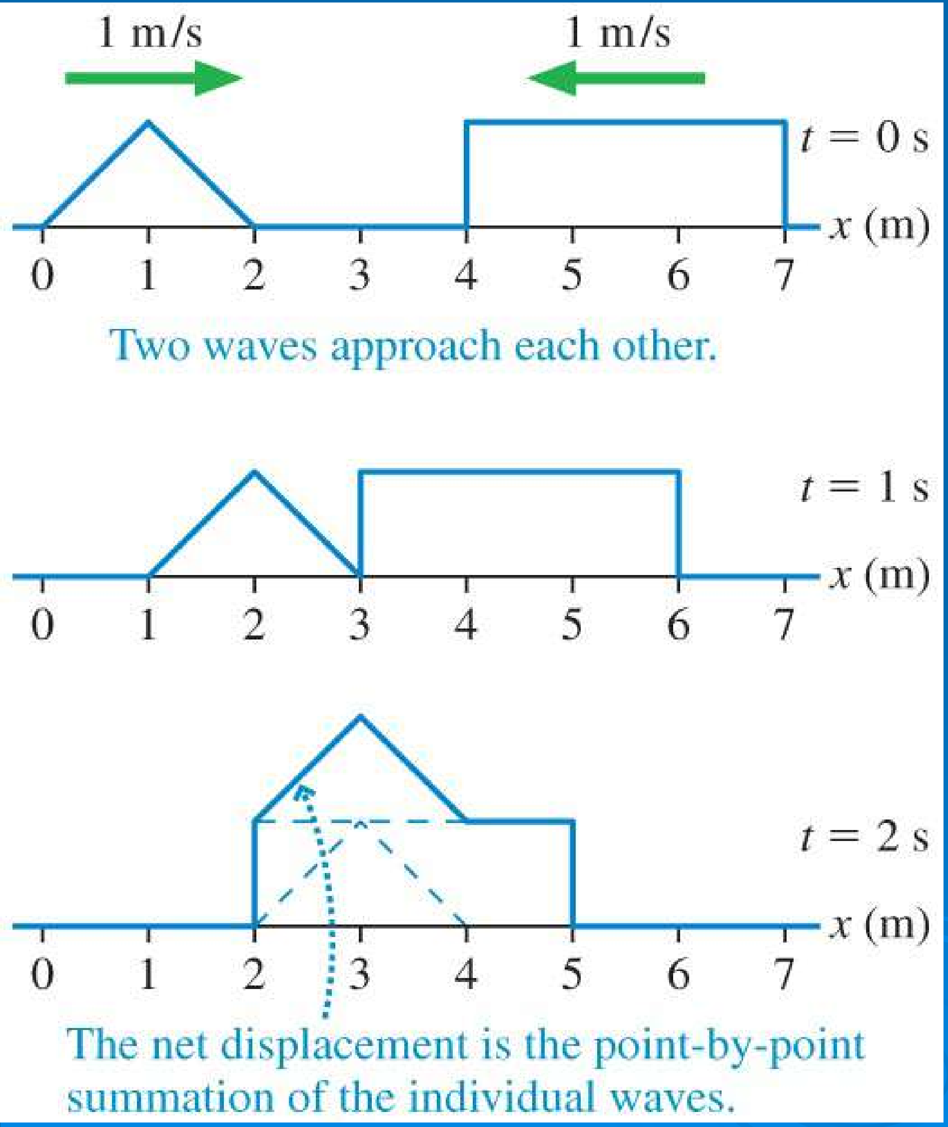
1. When two waves overlap, the displacement of the medium is the sum of the displacements of the two individual waves. This is the principle of __________.

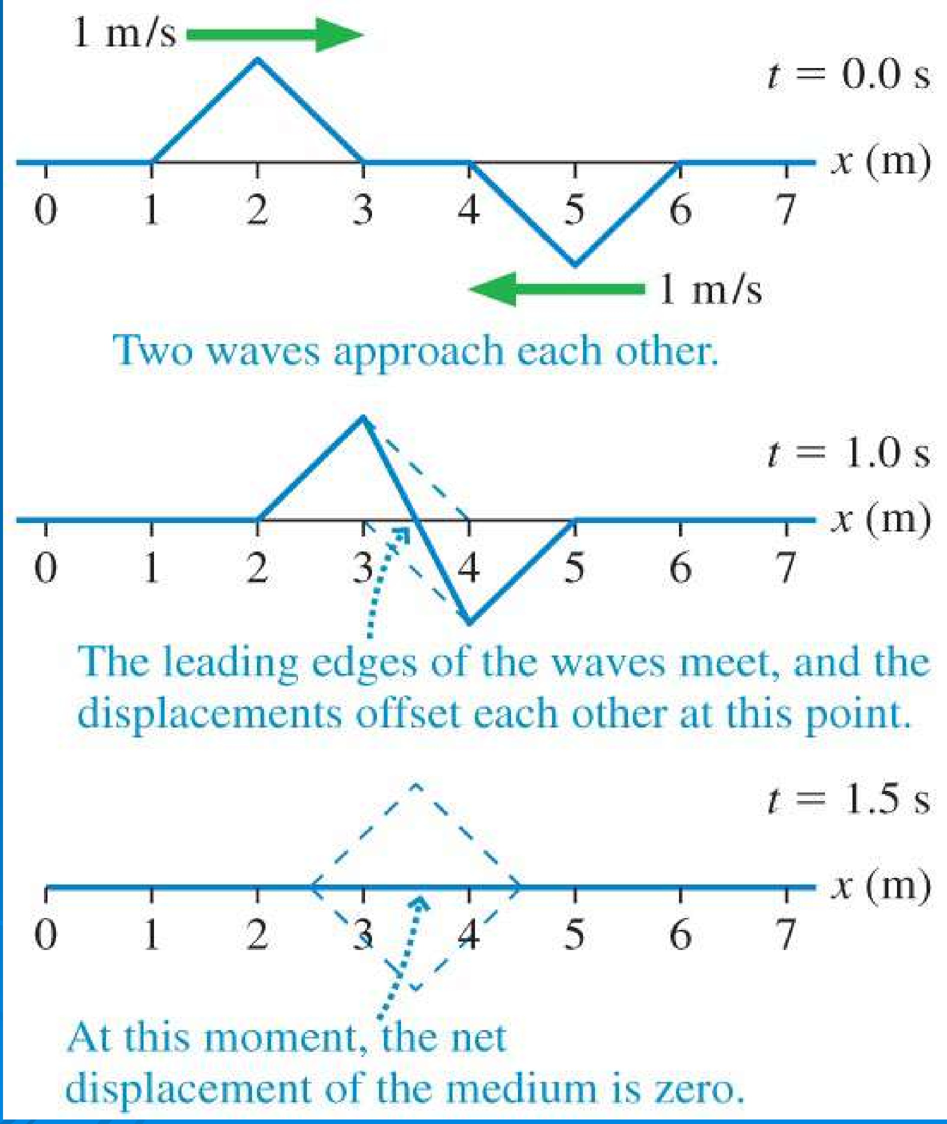
   A. constructive interference
   B. destructive interference
   C. standing waves
   D. superposition

