

# The Physics of Sailing

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We present a review of the physical phenomena that govern the motion of a sailing yacht. Motion is determined by force, and the forces on a sailing yacht depend on the interactions of the hull and keel of the yacht with the water and on the interactions of the sail or sails of the yacht with the air. We discuss these interactions from the perspective of fluid dynamics, governed ultimately by the Navier-Stokes equation, and show how forces such as lift and drag are achieved by the relative motion of a viscous fluid around a body. Additionally, we discuss phenomena that are exclusive to sailing yacht geometries.

## I. INTRODUCTION

In the year 1851, about a half century prior to the first successful flight of a fixed-winged aircraft, the New York Yacht Club's schooner yacht won the Royal Yacht Squadron Cup from the Royal Yacht Squadron (a British yacht club), and it was thereafter known as the America's Cup. This began a series of matches between the holder of the Cup and a challenger (or challengers) that today are known as the America's Cup sailing regattas. Interestingly, the New York Yacht Club held the cup for 132 years after its first victory until, in 1983, the Royal Perth Yacht Club challenged and won the Cup with their *Australia II* yacht. About 120 years after the first America's Cup regatta, advances in scientific knowledge had dramatically altered the world's understanding of winged craft; however, particular attention had been paid to aircraft while sailing craft had been left much less explored.

After an afternoon of recreational sailing with a friend, scientist and Boeing engineer Arvel Gentry fell for the sport and decided to apply his knowledge of aerodynamics and fluid mechanics to sailing yachts in an effort to better understand the physical principles that were at work on them. To his surprise, much of what he read in the existing literature on the subject was at least misleading, if not completely wrong. His first attempt to rectify the understanding of the interactions of a sailboat with the wind and water [1] was ill-received by many, but not all. Indeed, scientist and avid sailor John Letcher took notice of Gentry's work and began his own line of research involving the application of computational fluid dynamics (CFD) to sailing yachts, leading to the first velocity prediction programs (VPPs), which applied CFD to actual boat geometries and predicted their motion, or velocity, through the water. Later on, Letcher became the head scientist on Dennis Conner's team (of the San Diego Yacht Club) for the *Stars & Stripes 87* yacht, which took the America's Cup back from the Australians in 1987 [2].

Today, the 33<sup>rd</sup> America's Cup regatta is just around the corner, to be held in the Mediterranean Sea off the coast of Valencia, Spain in February, 2010. The regatta will be a "Deed of Gift" [3] match between the Société Nautique de Genève (defending the Cup with the *Al-*

*inghi 5* yacht, shown in figure 1a) and the Golden Gate Yacht Club (challenging the Cup with BMW Oracle Racing's *BOR 90* yacht, shown in figure 1b). These boats are direct products of an advanced understanding of the physics that governs the motion of a sailboat through the wind and water; respectively, the aero and hydrodynamics of sailing yachts.

This work reviews the physics, consisting mostly of fluid mechanics phenomena, that applies to sailing yachts and influences the design of modern sailing vessels, such as those to be raced in the 33<sup>rd</sup> America's Cup regatta. In section II, the forces that act on a sailing yacht are introduced and discussed. In section III, the fluid mechanics of viscous fluids flowing around fixed bodies is discussed to explain the origins of the forces that are introduced in section II. Section IV discusses contributions to the forces that are specific to yacht designs and are beyond the generalization of section III.

## II. FORCES ON A SAILING YACHT

Long before the invention of the mechanical engine or the understanding of fluid mechanics, people attached sails to boats in order to move them across the water. These sails acted like parachutes, catching the wind and moving the boat in the direction of the wind, thereby limiting the motion of the boat, more or less, to this direction [4]. While this limited direction of motion is certainly a drawback of such a design, another setback is that the boat speed can never exceed the wind speed with such a simple sail. Nevertheless, boats must sometimes sail downwind, in which case sails that resemble the more archaic parachute-like sails must be used. These sails are called "spinnakers" and, despite the relatively simple concept of downwind sailing, must be engineered with a nontrivial knowledge of fluid mechanics.

In addition to employing spinnakers for downwind sailing, modern sailing yachts use other types of sails for sailing in directions (points of sail) other than downwind. Taller, thinner mainsails and jibs can be used to achieve a "reach," or a point of sail that is at an angle to the wind direction. Such points of sail can be perpendicular to the wind direction or even have a component *against* the wind direction. Such sailing is called "windward"

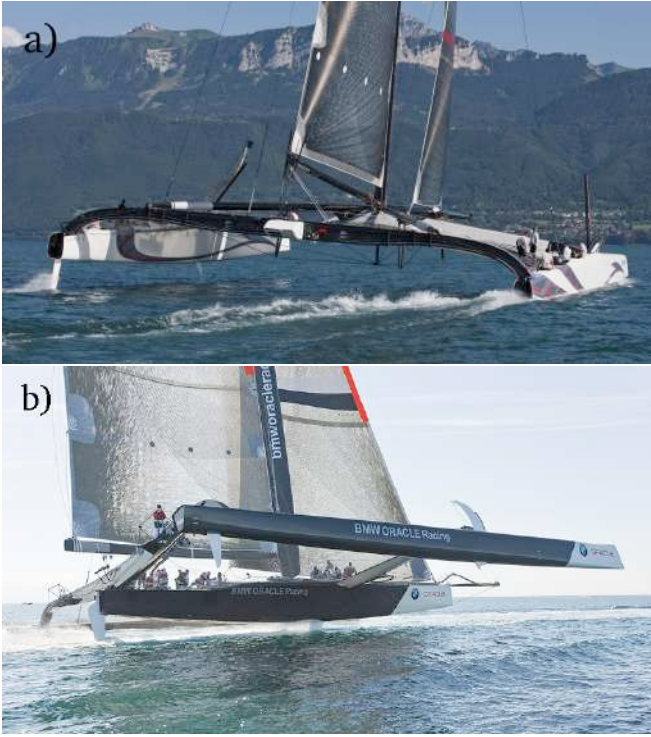


FIG. 1: (a) The two-hulled *Alinghi 5* yacht of the Société Nautique de Genève (<http://sailracewin.blogspot.com>) (b) The three-hulled *BOR 90* yacht of BMW Oracle Racing (<http://bmworaccleracing.com>)

sailing, and requires a component of force from the wind on the sail that is perpendicular to the direction of the wind. This force is called the aerodynamic lift force,  $\vec{L}$ . The force on the sail that is in the direction of the wind is called the aerodynamic drag force,  $\vec{D}$ . These forces are shown in the force diagram in Figure 2. The vectorial sum of  $\vec{L}$  and  $\vec{D}$  gives the total aerodynamic force on the sail(s),  $\vec{F}_T = \vec{L} + \vec{D}$ . This force can then be broken down into components parallel to and perpendicular to the point of sail, the driving force  $\vec{F}_R$  and the heeling force  $\vec{F}_H$ , respectively.

