Name: ______________________________ Date: ______________________

Annotated Formulas

Acceleration in g's

where a is acceleration.

$$g\text{'s} = \frac{a}{9.8 \text{ m/s}^2}$$

Circular Motion

Centripetal Acceleration

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

Circular Tangential Speed

$$v = \frac{2\pi r}{T}$$

Centripetal Force

$$F_c = ma_c = \frac{mv_c}{r} = \frac{m4\pi^2 r}{T^2}$$

where a_c is the centripetal acceleration, r is the radius of the path, T is the period, v is the tangential speed, F_c is the centripetal force, and m is mass.

Circumference of a Circle

$$C = 2\pi r$$

C is circumference, r is radius.

Coefficient of Static Friction

$$\mu_s = \frac{f_s}{N}$$

where μ_s is the coefficient of static friction, f_s is the maximum force of static friction, and N is the normal force pressing the two surfaces together.

Elevator/Spring Accelerometer

$$\boldsymbol{mg_s} + \boldsymbol{mg} = \boldsymbol{ma}, \quad \boldsymbol{F_s} = \boldsymbol{mg_s}$$

where m is the mass of the object, $\boldsymbol{a}$ is its acceleration, $\boldsymbol{g}$ is the acceleration due to gravity, $\boldsymbol{g_s}$ is the observed equivalent acceleration, and $\boldsymbol{f_s}$ is the calculated force.

Note: This formula is in vector form.

Figure 53. Spring Accelerometer

(continued)

Name: ______________________________ Date: ____________________

Force/Newton's Second Law

$$a = \frac{f}{m} \quad \text{or} \quad f = ma$$

where a is acceleration, f is force, and m is mass.

Gravitational Potential Energy

$$PE = mgh$$

where PE is gravitational potential energy, m is mass, g is the acceleration due to gravity, and h is the height above the base level.

Horizontal Acceleration—Accelerometer

$$a = g \tan \theta$$

where a is the horizontal acceleration, g is the acceleration due to gravity, and θ is the angle of deflection of a bob from the vertical.

Note: This is a scalar formula.

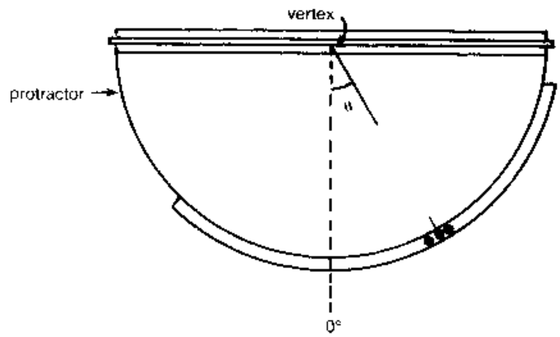


Figure 54. Horizontal Accelerometer

Ideal Angle for Curve Banking

$$\tan \theta = \frac{v^2}{gR}$$

where θ is the angle the banked curve makes with the horizontal, v is the velocity of the object going around the curve, g is the acceleration due to gravity, and R is the radius of the curve.

Impulse—Change in Momentum

$$f\Delta t = m\Delta v$$

where f is average force, Δt is the time of interaction, m is mass, and Δv is the change in speed.

(continued)

Name: ______________________________ Date: ______________

Kinematics

$$x = x_o + v_o t + (½)at^2$$

$$v = v_o + at$$

$$v_{ave} = \frac{v + v_o{}^2}{2}$$

$$v = \sqrt{v_o{}^2 + 2ax}$$

where x is elapsed distance, x_o is initial position, v is speed, v_o is initial speed, a is acceleration, t is elapsed time, and v_{ave} is average speed under uniform acceleration.

Time rate change of distance is called speed.

Time rate change of speed is called acceleration.

Time rate change of acceleration is called jerk.

Kinetic Energy

$$KE = (½)mv^2$$

where KE is kinetic energy, m is mass, and v is speed.

Momentum

$$p = mv$$

where p is momentum, m is mass, and v is speed.