Constructive and Destructive Interference

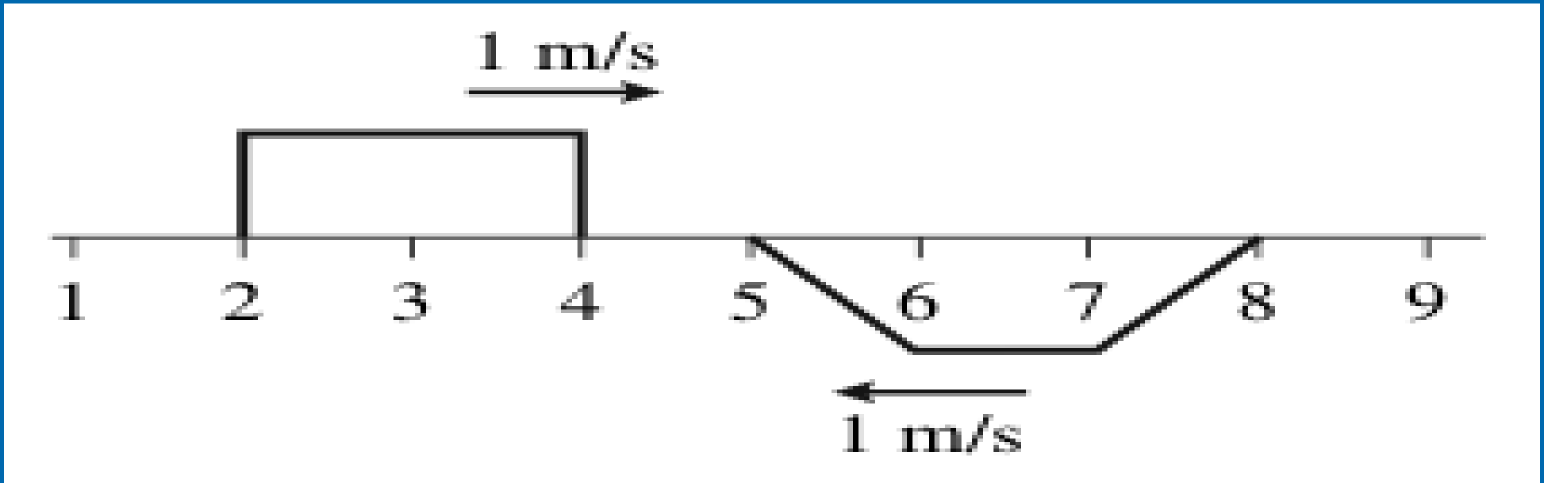
Constructive:
Displacements add

Destructive:
Displacements cancel

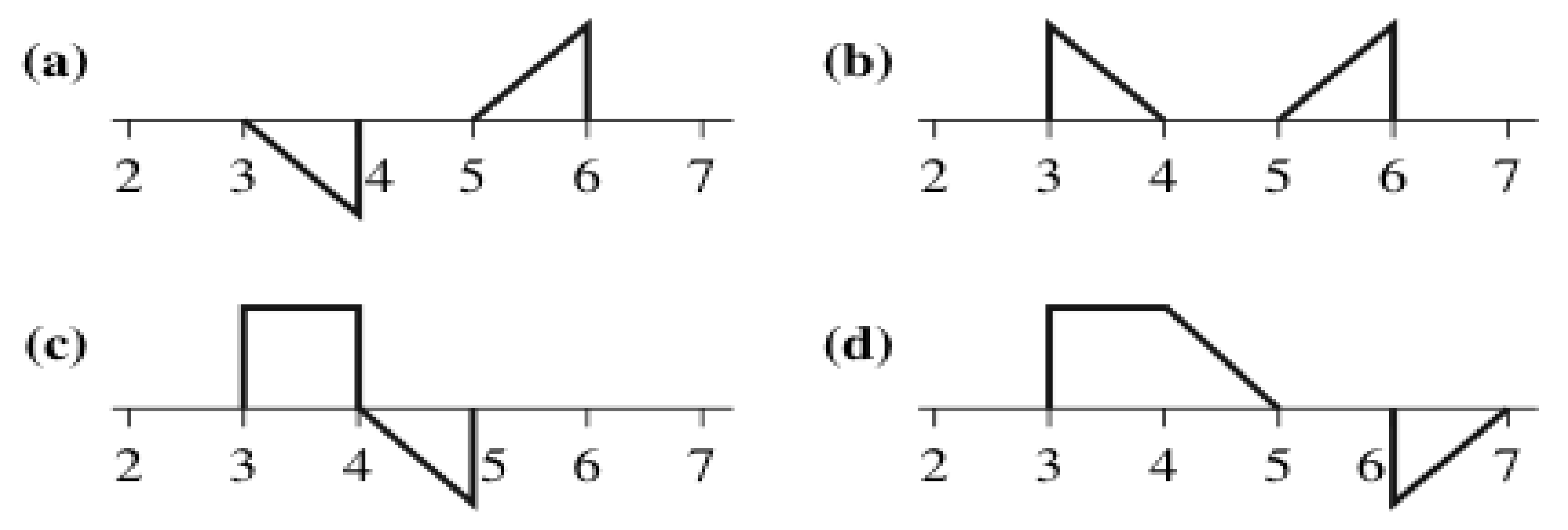




Two waves on a string are moving toward each other. A picture at t = 0 s appears as follows:

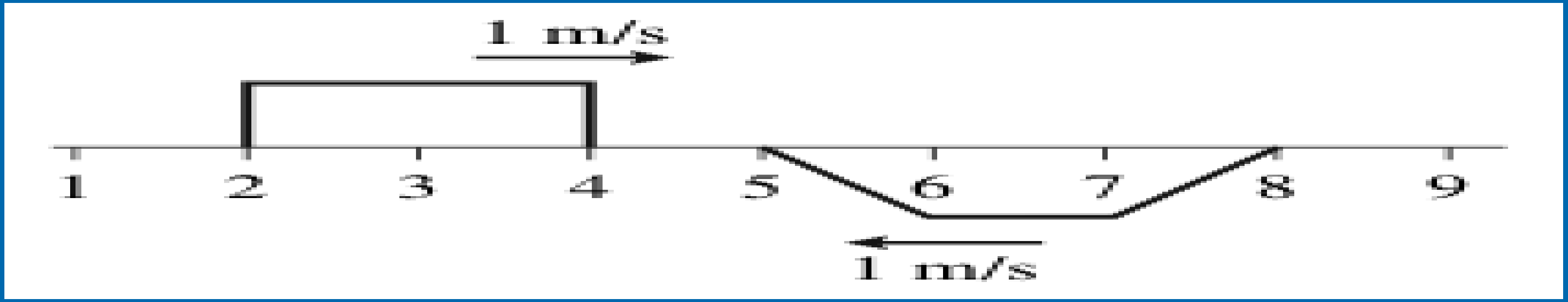


How does the string appear at t = 2 s?

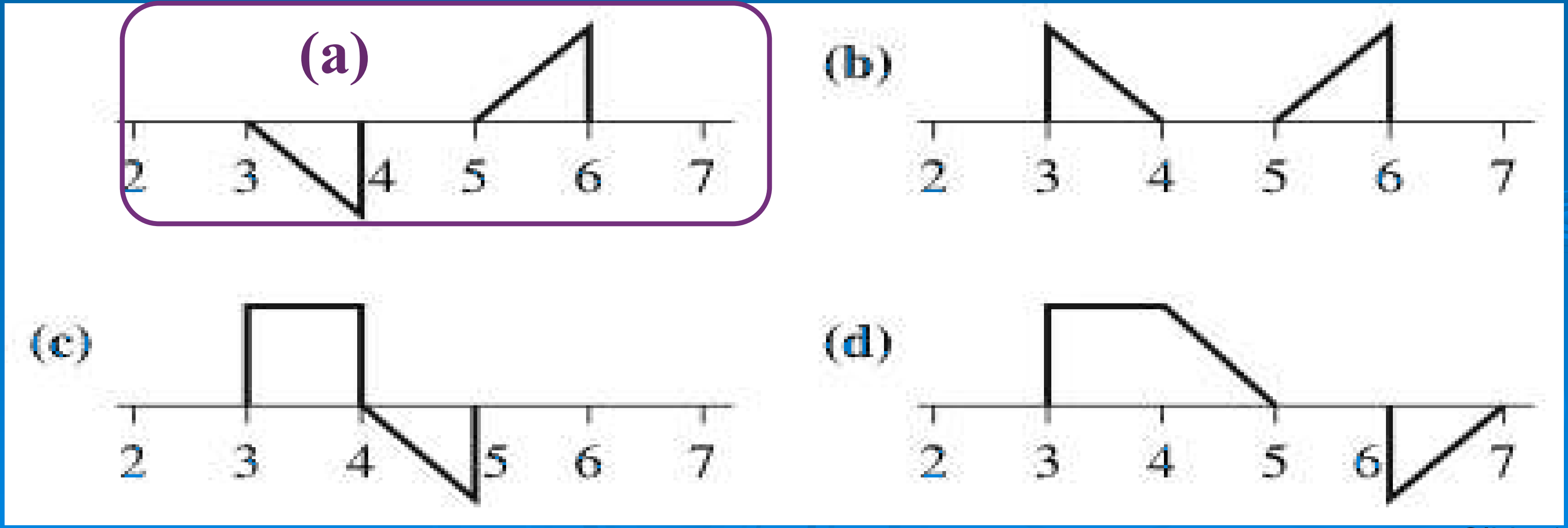


Answer

Two waves on a string are moving toward each other. A picture at t = 0 s appears as follows:



How does the string appear at t = 2 s?

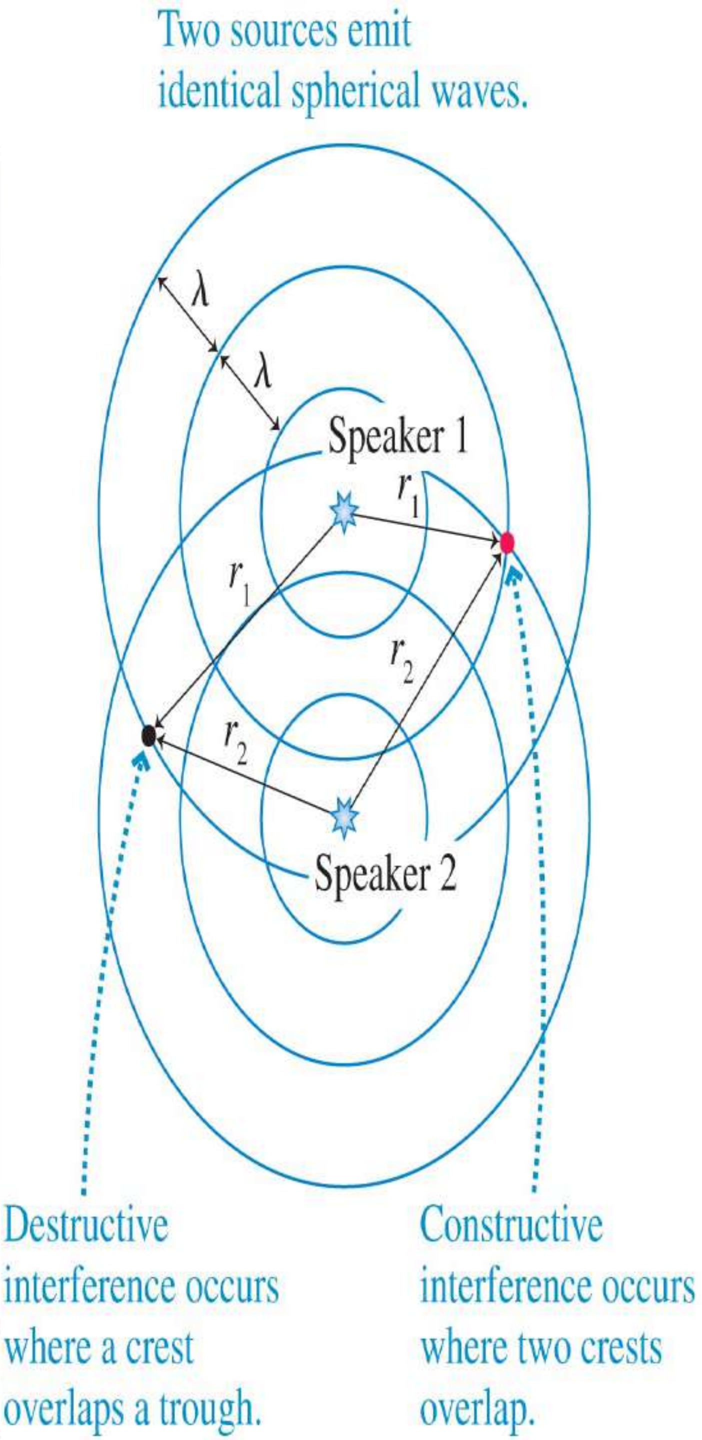


Interference of Spherical Waves

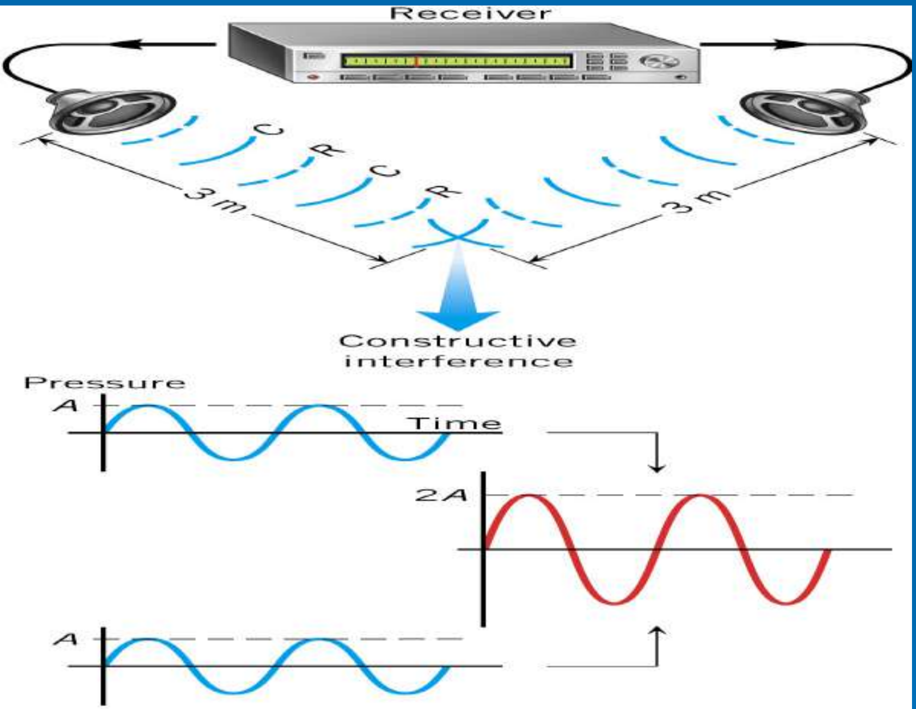
TACTICS BOX 16.1 Identifying constructive and destructive interference