Interestingly, properly designed keels can generate a similar lift force as they move through the water to enhance windward sailing. This lift is called the hydrodynamic side force,  $\vec{F}_S$ . The force on the keel (and the hull) that is parallel to the apparent motion of the water is called the hydrodynamic drag force,  $\vec{R}$ . These forces are shown in the force diagram in Figure 3. Like the aerodynamic forces, the hydrodynamic side force (hydrodynamic lift) and hydrodynamic drag can be summed to give the total hydrodynamic force on the keel and hull,  $\vec{R}_T = \vec{F}_S + \vec{R}$ . The origins of both the aero and hydrodynamic lift and drag forces are discussed in section III. The condition for sailing at a constant velocity is that the hydrodynamic and aerodynamic forces balance, or  $\vec{R}_T + \vec{F}_T = 0$ .

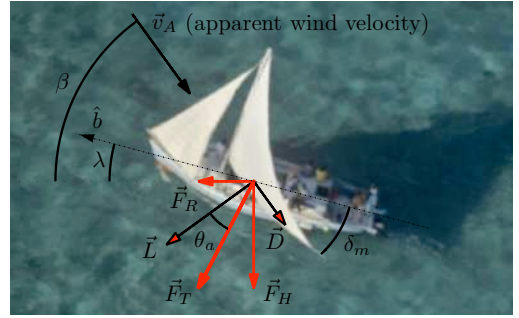


FIG. 2: The aerodynamic forces acting on a sailing yacht as illustrated for a windward point of sail. Here,  $\vec{L}$  is the aerodynamic lift on the sail and  $\vec{D}$  is the aerodynamic drag on the sail.

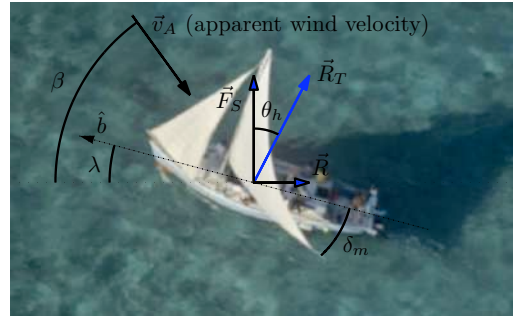


FIG. 3: The hydrodynamic forces acting on a sailing yacht as illustrated for a windward point of sail. Here,  $\vec{F}_S$  is the hydrodynamic side force, or lift and  $\vec{R}$  is the hydrodynamic drag. These forces are due to the interactions of both the hull and keel with the water.

Both aerodynamic and hydrodynamic forces are shown in Figure 4. In this Figure (and in Figures 2 and 3),  $\beta$  defines the angle between the point of sail and the origin of the apparent wind (the wind as measured in the boat's frame of reference),  $\lambda$  defines the leeway, or the angle between the boat axis  $\hat{b}$  and the the point of sail and  $\delta_m$  defines the angle of the sail relative to the boat axis  $\hat{b}$ . In terms of these angles, the angle of attack of the keel is simply  $\alpha_{\text{keel}} = \lambda$  and the angle of attack of the sail(s) is  $\alpha_{\text{sail}} = \beta - \lambda + \delta_m$ . The drag angles  $\theta_a$  and  $\theta_h$  characterize the aerodynamic and hydrodynamic forces, respectively, and are such that  $\cot \theta_a = L/D$  and  $\cot \theta_h = F_S/R$ .

Maximizing the aerodynamic drag  $D$  allows for more efficient downwind sailing while minimizing the aerodynamic drag and maximizing the aerodynamic lift  $L$  makes for more efficient windward sailing. While it is always beneficial to minimize hydrodynamic drag, it is important to design a keel that will generate significant hydrodynamic lift for windward sailing conditions, but not

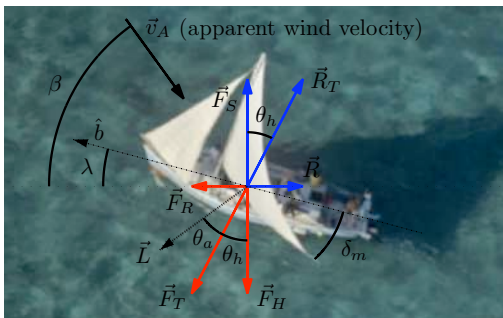


FIG. 4: The aero and hydrodynamic forces acting on a sailing yacht, shown for an equilibrium sailing condition when the total aerodynamic force plus the total hydrodynamic force vanishes.

so important to do so for downwind conditions. Figure 4 illustrates that, at an equilibrium sailing velocity,  $\cot \theta_h = F_H/F_R$ . Maximizing this ratio is necessary for maximizing performance for all points of sail, so minimizing  $\theta_h$  is always key to optimizing the performance of a sailing yacht. Notice that, when the forces are in equilibrium,  $\beta = \theta_h + \theta_a$  [5]. Thus, minimizing the drag angles, or equivalently maximizing the lift-to-drag ratios, maximizes the extent to which a yacht may sail to windward.

While the interaction of the wind and water with a sailing yacht influences its motion in the plane of the water, it also influences the vertical angle of the yacht, or the deviation of the yacht from the vector normal to the surface of the water,  $\hat{z}$ . This is called the heeling angle,  $\Theta$ , and is illustrated, with the heeling and righting torques, in Figure 5. The heeling torque  $M_H$ , due to the pressure of the wind on the sail, serves to tilt the yacht while the righting torque, due to the torque from gravity acting on the center of mass of the boat relative to the center of buoyancy and the hydrodynamic forces on the hull and keel, serves to move the boat more upright. The heeling torque, in terms of the aerodynamic forces, the sailing angle  $\beta$  and leeway  $\lambda$ , is given by

$$M_H = R_a [L \cos(\beta - \lambda) + D \sin(\beta - \lambda) \cos \Theta] \quad (1)$$

and the righting torque, in terms of the hydrodynamic forces and the leeway, is given by

$$M_R = -R_h [R \sin \lambda \cos \lambda (1 + \cos \theta) + F_S (\cos^2 \lambda - \sin^2 \lambda \cos \Theta)] + R_b F_b \sin \Theta \quad (2)$$

where  $F_b$  is the magnitude of the buoyancy (vertical) force  $\vec{F}_b = F_b \hat{z}$ , given by  $F_b = mg + (L - R \sin \lambda - F_S \cos \lambda) \sin \Theta$ . In Eqs. (1) and (2),  $R_a$  and  $R_h$  represent the distance from the center of mass of the sailing yacht to the center of effort of the sail and the center of effort of the keel and hull, respectively. The centers of