Period/Frequency

$$f = \frac{1}{T}$$

where f is frequency and T is period.

Power

$$P = \frac{W}{t}$$

where P is average power, W is work, and t is elapsed time.

(continued)

Name: ______________________________ Date: ______________

Projectile Motion

Horizontal Motion:

$$X = v_x t$$

$$v_x = v \cos \theta$$

Vertical Motion:

$$Y = v_y t - (1/2) g t^2$$

$$V_y = v \sin \theta$$

$$t = \frac{2 v_y}{g}$$

where X is horizontal distance, v_x is initial horizontal velocity, t is elapsed time of flight, v is muzzle velocity, θ is angle of elevation with respect to the horizontal, Y is vertical distance, v_y is initial vertical velocity, and g is the scalar acceleration due to gravity.

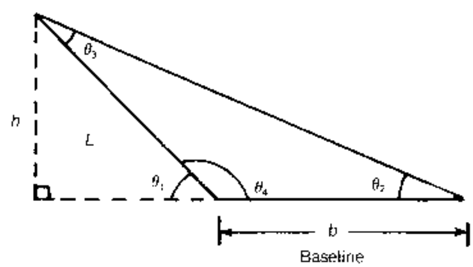
Relative Error

Relative error = (absolute error/accepted value) · 100%

Absolute error is the difference between the accepted and measured values.

Triangulation—Height

$$\text{Height} = \left[\frac{\sin \theta_1 \sin \theta_2}{\sin(\theta_1 - \theta_2)} \cdot b \right] + \text{observer's height}$$



Height Triangulation

where θ_1 and θ_2 are the base angle measurements and b is the baseline length.

Trigonometry Equations

$\sin \theta$ = sine = opposite/hypotenuse

$\cos \theta$ = cosine = adjacent/hypotenuse

$\tan \theta$ = tangent = opposite/adjacent

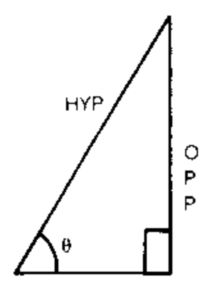


Figure 55.
Sides of a Right Triangle

(continued)

Name: ______________________________ Date: ______________________

Trigonometry Equations (*continued*)

Pythagorean Theorem (right triangles only)

$$c^2 = a^2 + b^2$$

where a and b are legs of the triangle and c is the hypotenuse.

Law of Cosines (all triangles)

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Law of Sines (all triangles)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where a, b, and c are legs of the triangle, and A, B, and C are the angles opposite each side a, b, and c respectively. Angle $C > 90°$.

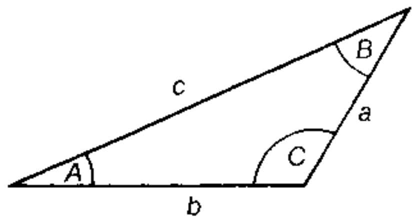


Figure 56.
Law of Sines

Weight

$$w = mg$$

where w is weight, m is mass, and g is acceleration due to gravity.

Work

$$W = fx = f_h x \cos \theta$$

$$\Delta \text{KE} = (1/2)mv^2 - (1/2)mv_o^2$$

where W is work, f is force, x is distance, m is mass, v is final speed, v_o is initial speed, f_h is horizontal force, and ΔKE is change in kinetic energy.