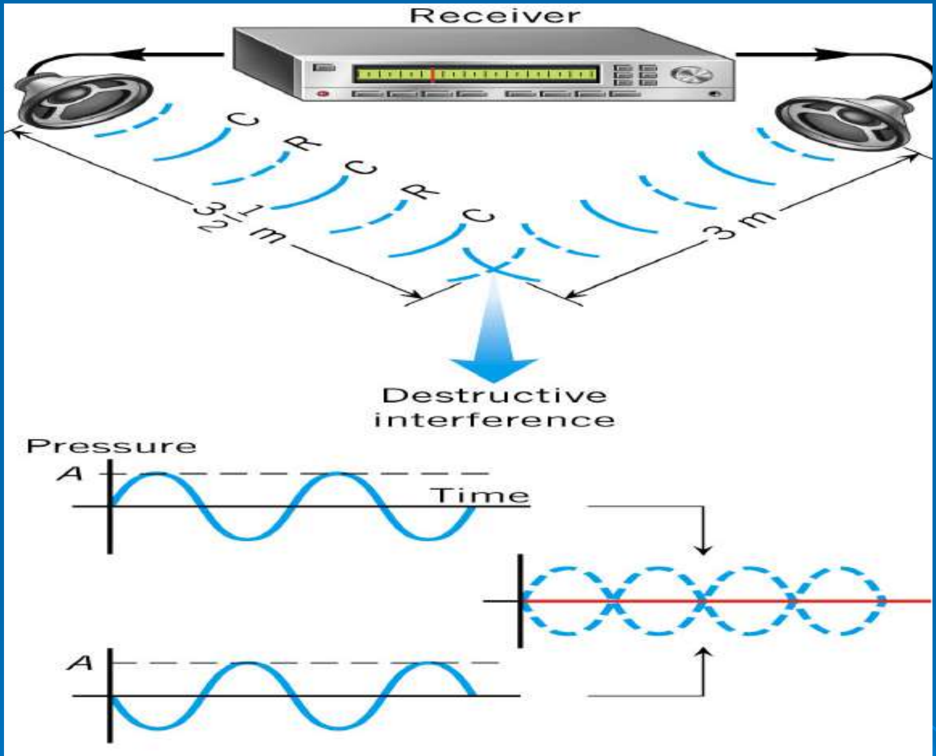
1. Identify the path length from each source to the point of interest. Compute the path-length difference $\Delta r = |r_2 - r_1|$.
2. Find the wavelength, if it is not specified.
3. If the path-length difference is a whole number of wavelengths ($\lambda, 2\lambda, 3\lambda, \ldots$), crests are aligned with crests and there is constructive interference.
4. If the path-length difference is a whole number of wavelengths plus a half wavelength $\left(1\frac{1}{2}\lambda, 2\frac{1}{2}\lambda, 3\frac{1}{2}\lambda, \ldots\right)$, crests are aligned with troughs and there is destructive interference.

Exercises 9,10



Constructive & Destructive Interference of Sound Waves

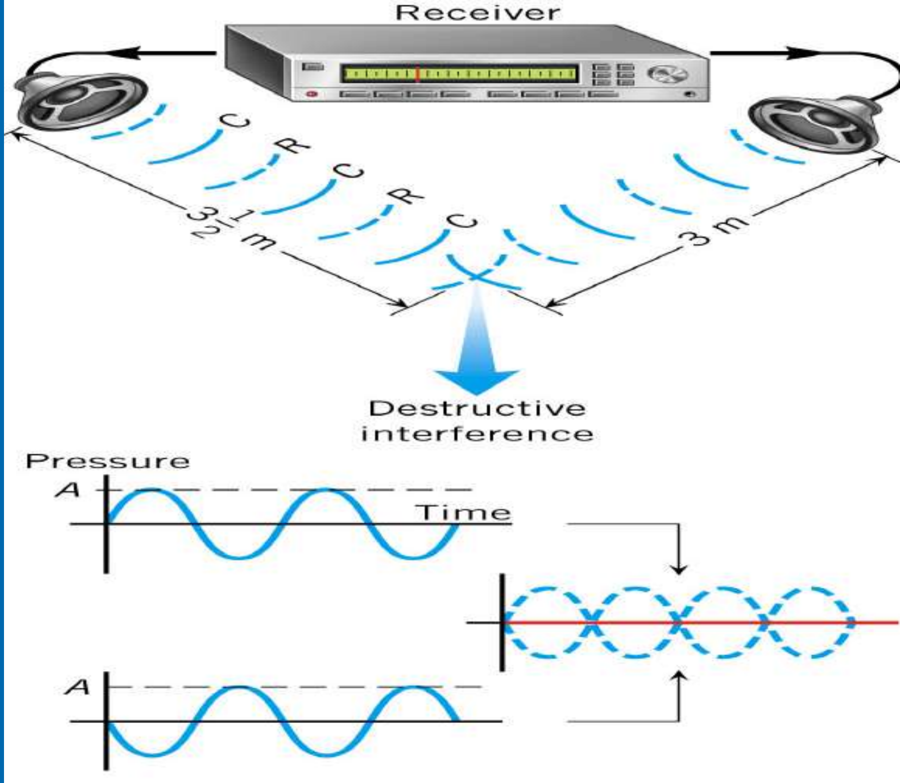


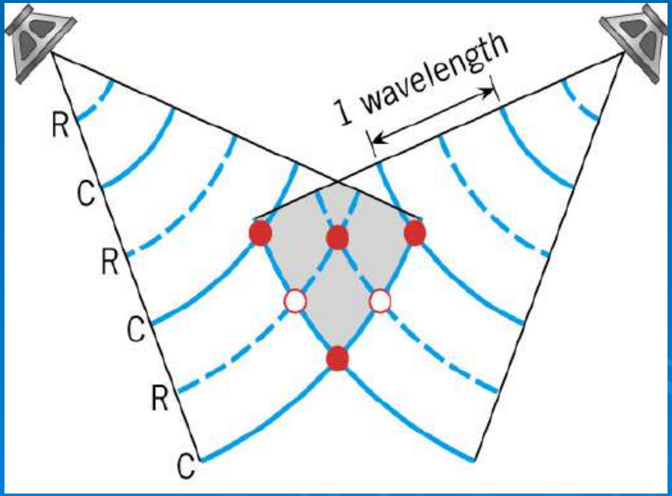


As a result of constructive interference between the two sound waves, each with amplitude A, a loud sound of twice the amplitude (2A) is heard at an overlap point equally distant from two in-phase speakers.

Even though these speakers vibrate in-phase, the left speaker is 1/2λ farther from the overlap point than the right speaker. Because of destructive interference, no sound is heard at the overlap point.

- If the left speaker were moved away from the overlap point by another ½ λ (or 4m away, instead of 3.5m away), the two waves would again be in phase & constructive interference would occur.
- For two wave sources vibrating in phase…
- **If the difference in path lengths is zero or a *whole* number integer of wavelengths (1λ, 2 λ, 3 λ, etc.), the result is *constructive interference*.**
- **If the difference in path lengths is a *half-integer* number of wavelengths(½λ, 1½λ, 2½λ, 3½λ, etc.), the result is *destructive interference*.**
- http://phet.colorado.edu/simulations/sims.php?sim=Sound