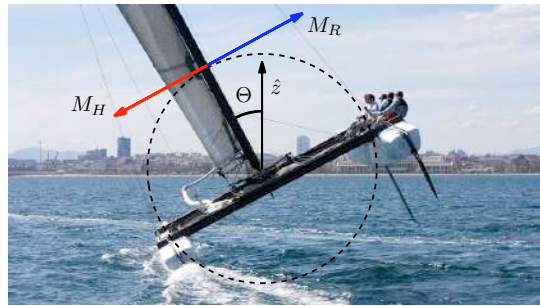


FIG. 5: A two-hulled yacht sailing at a heel angle  $\Theta$ . The heeling torque  $M_H$  (red) is due to the air acting on the sail and serves to tilt the yacht (increase  $\Theta$ ) and the righting torque  $M_R$  (blue) is due to the geometry of the boat and the action of the water on the yacht and serves to move the boat upright (decrease  $\Theta$ ). This image was taken from <http://www.sailjuice.com>.

effort describe the mean locations at which the aerodynamic and hydrodynamic forces act. Also, in Eq. (2),  $R_b$  is the distance between the center of mass of the yacht and the center of buoyancy of the yacht (being the center of mass of the water that would be present if the yacht were not there to displace it). To increase stability, many yachts are designed so that  $R_b$  increases with  $\Theta$  [5]. Multi-hulled yachts, such as those to be raced in the 2010 America's Cup, are designed in this way so that, for moderately small  $\Theta$ , the buoyancy contribution to the righting torque is very large. These yachts can withstand large heeling forces, or large aerodynamic lift, when sailing at an angle to the wind. In equilibrium sailing conditions, these torques balance themselves so that  $M_H + M_R = 0$ . To understand the nature of the forces and torques acting on a sailing yacht, it is essential to understand the physics of the fluids that generate these forces and torques.

### III. RELEVANT FLUID MECHANICS

#### A. Properties of Air and Water

All of the fluid mechanics that is relevant to the physics of sailing depends on the densities of air and water. At 10° C and atmospheric pressure ( $101.325 \times 10^3$  Pa), the density of (dry) air is  $1.247 \times 10^{-3}$  g/cm<sup>3</sup> and the density of pure water is 1.000 g/cm<sup>3</sup>. At 30° C, the density of (dry) air is  $1.164 \times 10^{-3}$  g/cm<sup>3</sup>, while the density of pure water is approximately the same.

When forces act on these fluids (air and water), their densities (or equivalently, volumes) may change. This is characterized by the isothermal compressibility of the fluid, or  $\kappa_T = -V^{-1}(\partial V/\partial p)_T$ , describing the unit change in volume of a fluid with a change in pressure at a fixed temperature  $T$ , where  $V$  is the volume of the

fluid being considered and  $p$  is the pressure exerted upon it [6]. At  $10^\circ$  C and atmospheric pressure, the isothermal compressibility of dry air is  $\kappa_{T\text{air}} = 9.9 \times 10^{-6} \text{ Pa}^{-1}$  and the isothermal compressibility of pure water is  $\kappa_{T\text{H}_2\text{O}} = 4.8 \times 10^{-10} \text{ Pa}^{-1}$  [7]; these values remain on the same order of magnitude with moderate changes in temperature and pressure.

If there is motion of an object through a fluid (or of a fluid around an object), there will be some response in the fluid surrounding this object. This is due to the transfer of momentum from particle to particle in the fluid and depends on the nature of the interparticle interactions. This phenomenon can be quantified, however, without the details of these microscopic interactions by considering the dynamic viscosity of the fluid,  $\mu$ , defined as the force per unit area necessary to change the velocity of the fluid by one unit per unit distance perpendicular to the direction of flow. Because of thermal and many-body effects in the fluid, this quantity must, in general, be determined empirically. At  $10^\circ$  C and atmospheric pressure, the viscosity of air is  $\mu_{\text{air}} = 1.77 \times 10^{-5} \text{ Pa s}$  and the viscosity of pure water is  $\mu_{\text{H}_2\text{O}} = 1.27 \times 10^{-3} \text{ Pa s}$ . At  $30^\circ$  C and atmospheric pressure, the viscosity of air is  $\mu_{\text{air}} = 1.87 \times 10^{-5} \text{ Pa s}$  and the viscosity of pure water is  $\mu_{\text{H}_2\text{O}} = 7.77 \times 10^{-4} \text{ Pa s}$ .

## B. Lift and Drag

Consider an inviscid, irrotational fluid with a steady velocity field  $\vec{V} = \vec{V}(x, y, z, t)$  (i.e.  $\partial V/\partial t = 0$  and  $\vec{\nabla} \times \vec{V} = 0$ ) describing the velocity of the fluid as a function of space and time, and let  $\rho$  be the mass density of the fluid and  $p = p(x, y, z, t)$  the pressure in the fluid. From the Navier-Stokes equation (derived in the appendix, Eq. A6),

$$\rho(\vec{V} \cdot \vec{\nabla})\vec{V} = -\vec{\nabla}p. \quad (3)$$

Note that  $(\vec{V} \cdot \vec{\nabla})\vec{V} = \vec{\nabla}V^2/2 - \vec{V} \times (\vec{\nabla} \times \vec{V})$  [8]. With  $\vec{\nabla} \times \vec{V} = 0$ , Eq. 3 can be rewritten as

$$\vec{\nabla} \left[ \frac{1}{2}\rho V^2 + p \right] \cdot d\vec{s} = d \left[ \frac{1}{2}\rho V^2 + p \right] = 0 \quad (4)$$

where  $d\vec{s}$  is an infinitesimal displacement vector. Thus, the scalar  $\frac{1}{2}\rho V^2 + p$  must be a constant. This constant is just the value in a background flow with pressure  $p_0$  and speed  $V_0$ . Thus, Eq. 4 can be written as

$$p = p_0 - \frac{1}{2}\rho\Delta V^2. \quad (5)$$

where  $\Delta V^2 = V^2 - V_0^2$ . This is Bernoulli's equation (for irrotational flow), and provides the basic relationship between pressure and velocity that gives rise to the phenomenon of lift [9], specifically, that flows with greater velocity exhibit lower pressure.

If a body is immersed in a moving fluid and the fluid moves faster along one side of the body than along the other side, Eq. 5 says that there will be a pressure gradient across the body, and thus a net force across the body. The element of this force that is perpendicular to the direction of the flow is the lift force  $L$ . Eq. (5) shows that  $L$  is proportional to the fluid density times the square of the flow velocity times an area. If  $A$  is the area of a body in the flow of a fluid with density  $\rho$  (say, for example, the area of a sail), then the lift force can be written as

$$L = \frac{1}{2}C_L\rho AV_\infty^2 \quad (6)$$

where  $V_\infty$  is the speed of the uniform fluid flow very far from the body and  $C_L$  is the dimensionless lift coefficient.  $C_L$  depends, in general, on the geometry of the body and on the orientation of the body in the fluid. Knowing  $C_L$  for a given object at a given orientation in a fluid flow provides all of the information necessary to describe the lift force on the object.