Name: ______________________________ Date: ______________________

Trigonometry Table

θ	SIN	COS	TAN
0	0.0000	1.0000	0.0000
1	0.0175	0.9999	0.0175
2	0.0349	0.9994	0.0349
3	0.0523	0.9986	0.0524
4	0.0698	0.9976	0.0699
5	0.0872	0.9962	0.0875
6	0.1045	0.9945	0.1051
7	0.1219	0.9925	0.1228
8	0.1392	0.9903	0.1405
9	0.1564	0.9877	0.1584
10	0.1736	0.9848	0.1763
11	0.1908	0.9816	0.1944
12	0.2079	0.9782	0.2126
13	0.2250	0.9744	0.2309
14	0.2419	0.9703	0.2493
15	0.2588	0.9659	0.2680
16	0.2756	0.9613	0.2868
17	0.2924	0.9563	0.3057
18	0.3090	0.9511	0.3249
19	0.3256	0.9455	0.3443
20	0.3420	0.9397	0.3640
21	0.3584	0.9336	0.3839
22	0.3746	0.9272	0.4040
23	0.3907	0.9205	0.4245
24	0.4067	0.9136	0.4452
25	0.4226	0.9063	0.4663
26	0.4384	0.8988	0.4877
27	0.4540	0.8910	0.5095
28	0.4695	0.8830	0.5317
29	0.4848	0.8746	0.5543
30	0.5000	0.8660	0.5774
31	0.5150	0.8572	0.6009
32	0.5299	0.8481	0.6249
33	0.5446	0.8387	0.6494
34	0.5592	0.8290	0.6745
35	0.5736	0.8192	0.7002
36	0.5878	0.8090	0.7265
37	0.6018	0.7986	0.7536
38	0.6157	0.7880	0.7813
39	0.6293	0.7772	0.8098
40	0.6428	0.7660	0.8391
41	0.6561	0.7547	0.8693
42	0.6691	0.7431	0.9004
43	0.6820	0.7314	0.9325
44	0.6947	0.7193	0.9657
45	0.7071	0.7071	1.0000
45	0.7071	0.7071	1.0000
46	0.7193	0.6947	1.0355
47	0.7314	0.6820	1.0724
48	0.7431	0.6691	1.1106
49	0.7547	0.6561	1.1504
50	0.7660	0.6428	1.1918
51	0.7772	0.6293	1.2349
52	0.7880	0.6157	1.2799
53	0.7986	0.6018	1.3270
54	0.8090	0.5878	1.3764
55	0.8192	0.5736	1.4282
56	0.8290	0.5592	1.4826
57	0.8387	0.5446	1.5399
58	0.8481	0.5299	1.6003
59	0.8572	0.5150	1.6643
60	0.8660	0.5000	1.7321
61	0.8746	0.4848	1.8041
62	0.8830	0.4695	1.8807
63	0.8910	0.4540	1.9626
64	0.8988	0.4384	2.0503
65	0.9063	0.4226	2.1445
66	0.9136	0.4067	2.2460
67	0.9205	0.3907	2.3559
68	0.9272	0.3746	2.4751
69	0.9336	0.3584	2.6051
70	0.9397	0.3420	2.7475
71	0.9455	0.3256	2.9042
72	0.9511	0.3090	3.0777
73	0.9563	0.2924	3.2709
74	0.9613	0.2756	3.4874
75	0.9659	0.2588	3.7321
76	0.9703	0.2419	4.0108
77	0.9744	0.2250	4.3315
78	0.9782	0.2079	4.7046
79	0.9816	0.1908	5.1446
80	0.9848	0.1736	5.6713
81	0.9877	0.1564	6.3138
82	0.9903	0.1392	7.1154
83	0.9925	0.1219	8.1444
84	0.9945	0.1045	9.5144
85	0.9962	0.0872	11.4301
86	0.9976	0.0698	14.3007
87	0.9986	0.0523	19.0811
88	0.9994	0.0349	28.6363
89	0.9999	0.0175	57.2900
90	1.0000	0.0000	∞

Clothoid Loops

When most people first view a looping roller coaster, they think that the loop is a circle. This is a common misconception. These vertical loops follow a special shape called a clothoid (also spelled klothoid). The clothoid loop is actually a section of a cornu spiral. Cornu spirals have applications in Fresnel diffraction problems and in the design of highway exit ramps. Figure 31 shows the clothoid loop, which is a cornu spiral section with its reflection, and resembles an upside-down teardrop.