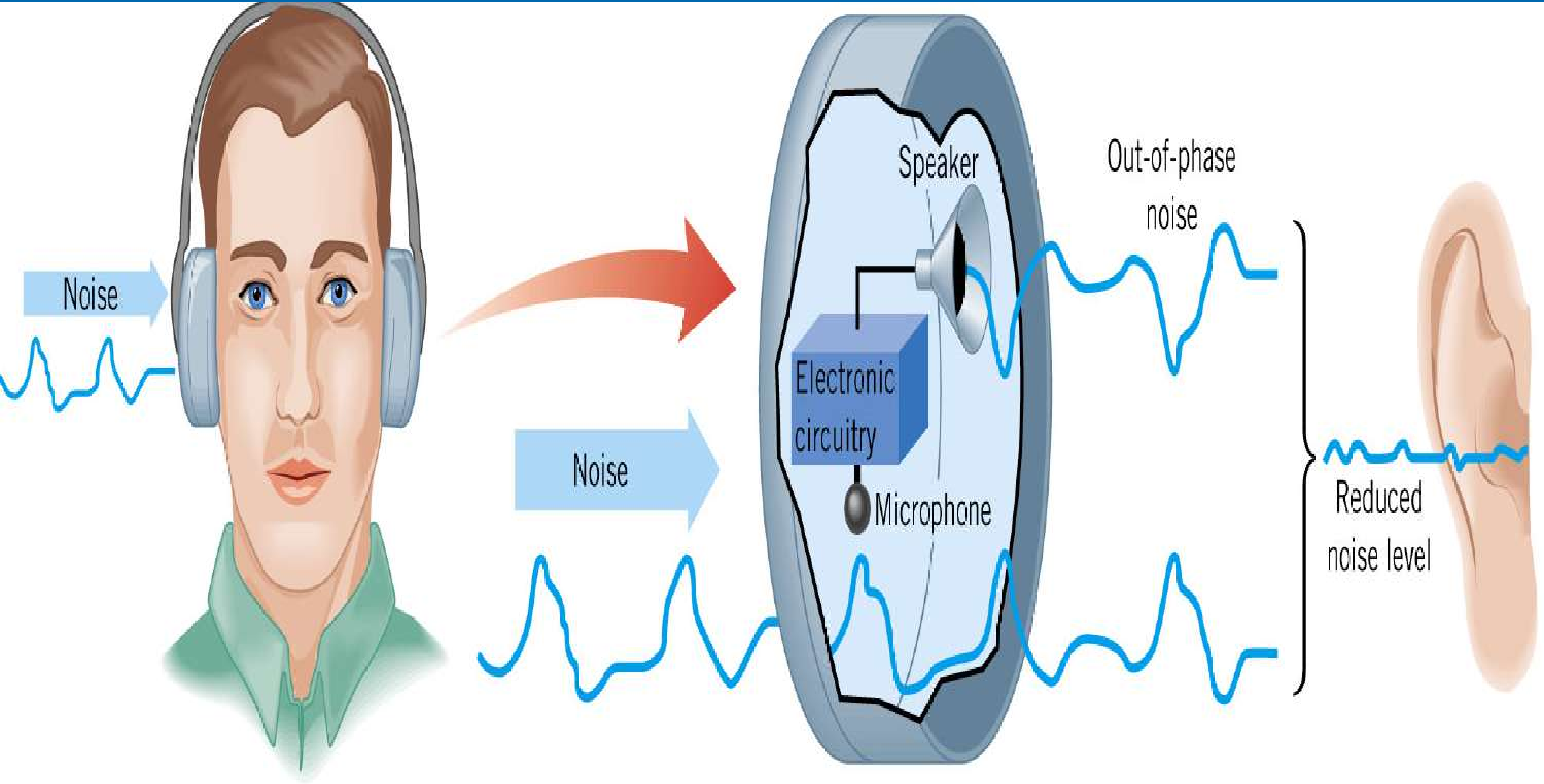




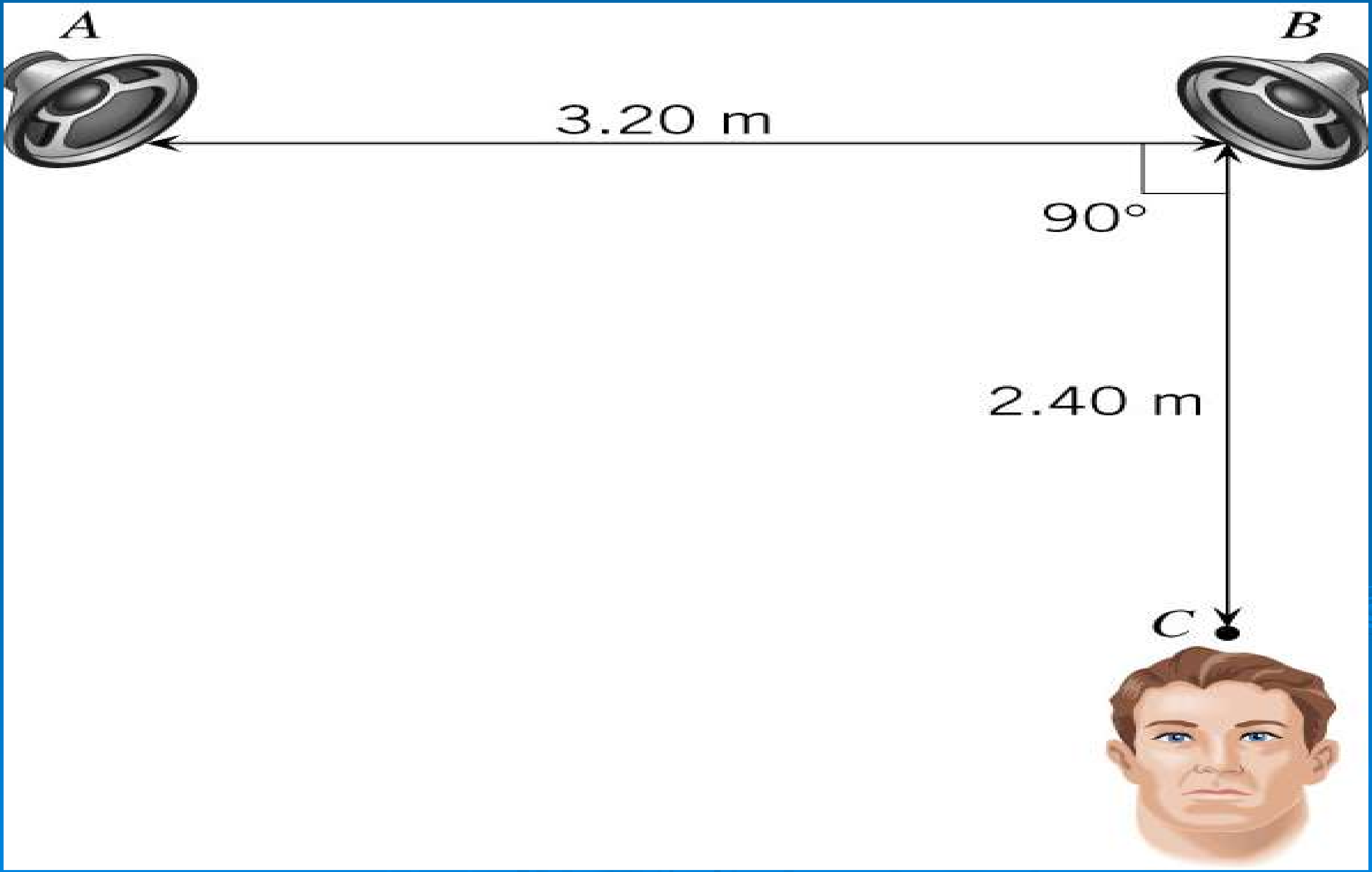
Solid red dot=constructive interference.

Open dot=destructive interference

Noise canceling head-phones utilize destructive interference…



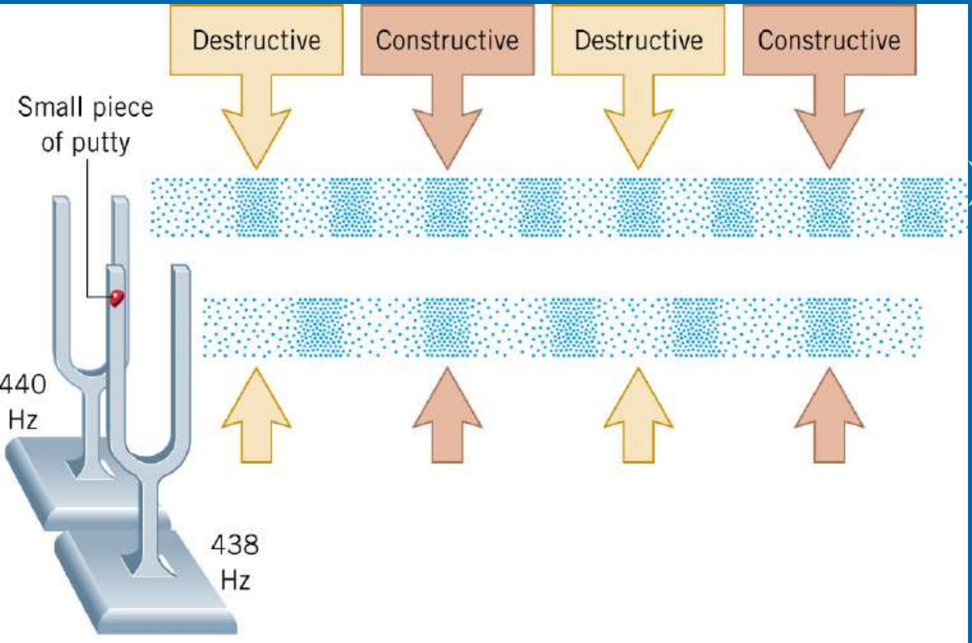
Both speakers are playing identical 214-Hz tones, and the speed of sound is 343 m/s. Does the listener hear a loud sound or no sound?

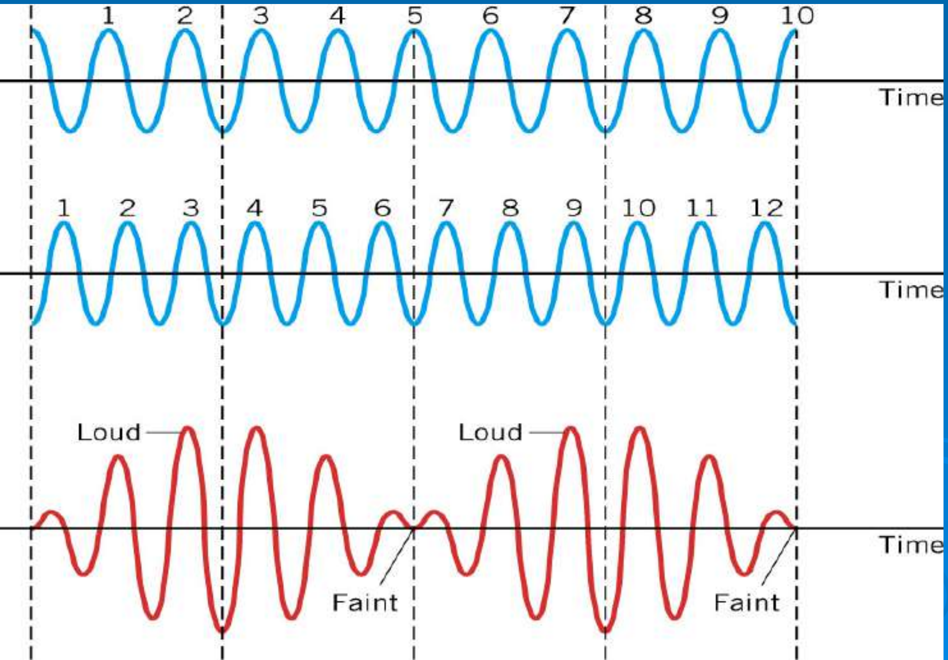


And survey says…

- $(3.20\ m)^2 + (2.40\ m)^2 = AC^2$
- AC = 4.00 m
- 4.00 m – 2.40 m = 1.60 m = the difference in travel distances for the wave.
- $\lambda = v/f$ = (343 m/s) / (214 Hz) = 1.60 m
- Voila!!
- Since the difference is exactly one wavelength, the interference will be constructive!!
- The listener hears the sound!