Similarly, a dimensionless drag coefficient can be defined to characterize the drag on a body in a fluid flow, or the component of the force on the body in the direction of the flow. Intuitively, the drag should depend linearly on the density of the fluid in which the body is immersed (because force depends linearly on mass) and linearly on the area of the body that is exposed to the flow because the volume of fluid that must be displaced as the body moves through it is proportional to this area. A dimensional argument then leaves the velocity dependent on the square of the fluid velocity. Thus, the drag force on the body can be written as

$$D = \frac{1}{2}C_D\rho AV_\infty^2 \quad (7)$$

where  $C_D$  is the dimensionless drag coefficient. Like  $C_L$ ,  $C_D$  depends on the geometry of the body and on the orientation of the body in the fluid.

## C. Effects of Viscosity on Drag

When a fluid has some viscosity, as do water and air, it adheres to the surface  $S$  of an immersed body so the fluid has no velocity at the surface of the body, or  $V|_S = 0$ , which is known as the “no slip” condition. Further from the surface, however, the fluid has some non-zero velocity that, because of viscosity, is affected by the motionless fluid at the surface of the body and the fluid that is moving at some distance from the body. This fluid creates what is called the boundary layer, being the layer of fluid near the surface of the body that is most affected by viscosity. This boundary layer can, in general, be either laminar or turbulent.

Laminar flow is a steady flow localized near the boundary of the immersed body and is such that the velocity field of the fluid is smooth along the plane of the surface

as though the flow consists of layers, or laminae. For this reason, laminar flow around a body is said to be “streamline.” Consider fluid flow with a constant velocity  $V_\infty \hat{x}$  in two-dimensions (2D) that encounters a boundary defined by  $y = 0$  at the point  $x = 0$  (as an approximate geometry of a keel or sail). The width of the laminar boundary layer in this case is given by, as a function of the distance  $x$  along the boundary, [9]

$$\delta = 5.2 \sqrt{\frac{\mu x}{\rho V_\infty}}. \quad (8)$$

For typical (America’s Cup) sailing speeds of  $\sim 10.3$  m/s, this gives, for a temperature of  $10^\circ$  C, pure water and air laminar widths of  $\delta_{\text{H}_2\text{O}} \simeq 0.60\sqrt{x}$  cm and  $\delta_{\text{air}} \simeq 2.0\sqrt{x}$  cm, respectively, which are quite small compared to the sizes of the sails and keels on typical sailing yachts.

The shearing of the fluid in the laminar boundary layer leads to a drag on the body known as frictional resistance. The magnitude of the force required to shear a fluid is defined, as described earlier, by the viscosity  $\mu$  of the fluid. Thus, the stress that gives rise to this laminar frictional force (in the 2D case described above) at the surface of the plane in the flow is the laminar shearing stress,

$$\tau_l = \mu \frac{\partial V_x}{\partial y} \Big|_{y=0}, \quad (9)$$

which, in three dimensions (3D), generalizes to the tensor  $\tau_{ij} = \mu (\partial V_j / \partial x_i + \partial V_i / \partial x_j)$ . While laminar flow dominates the boundary layer near the leading edge of the body (where the fluid first encounters its surface), it, at some point, may separate from the surface due to the development of an adverse pressure gradient along the surface of the body; this point is called the separation point [9]. Downstream from the separation point, the flow near the surface may reverse direction, leading, ultimately, to nonlaminar flow with vortices and strong time dependence which greatly widens the extent of the boundary layer. This flow is known as turbulent flow. In qualitative terms, flow in the direction of decreasing pressure is stable (laminar) but flow into increasing or adverse pressure is unstable, and will ultimately separate and become turbulent. As flow in the boundary layer resists this pressure increase, it moves away from the surface, expanding the boundary layer in the turbulent regime.

Because the thickness of the boundary layer for laminar flow is very small, it is a good approximation to treat the flow outside of the laminar boundary layer as nonviscous and compute the pressure along the surface of the body by applying Bernoulli’s equation to the flow velocity along the layer. Because the boundary layer downstream from the separation point is much wider than the laminar boundary layer, this approximation cannot be used in this region. The location of the laminar-turbulent transition can be estimated with accuracy by considering the

dimensionless Reynold’s number of the system, given by

$$\mathcal{R} = \frac{V_\infty x \rho}{\mu} \quad (10)$$

where  $x$  is the distance downstream along the body. Interestingly, fluid flow is found to turn turbulent when  $\mathcal{R} \simeq 5 \times 10^5$  for all viscous flows, depending, of course, on the nature of the surface in the flow (form, roughness, etc.). Using a background velocity of  $V_\infty = 10.3$  m/s and the densities and viscosities of pure water and air at  $10^\circ$  C, the distances at which flow turns from laminar to turbulent in pure water and air are, from a Reynold’s number analysis,  $x_{\text{H}_2\text{O}} \simeq 0.062$  m and  $x_{\text{air}} \simeq 0.68$  m, respectively. Clearly, with typical hull and keel sizes being much greater than 6 cm, hydrodynamic turbulence plays an important, unavoidable role in sailing. The downstream distance for the onset of turbulence in air, however, is more comparable to the downstream width (chord) of typical sails. In fact, clever design and usage of sails can lead to purely streamline airflow about sails in windward sailing conditions.

Turbulence serves to randomize the motion of the fluid and thereby flattens the average velocity profile. Because the “no slip” condition still holds at the surface, however, the partial derivative  $(\partial V_x / \partial y)_{y=0}$  is greater in the turbulent region than in the laminar region. Thus, the shear stress is greater in turbulent regions, which leads to a greater contribution of frictional drag in the turbulent boundary layer. This frictional drag can be estimated by an effective turbulent viscosity,  $\mu_t$ , so the shear stress on a surface in turbulence is given by  $\tau_t = \mu_t \partial V_x / \partial y|_{y=0}$  [10]. This is discussed further in section IV.

Additionally, the phenomenon of separation leads to another increase in drag known as pressure drag. Because the width of the boundary layer beyond the point of separation increases significantly, the pressure on the body cannot reach values comparable to those acting on the front of the body where the flow is laminar. Thus, there is a greater pressure contribution in the direction of the flow than there is opposing it, and a net “pressure” drag results. Because both friction and pressure drag depend strongly on the location of the separation point on a body, and the location of this separation point depends strongly on the Reynold’s number, the drag force on a body depends strongly on the Reynold’s number of the body, specifically, the drag force increases significantly beyond a critical Reynold’s number due to the phenomena of flow separation and turbulence [11].