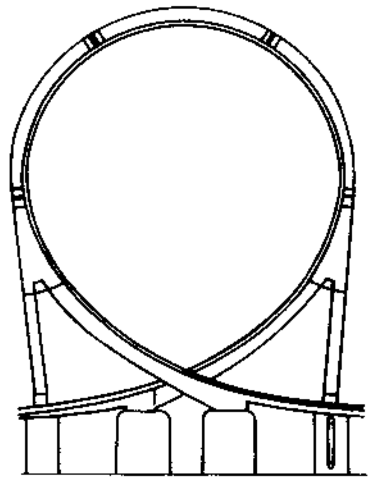


Figure 31.
Clothoid Loop

The design most commonly used in amusement parks involves a typical linear approach to the bottom of the loop, followed by a curve with a regularly decreasing radius. With respect to a vertical line bisecting the loop, the top ±65° approximates the arc of a circle. Specifically, the curvature of the clothoid is proportional to the arc length at any point on the curve. That is to say,

$$\frac{1}{r} = \frac{s}{a^2}$$

where: a is some constant that determines the tightness of the curve,

r is the radius at the point in question, and

s is the arc length (see Figure 32).

Safety and passenger comfort are the primary reasons for use of the clothoid loop. Normally in circular motion, if all other things are kept constant, as the radius reduces, the tangential velocity increases. We assume there is no friction here. A classic example of this is ice skaters or ballet dancers going into a tight pirouette, spinning faster as their arms and legs are brought in and more slowly as they extend their extremities. But the situation with the roller coaster is more complicated than that. As the roller coaster climbs, it loses kinetic energy to potential energy and therefore reduces its velocity. If the loop were a circle, there would be a *constant* centripetal acceleration as the coaster reduces its speed due to its change in height. If a clothoid track is chosen, the centripetal acceleration increases as you go up into the loop. So two things are at play here on the way up to the top of the loop: the increase

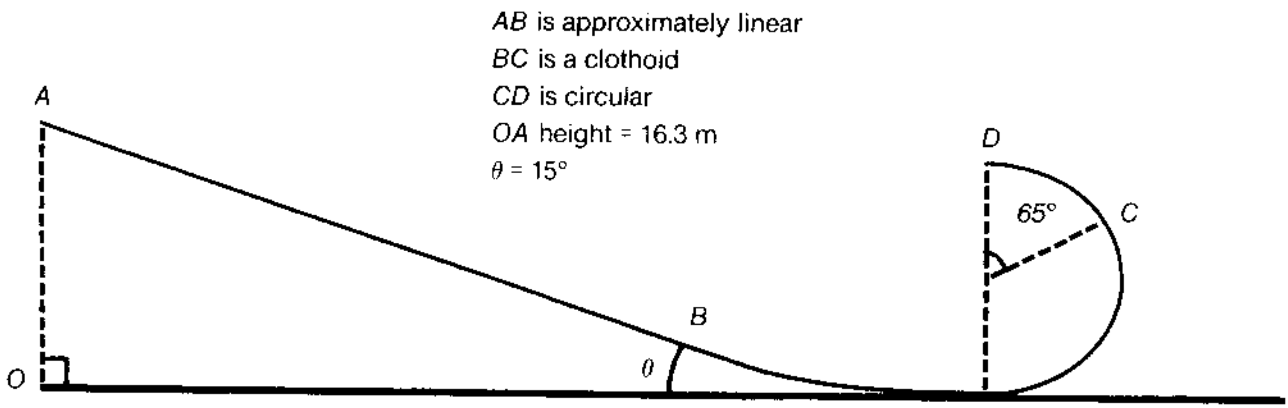


Figure 32.
Curvature of Clothoid

of speed due to the decreasing radius of the loop and the decrease of speed due to the decrease of kinetic energy.