Beat Frequency

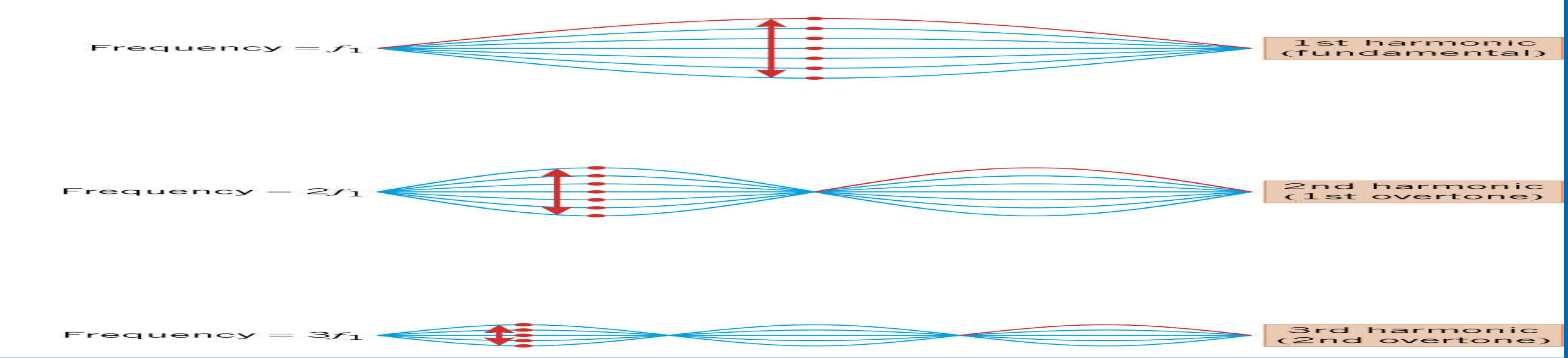




- Another phenomena due to superposition is the production of beats.
- Beats are produced from the interference of two sound waves w/slightly different frequencies, and heard as alternating loudness, faintness, loudness, faintness, etc.
- **W**AAAAA **W**AAAAA **W**AAAAA
- Beat frequency (Hz) = | $f_1 - f_2$ |
- http://www.school-for-champions.com/science/sound_beat.htm

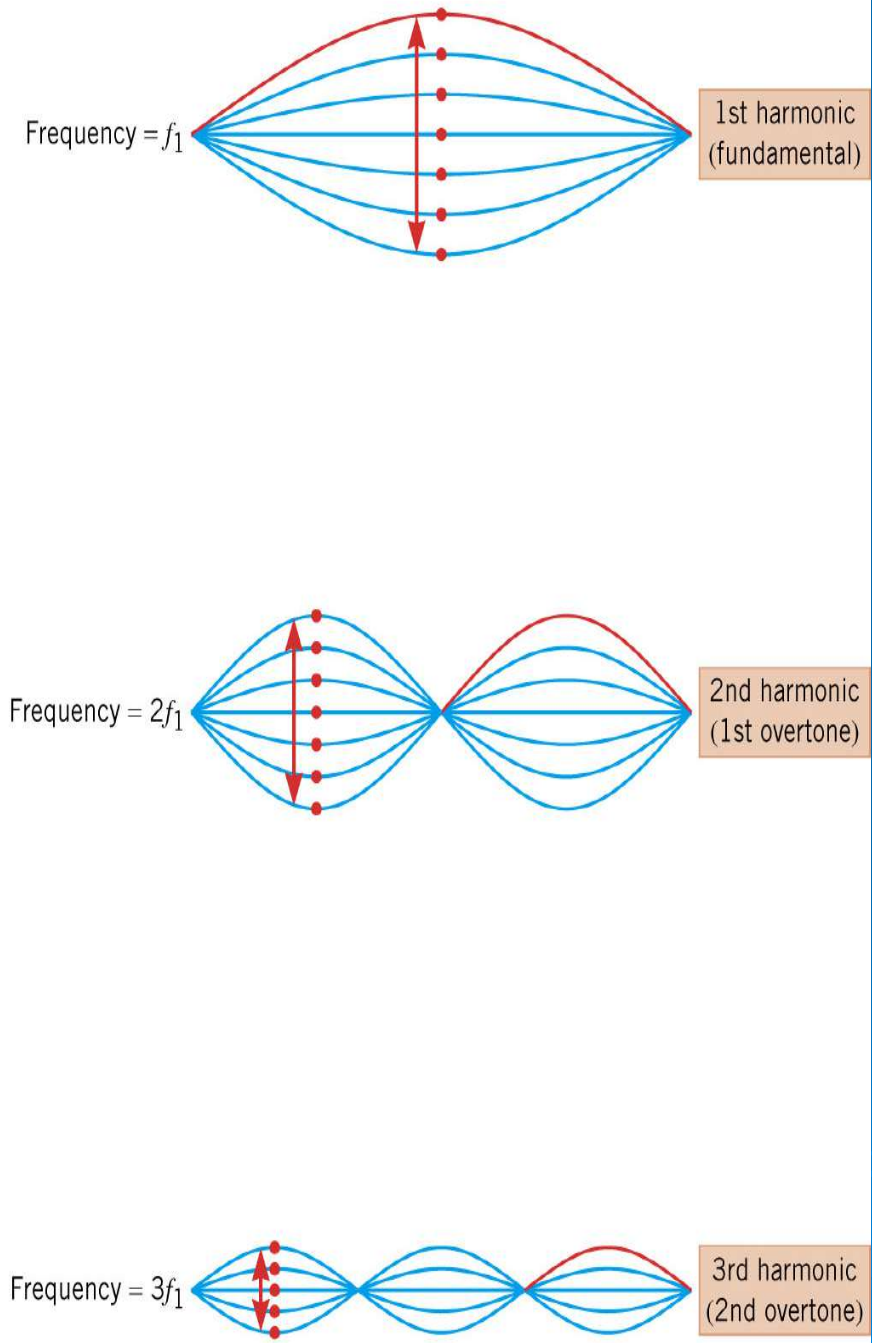
Transverse Standing Waves & Harmonics

- Transverse standing wave: identical waves travel through a medium in opposite directions and combine together (superimpose).
- Superposition creates a standing wave.
- There are different standing wave patterns.
- All have nodes (minimum disturbance) and antinodes (maximum disturbance).
- http://www.walter-fendt.de/ph14e/stwaverefl.htm

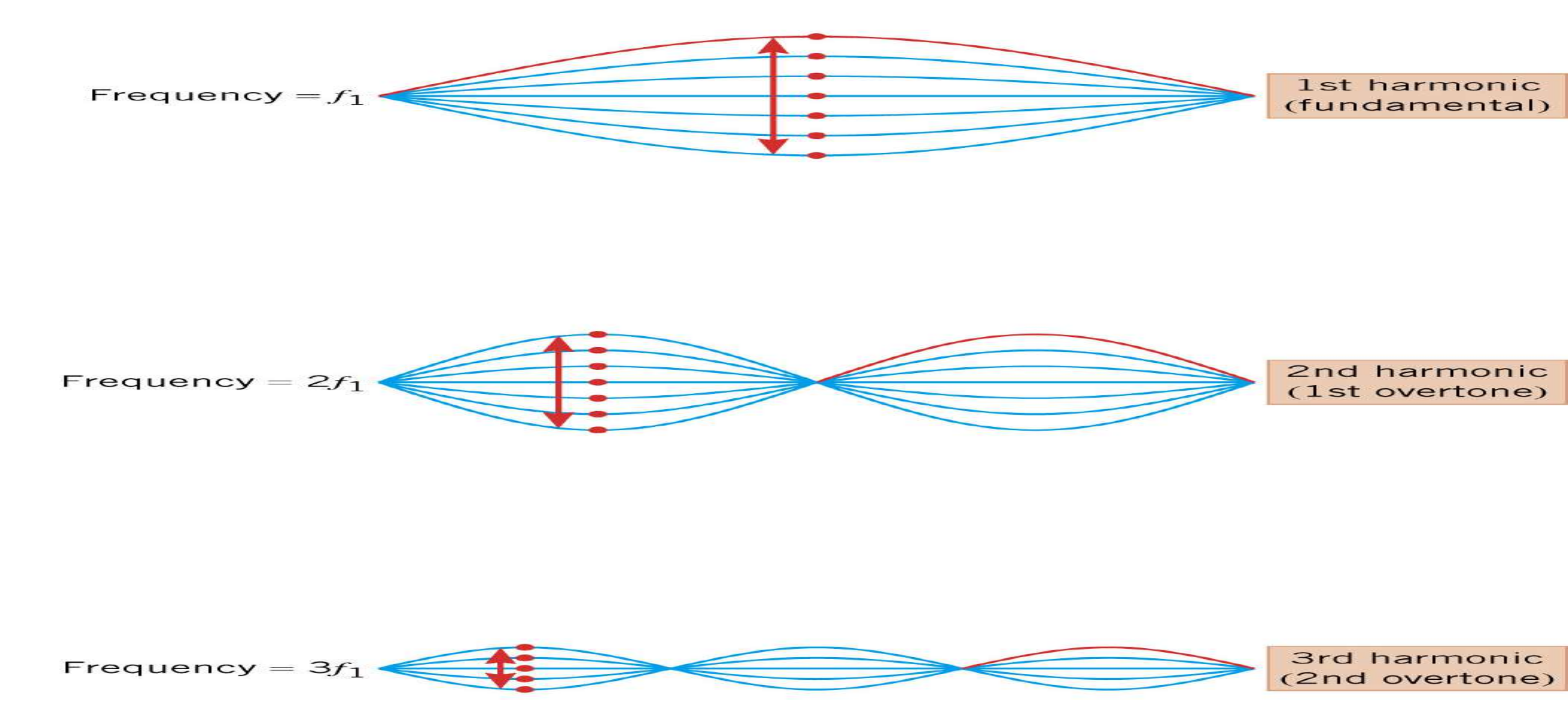


Standing wave pattern	Harmonic	Smallest frequency	Example of frequency	# of waves
1 loop	1st Harmonic (Fundamental Frequency)	f_1	10 Hz	½ wave
2 loops	2nd Harmonic (1st Overtone)	$2f_1$	20 Hz	1 wave
3 loops	3rd Harmonic (2nd	$3f_1$	30 Hz	1½ wave