#### D. Effects of Viscosity on Lift

To generate a net lift force in a viscous fluid flow, a body must exhibit an asymmetry about the direction of the external flow. An example of such an asymmetry is the rotation of an otherwise symmetric object about its

axis, such as a cylinder immersed in a flow with its symmetry axis perpendicular to the direction of the flow. Because of the no-slip condition at the surface of the cylinder, the flow is biased to move in one direction around the cylinder due to the presence of a circulatory flow field generated by the rotation. In this case, the lift force (per unit distance along its symmetry axis) on the cylinder is given by  $L = \rho V_\infty \Gamma$  where  $\Gamma$  is the circulation of the fluid around the cylinder given by the line integral  $\Gamma = \oint_C \vec{V} \cdot d\vec{s}$  where  $C$  is a line that encloses a cross-section of the cylinder and lies outside of its boundary layer [12]. This is known as the Kutta-Jukowski theorem, and it generalizes to bodies of arbitrary geometry. Thus, for a body to experience any lift in a viscous flow, there must be some non-zero circulation around the body.

Indeed, the designs of sails and keels are such that, when oriented appropriately, they develop a circulation flow field around them and thus experience a net lift. As a simple example of this, consider a flat plate in a 2D (or cross-sectional 3D) geometry tilted at a small angle relative to an external viscous flow. As the flow encounters the leading edge of the plate, it will split and some will move across the near (upstream) side of the plate while the rest will move around and across the far (downstream) side of the plate. Because of viscosity, the fluid moving on the downstream side (assuming that the flow is laminar) will be retarded and travel a shorter distance than the fluid moving along the upstream side. This will leave a gap at the end of the downstream side of the plate and the fluid on the upstream side will bend around the trailing edge of the plate to fill in this gap. Viscosity prevents the sharpening of this bend and results in the creation of a vortex at this trailing edge, which has some angular momentum. The flow sweeps this vortex downstream while another vortex, the circulation flow around the plate, is formed to conserve the angular momentum in the system. Because of the Kutta-Jukowski theorem, this circulation creates a lift force on the plate [13]. This phenomenon is illustrated in Figure 6. If there were no circulation about the plate, regardless of its angle of attack, the net lift on the plate would be zero. Additionally, the direction of the induced circulation about the plate serves to speed up the flow on the downstream side and slow down flow on the upstream side, consistent with lift as is explained by Bernoulli's equation. It is worth noting that Bernoulli's principle and the Kutta-Jukowski theorem are completely consistent with one another in explaining the phenomenon of lift.

In general, however, this lift force will be oriented perpendicular to the body and not to the external flow, with the body tilted at an attack angle to the external flow, as described for keels and sails in section II. This results in a lift-induced drag (per unit distance perpendicular to the flow) with force coefficient  $C_{D,i} = 2\Gamma \sin \alpha / V_\infty A$  [9].

In addition to affecting the drag force (as discussed in section III C), flow separation affects the lift force on a body, as well. In a laminar boundary layer, the flow is steady and close to the surface of the body so the effects

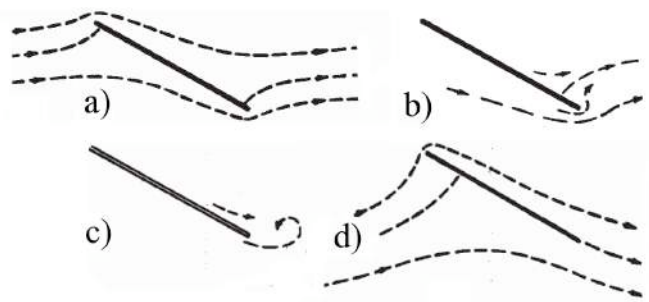


FIG. 6: The formation of the starting vortex on a flat plate tilted at an angle to an otherwise steady flow. (a) The flow lines on the plate if the fluid were to have zero viscosity. (b) With some viscosity, the fluid makes a turn from the upstream side around the trailing edge of the plate to meet the flow on the downstream side. (c) This bend results in the creation of a vortex that is swept downstream. To conserve angular momentum, a circulation is induced around the plate. (d) The resulting flow lines with the circulation flow field present. Because the flow lines are closer together on the top of the plate, the velocity is faster here, and thus, by Bernoulli's equation, the pressure is less here and there is a net lift on the plate. These images were taken from Ref. [13].

of the decrease in pressure due to increase in velocity are well translated to the body itself. When separation occurs, this pressure is not well translated and the overall circulation around the body significantly decreases due to the appearance of turbulence beyond the separation point, and thus, by the Kutta-Jukowski theorem, decreases the lift on the body. Additionally, the turbulent region beyond the point of separation has greater pressure than it would if the flow were laminar due to the decrease in average velocity in this region. This serves to increase the pressure (by Bernoulli's equation), and thus decrease the lift on the body. The phenomenon of flow separation on the leeward side of a sail is known as "stalling," and, as mentioned above, significantly reduces the lifting ability of a sail.

#### IV. APPLICATION OF FLUID MECHANICS TO SAILING YACHTS

The examples given above consider 2D flows, or cross sections of 3D flows that are homogeneous in one direction. However, we exist in a 3D world, and so do sailboats, and this fact changes things, sometimes slightly, and sometimes significantly. Nevertheless, in both 2D and 3D, viscous, turbulent fluid dynamics are incredibly complicated to understand; however, simple physical arguments and observations can provide a basic framework of how certain bodies should be designed in order to behave predictably in the presence of viscous fluids.

## A. Computational Tools

Beyond these arguments, which are discussed in the following sections, the determination of the lift and drag forces (or equivalently the lift and drag coefficients) on a body in a viscous flow must be calculated numerically or deduced empirically. The task of calculating force coefficients numerically (computationally) is very non-trivial. The main challenge is to model the presence of turbulence in the flow around the complicated shapes of a yacht's keel, hull and sail(s). One way to do this is to approximate turbulent effects in the velocity field and pressure manifolds in the Navier-Stokes equation using the Reynolds-Averaged Navier-Stokes (RANS) equation. The RANS equation is derived by assuming that the velocity and pressure fields can be decomposed into time-independent mean values and time-dependent fluctuations about this mean value,

$$\begin{aligned} V_i(x, y, z, t) &= \bar{V}_i(x, y, z) + V'_i(x, y, z, t) \\ p(x, y, z, t) &= \bar{p}(x, y, z) + p'(x, y, z, t), \end{aligned} \quad (11)$$

and then plugging these values into Eq. (A7) and averaging over time, which gets rid of the fluctuations that average to zero over time, ultimately giving an equation to describe time-averaged turbulence phenomena in the system. Solving the Navier-Stokes equation exactly for a given system, on a numerical grid, would require a number of grid points on the order of  $\mathcal{R}^{9/4}$  [14]. With Reynolds numbers easily exceeding  $10^6$  for typical sailing yachts, this problem becomes incredibly computationally expensive.

In addition to the RANS formalism, methods have been developed to model turbulent flow by calculating the average turbulent dissipation  $\epsilon$  and kinetic energy  $k$  of a flow. This is known as the  $k\epsilon$  model, and has been shown to be accurate and computationally accessible [15]. In this model,  $k$  and  $\epsilon$  are determined by solving coupled transport equations and then the effect of turbulence is modeled by an effective turbulent viscosity,  $\mu_t$ , which can be written in terms of  $k$  and  $\epsilon$ .