With this background, we only begin to understand the problem. We still have not answered the question "Why not just use a circular track?" Most people begin to feel uncomfortable at accelerations in excess of 3.5 g's. Beyond 5 g's, many people begin to lose consciousness. From the amusement park's point of view (and that of many passengers), this is an undesired side effect! The derivation that follows shows that if a circular loop were used, the minimum acceleration at the bottom of a circular loop would be in the range of 6 g's—not good. Fortunately, physics comes to the rescue. If we start with a curve that is greater than what is needed, our change in velocity would be less. This leads to two problems—enormous circles (not enough room and too expensive to build) and running out of kinetic energy before reaching the top of the loop (thus, not going upside down). As the speed falls due to climbing, we can counteract that by tightening the curvature of the circle. Thus, decreasing speed due to the loss of kinetic energy (the climb) is countered with the increase in velocity due to the reduced radius. The clothoid shape satisfies these conditions so that there is enough energy to go over the top (upside down) with accelerations that rarely exceed 3.7 g's.

The descending trip is just the reverse of the ascending trip. As we increase our speed due to falling, the curvature of the loop is increased so that the resulting acceleration when hitting the bottom of the loop is within acceptable limits. In short, the change in speed due to the change in radius of the curve is offset by the change in speed due to height.

Investigating a frictionless *circular* loop of radius R with the condition that the train remain on the track at the top of the loop, we find that the weight mg must be equal to the centripetal force due to the circular motion:

$$mg = \frac{mv^2_{(top)}}{R}$$

Solving for the velocity at the top, we have:

$$v^2_{(top)} = Rg$$

The potential energy at the top of the circular loop is:

$$PE = mg2R,$$

since the diameter of the circle, or $2R$, is the height climbed. The total energy at the top is the gravitational potential energy due to height and the kinetic energy just moving the train over the top. Hence:

$$mg2R + (½)mgR = 2.5mgR$$

In a frictionless system, this must be equal to the potential energy of the original roller coaster hill, height h, before entering the circular loop; hence:

$$mgh = 2.5mgR$$

Therefore, the minimum height of the starting hill that drops the roller coaster into the circular loop must be $2.5R$ of the circular loop. Again, assuming a frictionless system, the potential energy at the top of the starting hill must be equal to the kinetic energy at the bottom of the loop:

$$mgh_{(top)} = 2.5Rmg = (½)mv^2_{(bottom)}$$

Solving for v^2, we have:

$$v^2_{(bottom)} = 5Rg$$

Since the track must not only hold up the train's weight (mg) but also keep the train moving in circular motion ($mv^2_{(bottom)}/R$), the forces at the bottom must be:

$$F = ma = mg + \frac{mv^2_{(bottom)}}{R}$$

or:

$$ma = mg + \frac{m(5Rg)}{R} = mg + 5mg = 6mg$$

Simplifying, $a = 6g$ at the bottom.

This situation does not exist, since we have discounted the effects of friction. Two ways to solve this difficulty: either start from a greater height or catapult the train. This would increase the accelerations at the bottom, entering the loop at an unacceptable range, somewhat higher than 6 g's. Again, this high speed, which results in a large acceleration in a circular system, is necessary to keep the train from falling off at the top.

Thus the elegance of the clothoid loop. Due to its changing radius of curvature, smaller accelerations are experienced at the bottom than if a circular track were used.

Dual-Axis Turning Rides

Dual-axis turning rides are particular cases of epicycle motion. The "Triumph of Mechanics" section in *Project Physics* has an excellent explanation of this coupled with film loops on retrograde and epicycle motion.

In analyzing dual-axis turning rides a major source of confusion is choosing a poor frame of reference. Another is unwittingly changing the frame of reference midway through the solution of a problem.

In the frame of the earth:

R = length of center arm

r = length of cluster arm

α = angle rider makes at the center

β = angle rider makes with cluster center

and: $\omega_{hub} = d\alpha/dt$

$\omega_{cluster} = d\beta/dt$

(See Figure 33.)