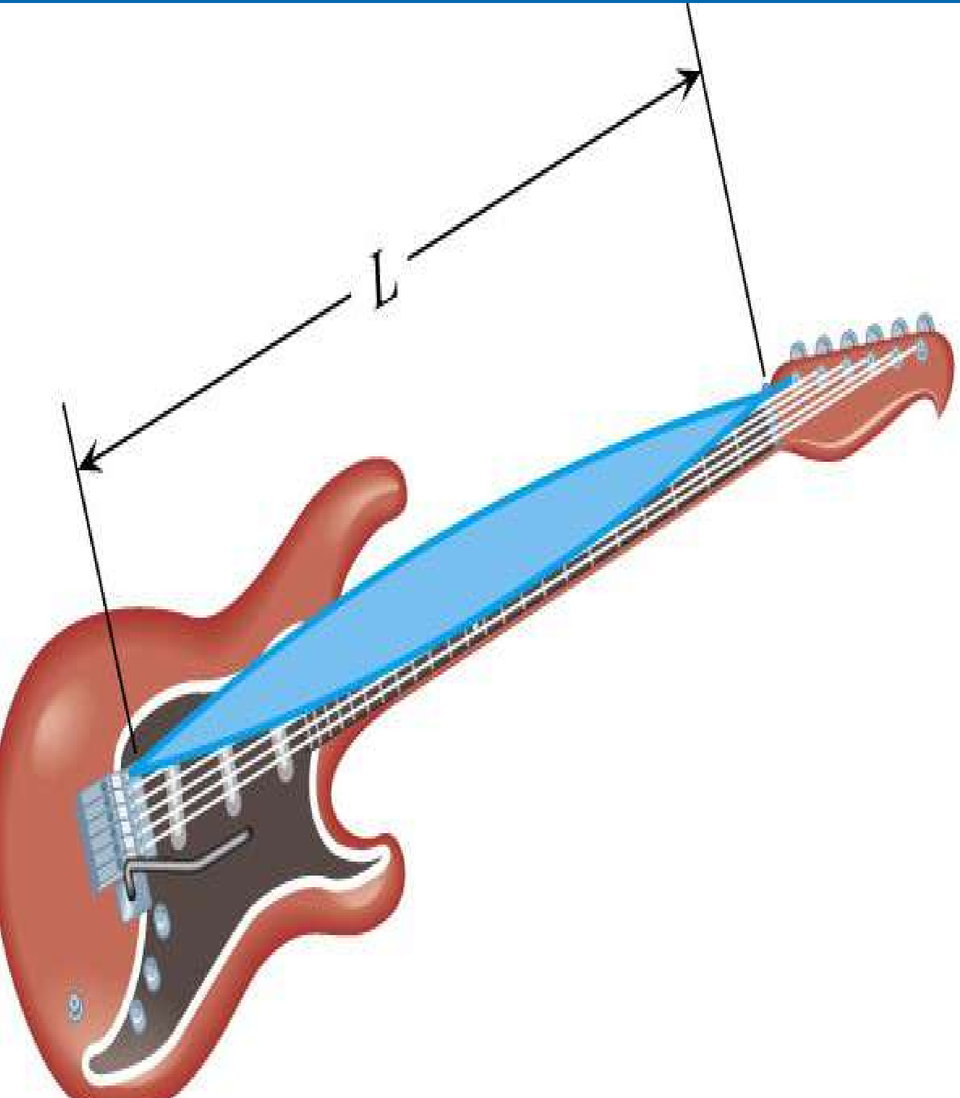
http://www.acoustics.salford.ac.uk/feschools/waves/standing_waves.php



- Notice that standing waves for a certain length of medium will form only in ½ λ increments (or increments of half a wave).
- For that medium, these lengths— ½ λ, λ, 1 ½ λ, 2 λ, etc.—correspond to the natural frequency of that medium.
- Most mediums have only one natural frequency.
- Strings (as mediums) have more than one natural frequency.



- $f = v/\lambda$
- 1st Harmonic: $f_1 = v/\lambda = v/(2L)$
- 2nd Harmonic: $f_2 = v/\lambda = v/L$
- 3rd Harmonic: $f_3 = v/\lambda = v/(2/3\ L)$
- 4th Harmonic: $f_4 = v/\lambda = v/(1/2\ L)$ and so on…

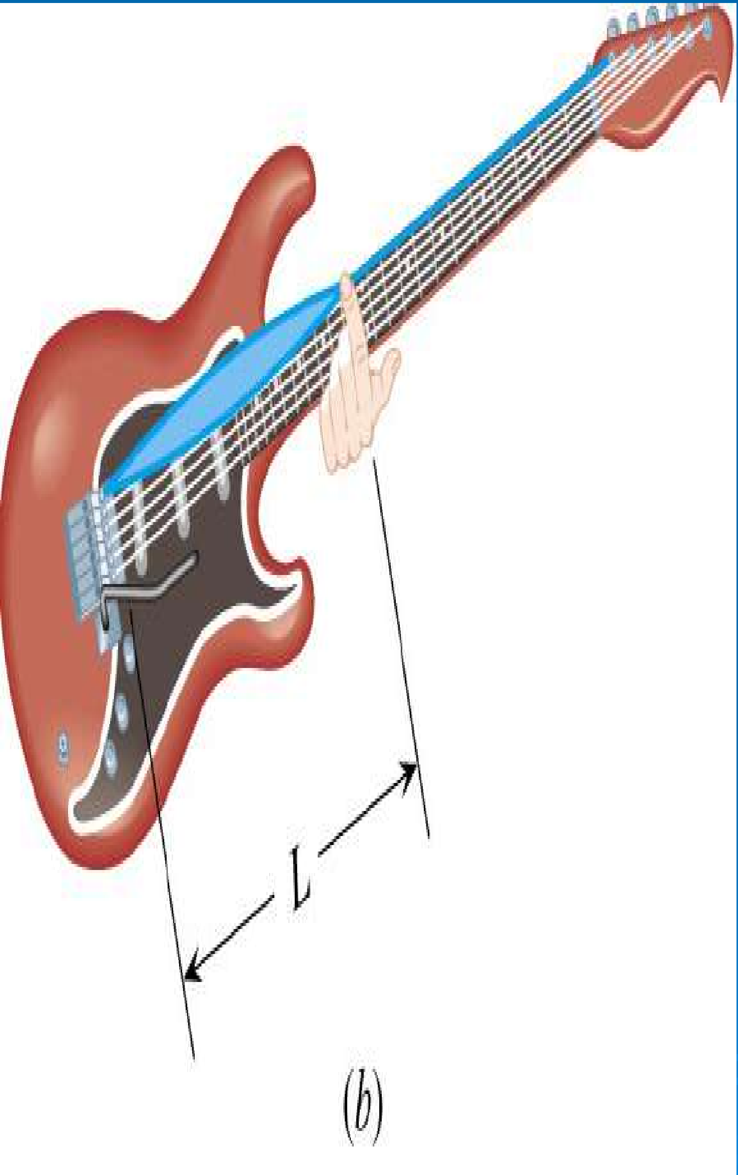


(a)

- The heaviest string on an electric guitar has a linear density of 5.28 X 10^{-3} kg/m and is stretched w/a tension of 226N.
- This string produces the musical note E when vibrating along its entire length in a standing wave at the fundamental frequency of 164.8 Hz.
- Find the length L of the string between its two fixed ends (see drawing).