## B. Downwind Sailing

As mentioned in section II, efficiency in downwind sailing requires a minimization of hydrodynamic drag (both from the keel and the hull) and a maximization of aerodynamic drag. Maximization of aerodynamic drag for downwind sailing is achieved in a number of ways, but most simply by employing a sail with very large area cast perpendicular to the direction of the wind, since the drag force scales with the area of the body being considered. These sails are known as spinnakers.

However, it is unlikely that a sailor wishes to sail directly downwind. Instead, downwind sailing typically deviates from *purely* downwind by some small angle. Figure 7 shows the maximum attainable velocities, calcu-

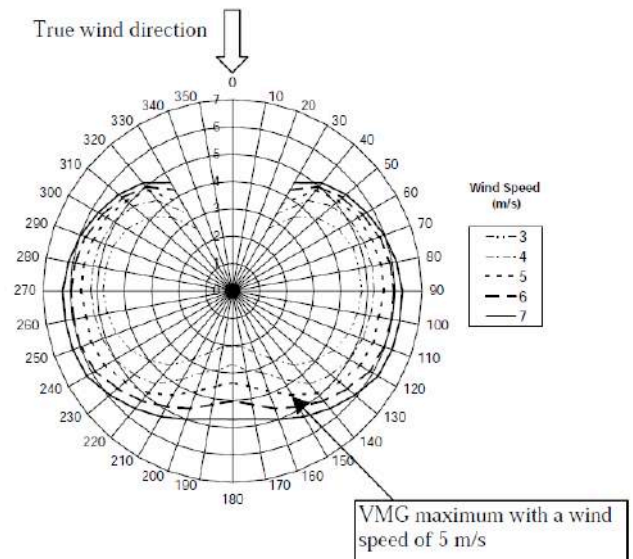


FIG. 7: The maximum speed of an IACC yacht, calculated using a VPP, as a function of point of sail relative to the true wind and true wind speed. Notice that, due to lift effects, the maximum velocity attainable is not down wind ( $\beta = 180^\circ$ ), but at a reach almost perpendicular to the true wind direction. Image taken from Ref. [16].

lated using a VPP (solving RANS), for a typical International America's Cup Class (IACC) yacht sailing at various angles to the true wind. From this data, an apparent wind speed, being  $|\vec{v}_a| = |\vec{v}_t - \vec{v}_b|$  where  $\vec{v}_t$  is the true wind speed and  $\vec{v}_b$  is the velocity of the yacht and an apparent wind angle,  $\beta$ , can be extracted. These are shown in Figure 8. Figure 8(a) shows that the apparent wind speed changes very little with small deviations in angle from  $180^\circ$  (being purely downwind). However, Figure 8(b) shows that there are strong deviations in apparent wind direction with these small deviations in angle. Thus, at points of sail close to the direction of the wind, a spinnaker must produce significant drag but must also produce significant lift in order to maintain course and velocity [16]. For a given point of sail, then, there exists an optimum  $C_L/C_D$  ratio for maintaining course, and many spinnakers are designed asymmetrically to achieve this ratio.

Nevertheless, to reach small apparent wind speed, the hydrodynamic drag must be sufficiently small so the aerodynamic forces may dominate the downwind motion of the yacht. To reduce the hydrodynamic drag on the hull, appealing to the arguments discussed above, the hull should be as streamline as possible so as to reduce turbulent friction drag and pressure drag due to separation. Also, the yacht itself should be as light as possible so as to minimize the amount of water that the hull displaces and thus minimize the surface area of the hull that interacts with the water. Additionally, hulls interact with the water surface and thus create surface waves, which also contribute to drag. Hull geometries can be generalized

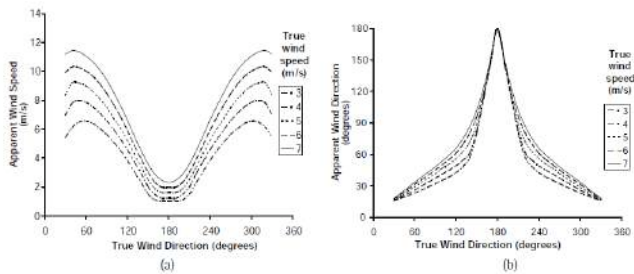


FIG. 8: Calculated from the data in Figure 7, the apparent wind speed and apparent wind direction as functions of the true wind direction for a typical IACC yacht. Image taken from Ref. [16].

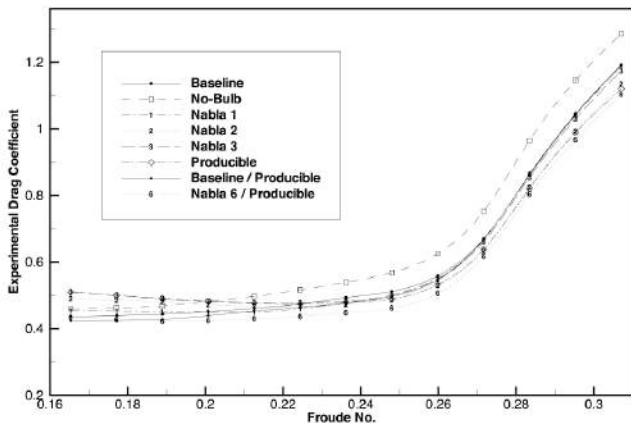


FIG. 9: The experimental drag coefficient for a number of sailing yacht monohulls as a function of the Froude number. The coefficient increases significantly at a critical Froude number due to the hull speed phenomenon. Image taken from Ref. [17].

by a Froude number,

$$F = \frac{V_\infty}{\sqrt{gL_{\text{ref}}}}, \quad (12)$$

where  $V_\infty$  is the speed of the yacht,  $g = 9.8 \text{ m/s}^2$  and  $L_{\text{ref}}$  is the characteristic downstream length of the hull. Figure 9 shows experimentally measured drag coefficients for a number of monohulls as a function of Froude number [17].