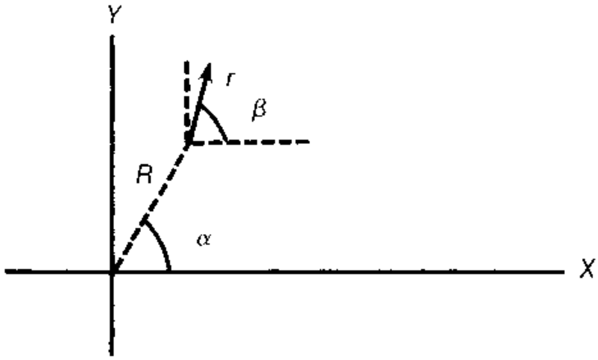


Figure 33.

The rider path is generated by the following equations:

$$X = R\cos\alpha + r\cos\beta$$

$$Y = R\sin\alpha + r\sin\beta$$

The equations for velocity are:

$$V_X = dX/dt \qquad V_Y = dY/dt$$

The equations for acceleration are:

$$A_X = dV_X/dt \qquad A_Y = dV_Y/dt$$

The actual path of a rider may look as shown in Figure 34 (next page).

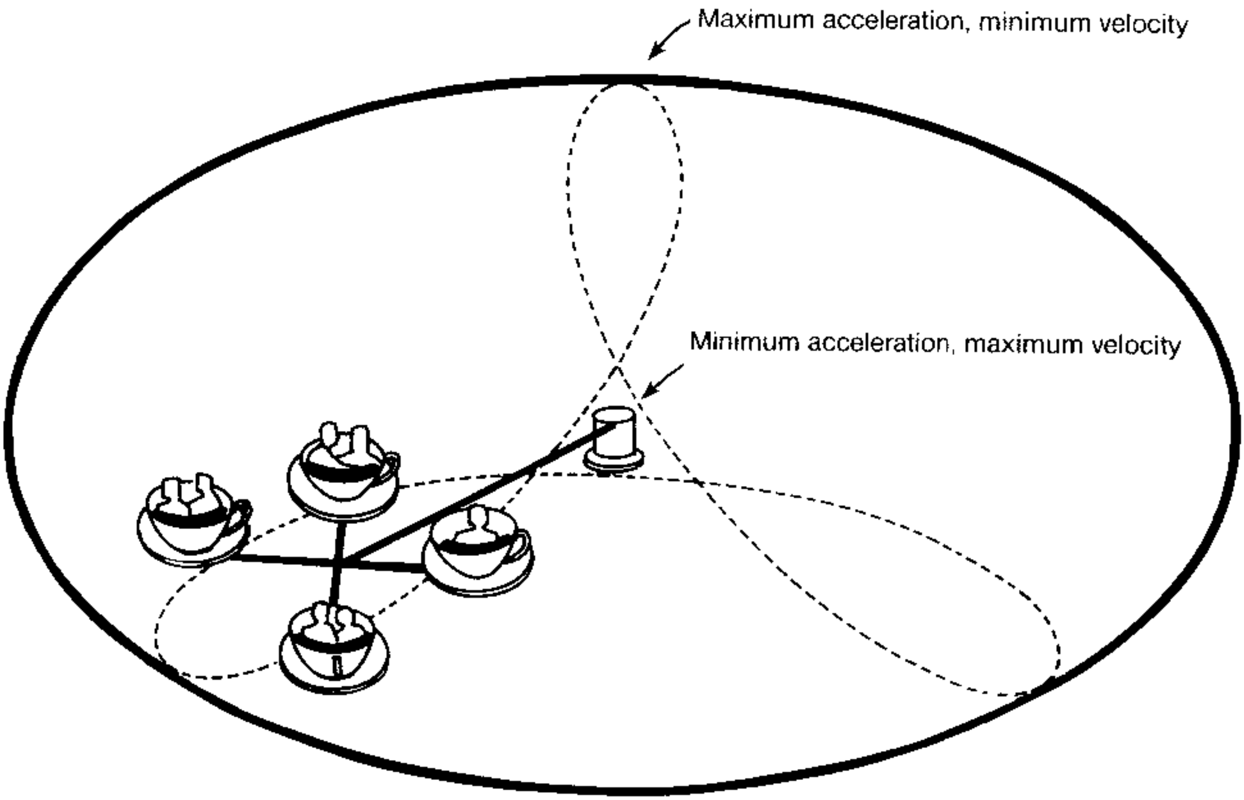


Figure 34.
Path of Rider

Name: ______________________________ Date: ______________________

Triangulation Practice Problems

Note: Solve these problems either graphically or trigonometrically, or both, as directed by your teacher.