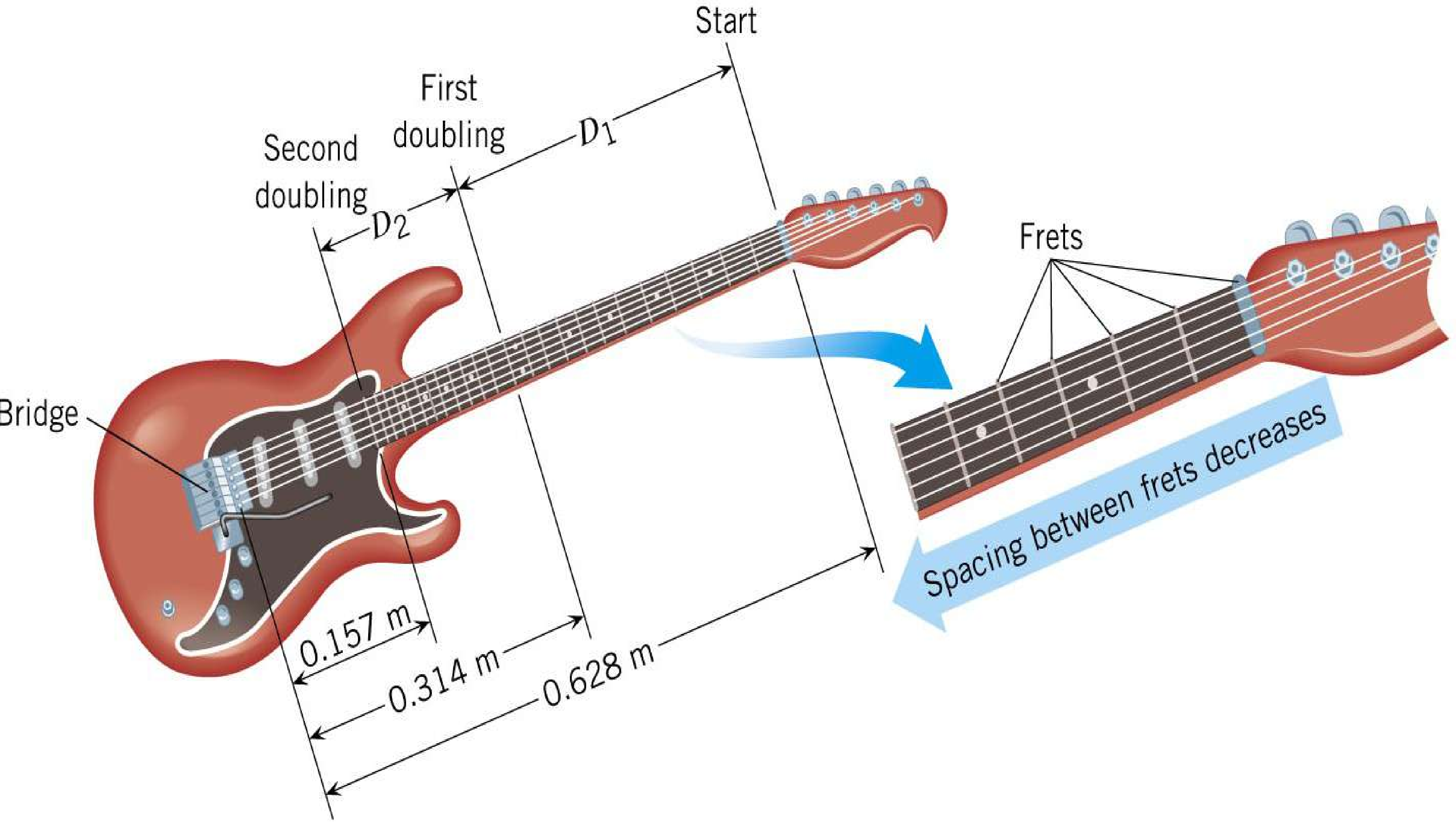
Solution…

- $m/L = 5.28 \times 10^{-3}$ kg/m
- $F_T = 226N$
- f = 164.8 Hz
- L = ?
- Since this wave is vibrating at the fundamental frequency, we know $f_1 = v/\lambda = v/(2L)$
- And, since this is a string with linear density, m/L, vibrating with tension, F_T, we know the speed of the wave is found with $v = \sqrt{[F_T/(m/L)]}$
- $v = \sqrt{[F_T/(m/L)]} = \sqrt{[226N/(5.28 \times 10^{-3}\ kg/m)]} = 207$ m/s
- Rearrange $f_1 = v/\lambda = v/(2L)$ for L:
- $L = v/(2 f_1) = (207\ m/s) / (2 \cdot 164.8Hz)$
- $L = (207m/s)/329.6Hz$
- **L = 0.628 m**



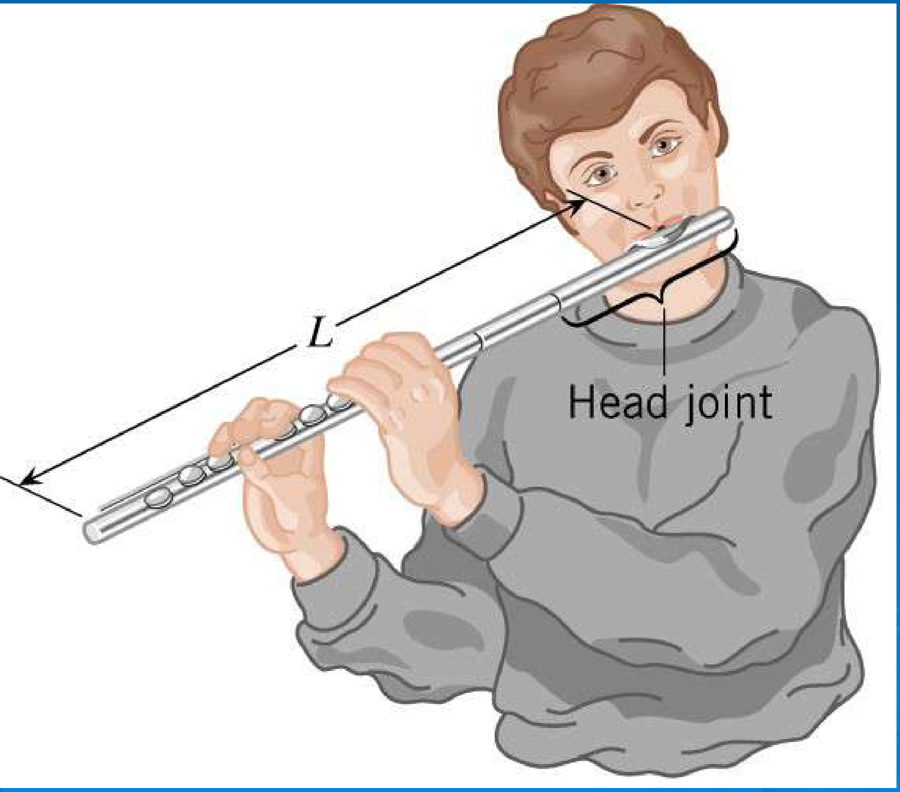
- The same guitar player wants to play the musical note E one octave higher than in the previous example.
- ∴ the string to needs to vibrate at a fundamental frequency of 2 X 164.8 Hz, or 329.6 Hz, in order to produce an octave higher.
- To accomplish this, he presses the string against the proper fret and then plucks the string (see picture).
- Find the distance L between the fret and the bridge of the guitar.
- How does this distance relate to the distance in the previous example?
- $L = v/(2 \cdot f) = (207 m/s)/(2 \cdot 329.6 Hz)$
- **L = 0.314 m** (…half the original length)

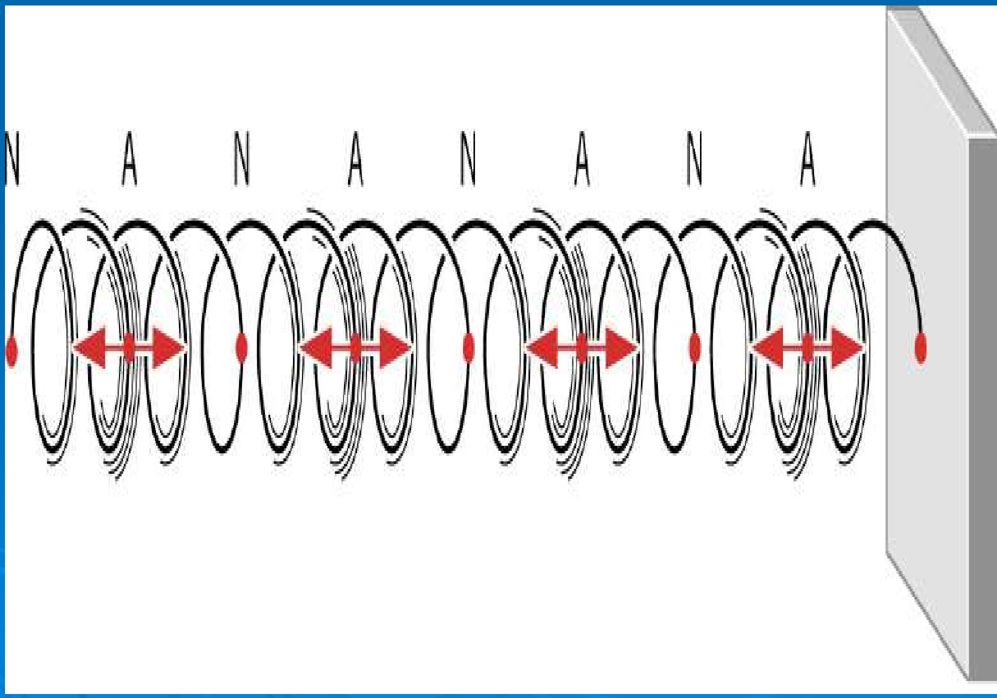
Why do frets get closer together as going down the neck toward the bridge?



Longitudinal Standing Waves

- Musical wind instruments (flute, trumpet, clarinet, pipe organ, etc.) depend on longitudinal standing waves– columns of vibrating air with nodes and antinodes.
- http://www.walter-fendt.de/ph14e/stlwaves.htm