Objects with drag coefficients that scale in this way are said to “Froude scale,” meaning their drag coefficients scale with velocity. Notice that at  $F \simeq 2.6$ , corresponding to  $V_\infty \simeq 2.6\sqrt{gL_{\text{ref}}}$ , there is a significant increase in drag. This phenomenon can be understood by considering the nature of the waves created by the hull of a sailing yacht as it moves through the water. As the bow of the yacht encounters water, it displaces a volume of water at a rate  $\propto V_\infty A$  where  $A$  is the cross-sectional area of the hull. This water must ultimately be displaced, but the effect is delayed so that a “bow wave” forms at the bow

of the yacht. This bow wave generates a trailing surface wave, which is dispersive and has velocity (for small wave amplitude)

$$v_{\text{wave}} = \sqrt{\frac{g\lambda_{\text{surf}}}{2\pi}} \quad (13)$$

where  $\lambda_{\text{surf}}$  is the wavelength of the surface wave [18]. When the length of the hull  $L_{\text{ref}}$  is about the same as  $\lambda_{\text{surf}}$ , the yacht sits nicely atop the peaks of the wave. However, if  $\lambda_{\text{surf}}$  exceeds  $L_{\text{ref}}$ , the stern of the yacht will sink into the trough of the bow wave, greatly increasing the cross-sectional area exposed to the water and thus greatly increasing the drag on the hull. The boat speed above which this drag increases is the hull speed,  $v_{\text{hull}} = \sqrt{gL_{\text{ref}}/2\pi}$ . From Eqs. (12) and (13), the Froude number of the yacht when traveling at hull speed is  $F_{\text{hull}} = \sqrt{1/2\pi} = 0.399$ , just above the observed increase in hull drag shown in Fig. 9. The discrepancy here lies in the fact that the center of gravity of the hulls tested and reported in Figure 9 are near the center of the hull at  $\sim 0.5L_{\text{ref}}$ . Thus, while the bow wave is being created at the bow, gravity is acting near the center of the boat. If the effective length due to the location of the center of gravity is approximated to be  $L_{\text{eff}} = 0.5L_{\text{ref}}$ , the onset of drag due to the hull speed phenomenon should occur at  $F \simeq 1/\sqrt{4\pi} = 0.282$ . This shows that yachts with longer hulls will experience a significant increase in drag due to the hull speed phenomenon at greater velocities, and thus are typically faster than shorter yachts.

### C. Windward Sailing

While the minimization of hydrodynamic drag remains important for windward sailing, the minimization of aerodynamic drag is important, as well. In fact, maximizing the lift to drag ratios  $C_L/C_D$  for both the hydrodynamic and aerodynamic forces is essential for optimizing windward sailing, as was shown in section II.

In addition to friction and pressure drags on the keel and sail(s) of a yacht, drag can arise due to the lift force having a component in the direction of the water or apparent wind, respectively. This lift induced drag was shown to be proportional to the total circulation around the body (keel or sail) in section III D. Indeed, if the circulation is perfectly elliptical, the lift induced drag coefficient can be shown to be  $C_{D,i} = \frac{1}{\pi\gamma}C_L^2$ , where  $\gamma$  is the aspect ratio of the body, or the ratio of its downstream length (chord) to its width [11]. For bodies that do not exhibit perfectly elliptical circulation flows, the proportionality still holds,

$$C_{D,i} \propto \frac{1}{\pi\gamma}C_L^2, \quad (14)$$

where the proportion is well within an order of magnitude [9]. From the arguments presented in section III,



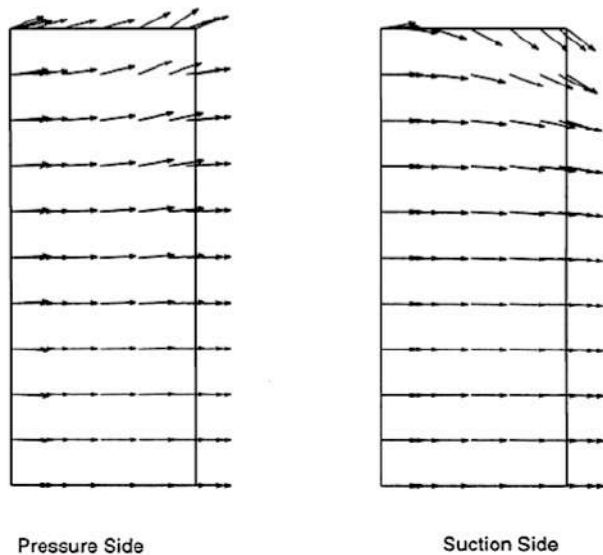


FIG. 10: The velocity field  $\vec{V}$  calculated using a RANS program for a wing with  $\mathcal{R} = 2.1 \times 10^5$  and an angle of attack  $\alpha = 9.45^\circ$ . The movement of the air from the high pressure side to the low pressure side is apparent, the phenomenon that is responsible for wing-tip vortex formation, a form of lift-induced drag. In this figure,  $\vec{V}_\infty = V_\infty \hat{x}$  and there is a boundary condition at  $y = 0$  such that  $\vec{V}_{y=0} = V_{y=0} \hat{x}$ . Image taken from Ref. [11].

it is clear that the lift and drag forces discussed therein will scale with unit distance perpendicular to the cross-section. However, Eq. (14) shows that, since the increase in unit distance of the body perpendicular to the flow increases the aspect ratio  $\gamma$ , it decreases the lift-induced drag force. Thus, taller, thinner sails and keels will tend to have larger lift-to-drag ratios and perform better in windward conditions. For example, Ref. [19] reports the computation, using RANS, of a lift-to-drag ratio of  $C_L/C_D \simeq 12$  for a keel at an attack/leeway angle of  $\alpha = 5^\circ$  on a IACC yacht in standard windward conditions.

In addition to influencing the lift-induced drag due to the circulation, 3D effects give rise to other drags related to the phenomenon of lift. Clearly, the circulation about an object such as a sail or keel will be influenced by its finite size. In particular, higher pressure on the downwind side of the sail or keel will force fluid (water or air) around the tip of the body to the other side, as is shown for a flat rectangular wing at an attack angle of  $\alpha = 9.45^\circ$  in Figure 10. These flow lines were calculated using a RANS computational program [11]. Such flow lines give rise to the formation of wing-tip vortices at the ends of keels and sails, as are seen in the wakes of yachts sailing in the 2001-2002 Volvo Ocean Race in Figure 11. These wing-tip vortices are present due to the pressure gradient across a body and are lift-induced and thus contribute to the lift-induced drag on the body from which they originate. They also, however, are related to lift-



FIG. 11: Tip vortices, a form of lift-induced drag, created by the sails of yachts sailing on a close haul, made visible by the fog during the 2001-2003 Volvo Ocean Race. Photo by Daniel Forster, taken from Ref. [20].

reduction because their formation reduces the velocity difference (and thus the pressure difference) on opposing sides of the keel or sail near its tip. Certainly, there is a boundary condition that the pressures, and thus the velocities of the flow on opposing sides of a sail, are equal at edges of the sail.

Keels, however, are able to counter this wing-tip vortex effect by having a bulb attached at their base. This bulb not only discourages the transfer of fluid from the higher pressure side of the keel to the lower pressure side, thus discouraging wing-tip vortex formation, but it also contributes to the righting torque  $M_R$  of the yacht because it can be heavy and significantly far from and below the center of buoyancy of the yacht. For example, the keel bulb on the BMW Oracle yacht to be raced in the 33<sup>rd</sup> America's Cup accounts for 19 of the yacht's 24 tons [21].