1. The grand country of Gillikin built itself a fabulous new amusement park. During the opening ceremony, Glinda the Good Witch announced that the new Ferris wheel was the tallest structure in Gillikin. To break the boredom of the witch's long-winded speech, Dorothy, who was 1.3 m in length from her eye to the ground, and the Scarecrow decided to measure the height of this new device. Earlier in the day, they had measured the height of the tallest castle in Gillikin and found it (using a barometer) to be 10.3 m tall. After setting a 12.0 m baseline, they measured the angle of elevation for the Ferris wheel from the far end of the line to be 14° and from the near end, 19°. They gave these data to Professor H. M. Wogglebug, T.E., for analysis. Was Glinda the Good's claim correct? Please support your answer.

2. The Cowardly Lion was talked into riding the Blue Munchkin Streak Roller Coaster with Jack the Pumpkin Head. The Cowardly Lion could jump down safely from trees that were 7.0 m tall. Being afraid of anything taller than a tree, the Lion asked the Tin Woodsman to help him measure the height of the Blue Munchkin Streak. Jack measured a 9.0 m baseline while the 1.6-m-tall Tin Woodsman measured the angles of elevation at the ends of the baseline. He found them to be 8° and 10°. Would the Cowardly Lion have enough courage for this new thrill machine? Please support your answer.

3. While traveling westward on the Road of Yellow Brick, Dorothy and her friends came to the Munchkin River. The Tin Woodsman found the tallest tree around, which was 18 m high. He thought they might be able to use it as a bridge. While the Woodsman was out in the forest, the rest of the gang surveyed the width of the river. After setting up an 11 m baseline, they measured the angles from the ends of the baseline to a point on the opposite shore. The angle from the north end of the baseline was 75°, while the angle from the south end was 88°. Was the tree tall enough, or did they need to build a raft? Please support your answer.

Name: ______________________________ Date: ______________

Horizontal Accelerometer and Spring Accelerometer Practice Problems

1. Holy Smokes! While on the Rotor, the BB's on my accelerometer deflected 76°. What is the measured acceleration? How many g's is this?

2. While riding the outside horse on a merry-go-round, Batman's horizontal accelerometer, when pointing toward the hub of the merry-go-round, indicated a reading of 11°. On the inside track, Robin's accelerometer read 9°. Calculate the difference in accelerations. Why is there a difference?

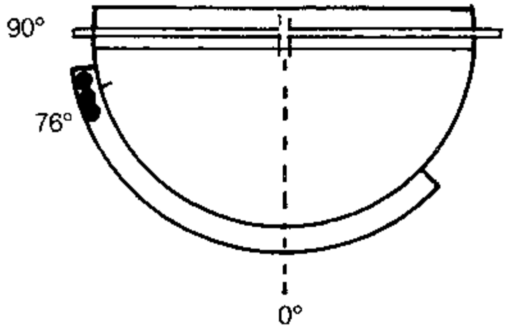


Figure 26. Problem 1

3. On the same merry-go-round as in Question 2, a 150 g mass is hanging from a spring scale. The Masked Wonder's scale reads a maximum value of 1.7 N when the horse is going up and a minimum value of 1.3 N when the horse is going down. Using these values, find the acceleration of the horse going up and going down. In which of these is the magnitude of acceleration the greater?

4. The Joker was bouncing up and down on his pogo stick. At the top of his bounce, his effective gravitational field was zero, while at the bottom of his bounce he measured 2.5 g. If his mass is 65 kg, what is his perceived weight at the top of his bounce, and at the bottom?