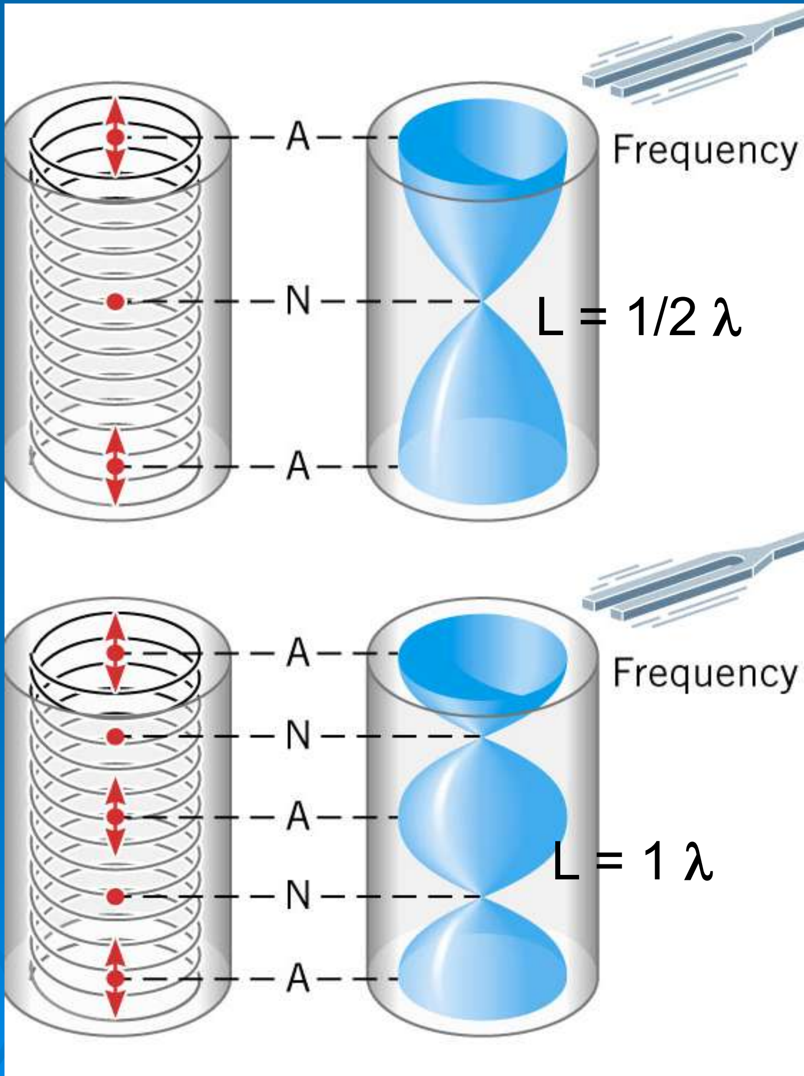




Tube open at both ends

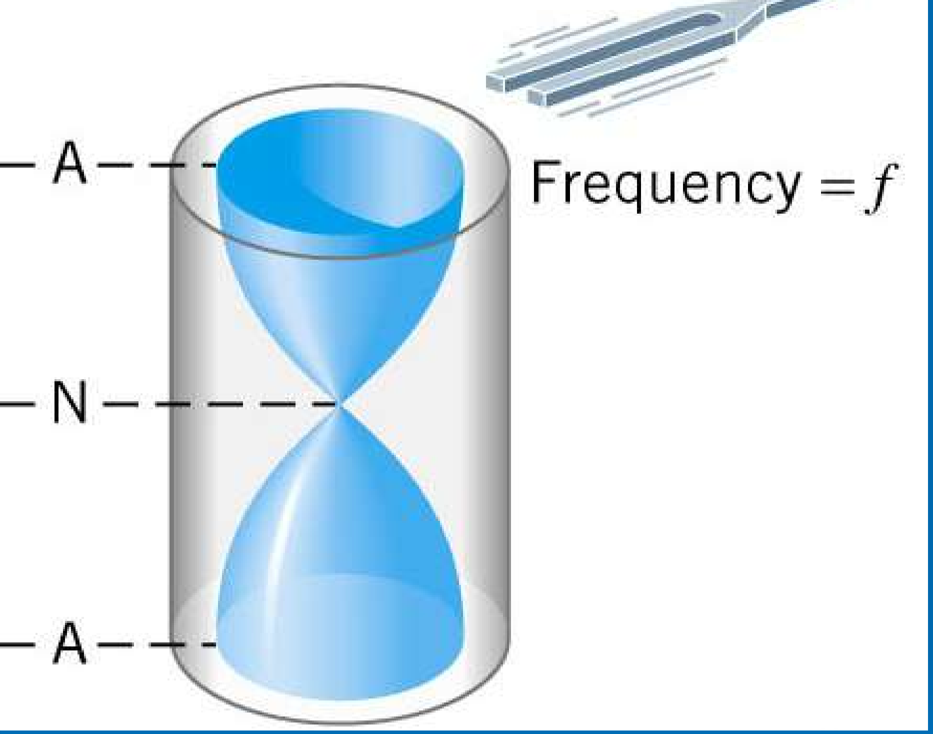
If the frequency of the tuning fork matches one of the natural frequencies of the air column, the downward and upward traveling waves combine to form a standing wave, and the sound of the tuning fork is noticeably louder.

- As in transverse standing waves, the distance between two successive antinodes is ½ of a wavelength
- ⇨ the length, L, of a tube must be an integer number, n =1,2,3,4, etc., of half-wavelengths:
- In other words, at both ends of the tube there will be antinodes (places of maximum disturbance)
- $f = v / \lambda$
- f = the natural frequency of the vibrating air in the column
- v = speed of sound

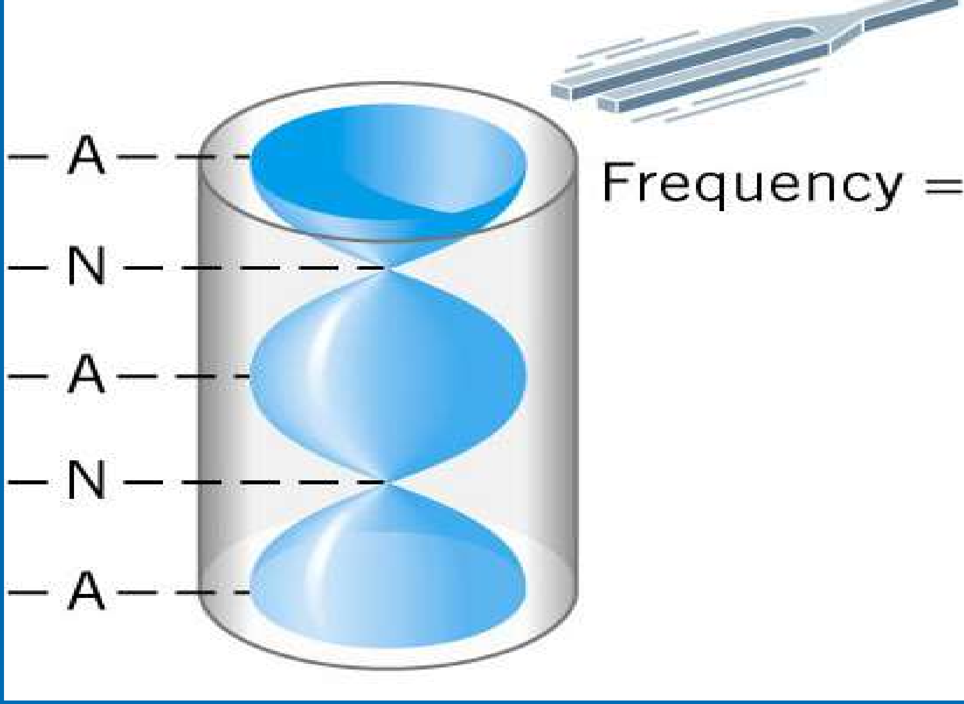


$f = v/\lambda$

A to N is ¼ of a wave… ∴ one-half of a wave is created below.↓



A to N is ¼ of a wave… ∴one wave is created below. ↓



$f = v / \lambda$

since there is one-half wave in the length L of the tube.

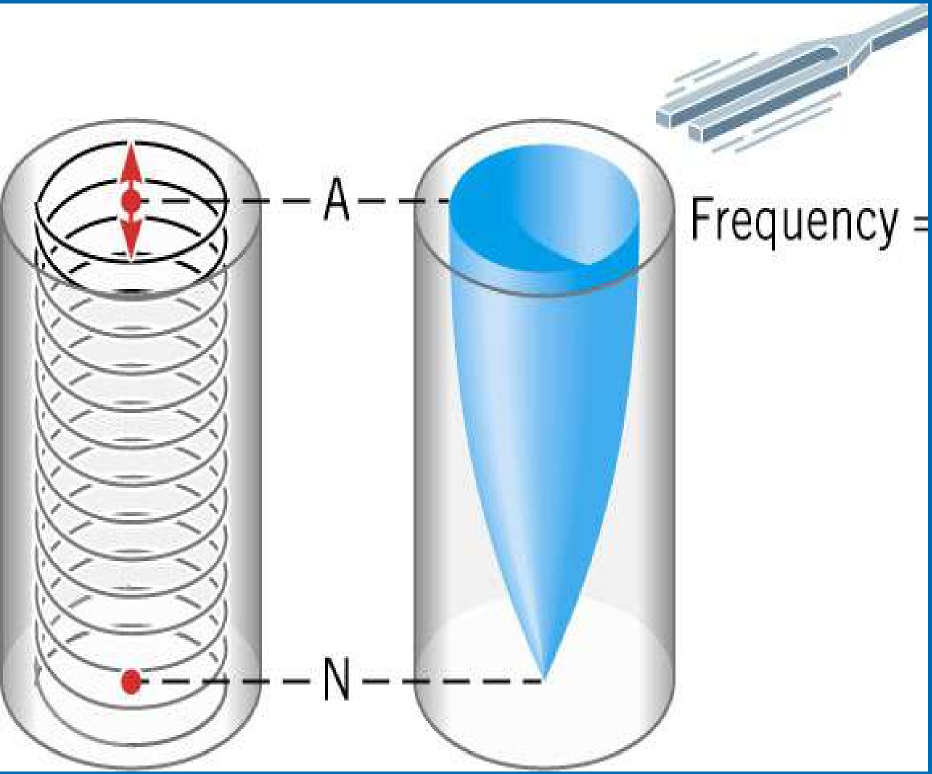
$f = v/(2L) = ½\ v/L$

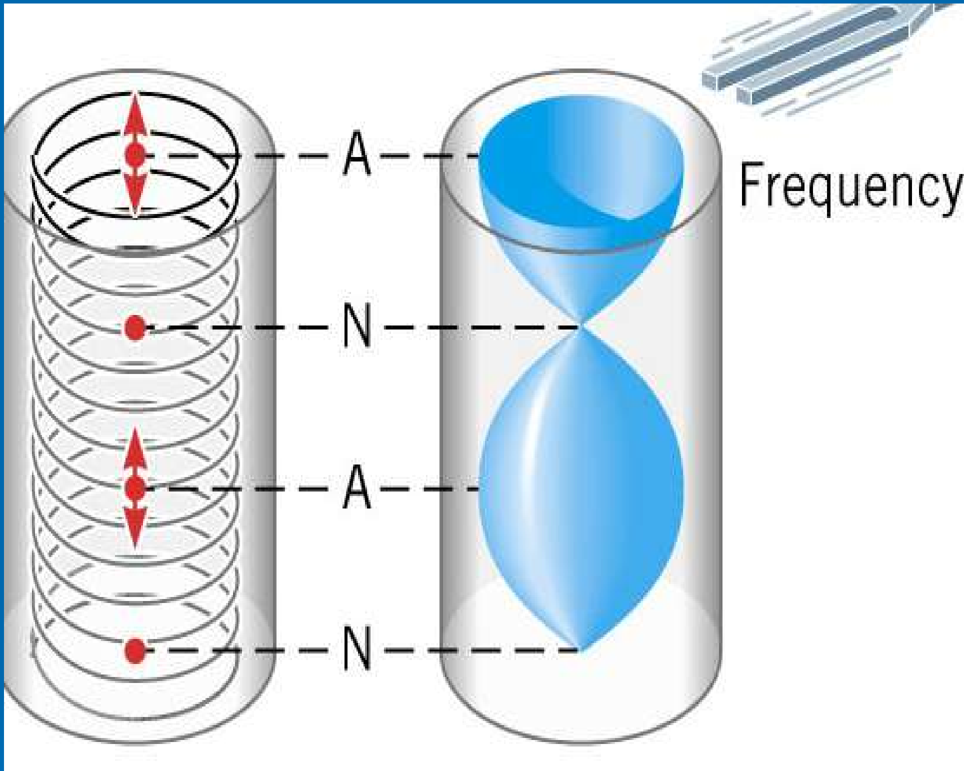
$f = v / \lambda$

since there is one wave in the length L of the tube.

$f = v/\lambda = v/(1L) = v/(L) = 1v/L$

f = v/λ & Tube open at one end:





f = v/λ = v/(4L)

since there is ¼ of a wave in the length L of the tube.

λ = length of tube X 4

f = v/λ = v / (4/3L)

since there is ¾ of a wave in the length L of the tube.

λ = length of tube X 4/3