In windward sailing conditions, sails may counter the loss of lift and the increase of drag by working together. Typically, IACC yachts use a mainsail (downwind of the mast) and a jib (upwind of the mast) simultaneously. In windward conditions, and if trimmed properly, the presence of the mainsail serves to increase the flow velocity (and therefore decrease the pressure) on the leeward side of the jib sail and the jib serves to prevent flow separation from the leeward side of the mainsail. This proper trimming requires setting the contours of the sails so that they follow the natural flow lines of the air, discouraging separation. Indeed, this effect has been verified experimentally and computationally using the  $k\epsilon$  model [10].

## V. CONCLUSION

Ultimately, as has been mentioned, the determination of the lift and drag coefficients for actual yacht components must be left to experiment or numerical computation. These methods, if employed correctly, provide

these coefficients for keels, hulls and sails as functions of their orientation parameters (angles) in various flow velocities, i.e., they are functions of the leeway angle  $\lambda$ , the sail angle (relative to the boat)  $\delta_m$ , the point of sail  $\beta$  and the heeling angle  $\Theta$ . For Froude scaling objects, such as hulls, the coefficients can depend on velocity, as well. The knowledge of these coefficients and how they vary with these parameters is all that is needed to describe the motion of a sailing yacht, simply, by integrating the equation of motion  $\partial_t \vec{V}_{\text{yacht}} = \frac{1}{m} \vec{F}$ , where  $\vec{V}_{\text{yacht}}$  is the velocity of the sailing yacht being considered and  $\vec{F}$  depends on the lift and drag coefficients and the angles of attack. This motion, due to the quirks of the physics of fluids, can, theoretically, be at any direction *except* directly into the wind. Indeed, as is seen in Figure 7, IACC yachts can perform very well at angles as low as  $30^\circ$  from windward. Without a doubt, this is due to design that has been informed by the nature of fluids and the ways in which they interact with bodies, namely, the fluid mechanics and physics of sailing yachts.

### APPENDIX A: DERIVATION OF NAVIER-STOKES EQUATION FOR INCOMPRESSIBLE FLUIDS

Consider a velocity field for a fluid,  $\vec{V} = \vec{V}(x, y, z, t)$ , describing the velocity of the fluid as a function of space and time, and let  $\rho$  be the mass density of the fluid and  $p = p(x, y, z, t)$  the pressure in the fluid. Additionally, let the fluid be incompressible, such that the mass density  $\rho$  of the fluid is constant in space and time, so that it obeys the continuity equation

$$\vec{\nabla} \cdot \vec{V} = 0. \quad (\text{A1})$$

Newton's 2<sup>nd</sup> Law, describing the conservation of momentum for a particle of mass  $m$  in a fluid occupying a volume  $\Delta x \Delta y \Delta z$ , is given by

$$m \frac{d}{dt} \vec{V} = \vec{F} \Delta x \Delta y \Delta z \quad (\text{A2})$$

where  $\vec{F}$  is the force per unit volume acting on the fluid. The evaluation of the time derivative of the fluid velocity field requires careful consideration. Let the velocity field be that of a steady flow so that, at time  $t$ , the motion of a particle in the fluid is described by  $V(x, y, z, t) = u\hat{x} + v\hat{y} + w\hat{z}$ . After a small change in time  $\Delta t$ , the particle is described by  $V(x + \Delta x, y + \Delta y, z + \Delta z, t + \Delta t) = (u + \Delta u)\hat{x} + (v + \Delta v)\hat{y} + (w + \Delta w)\hat{z}$  where, by the chain rule,  $\Delta u = \frac{\partial u}{\partial x} \Delta x + \frac{\partial u}{\partial y} \Delta y + \frac{\partial u}{\partial z} \Delta z$ , where similar equations exist for  $\Delta v$  and  $\Delta w$ . Thus,  $\Delta u / \Delta t = u \frac{\partial u}{\partial x} +$

$v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$ . Writing similar equations for  $\Delta v / \Delta t$  and  $\Delta w / \Delta t$ , it is seen that the acceleration of a particle in the steady velocity field is given by  $\vec{a}_s = (\vec{V} \cdot \vec{\nabla}) \vec{V}$ . Of course, in this steady flow,  $\partial \vec{V} / \partial t = 0$ . However, if the flow at some fixed point is changing as a function of time, this partial derivative is non-zero. Thus, we see that, in general,

$$\frac{d}{dt} \vec{V} = \left( \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla} \right) \vec{V}. \quad (\text{A3})$$

Thus, we can write Eq. A2, with  $m = \rho \Delta x \Delta y \Delta z$ , as

$$\begin{aligned} \rho \Delta x \Delta y \Delta z \left( \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla} \right) \vec{V} &= \vec{F} \Delta x \Delta y \Delta z \\ \rho \left( \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla} \right) \vec{V} &= \vec{F}. \end{aligned} \quad (\text{A4})$$

The force per unit area  $\vec{F}$  can be rewritten in terms of the stresses on the fluid volume element being considered. These stresses break down into “shear” and “normal” stresses, being the forces per unit area that are parallel and normal to the surfaces of the volume element, respectively. The shear stresses are characterized by the stress tensor

$$\tau_{ij} = -p \delta_{ij} + \mu \left[ \frac{\partial V_j}{\partial x_i} + \frac{\partial V_i}{\partial x_j} \right] \quad (\text{A5})$$

where  $\delta_{ij}$  is the Kronecker delta function and  $\mu$  is the viscosity of the fluid. The force per unit volume is written in terms of the stress tensor as  $\vec{F} = \vec{\nabla} \cdot \vec{\tau}$ . Thus, we can rewrite Eq. A4 as

$$\rho \left( \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla} \right) \vec{V} = -\vec{\nabla} p - \mu \vec{\nabla} \times (\vec{\nabla} \times \vec{V}). \quad (\text{A6})$$

Eq. A6 is the vector form of the Navier-Stokes equation for viscous, incompressible fluids. By rewriting the  $\vec{V} \cdot \vec{\nabla}$  term (see section III), this equation can be written as

$$\frac{\partial}{\partial t} \vec{V} = -\frac{1}{\rho} \vec{\nabla} p - \frac{1}{2} \vec{\nabla} V^2 + \vec{V} \times (\vec{\nabla} \times \vec{V}) + \frac{\mu}{\rho} \nabla^2 \vec{V}. \quad (\text{A7})$$

If a characteristic length  $x$  is used to rescale the lengths  $x_i \rightarrow x_i/x$  and a characteristic velocity is used to rescale the velocities  $V_i \rightarrow V_i/v$ , Eq. (A7) can be rewritten as

$$\frac{\partial}{\partial t} \vec{V} = -\frac{1}{\rho} \vec{\nabla} p - \frac{1}{2} \vec{\nabla} V^2 + \vec{V} \times (\vec{\nabla} \times \vec{V}) + \frac{1}{\mathcal{R}} \nabla^2 \vec{V} \quad (\text{A8})$$

where  $\mathcal{R} = vx\rho/\mu$  is the dimensionless Reynold's number of the system.

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