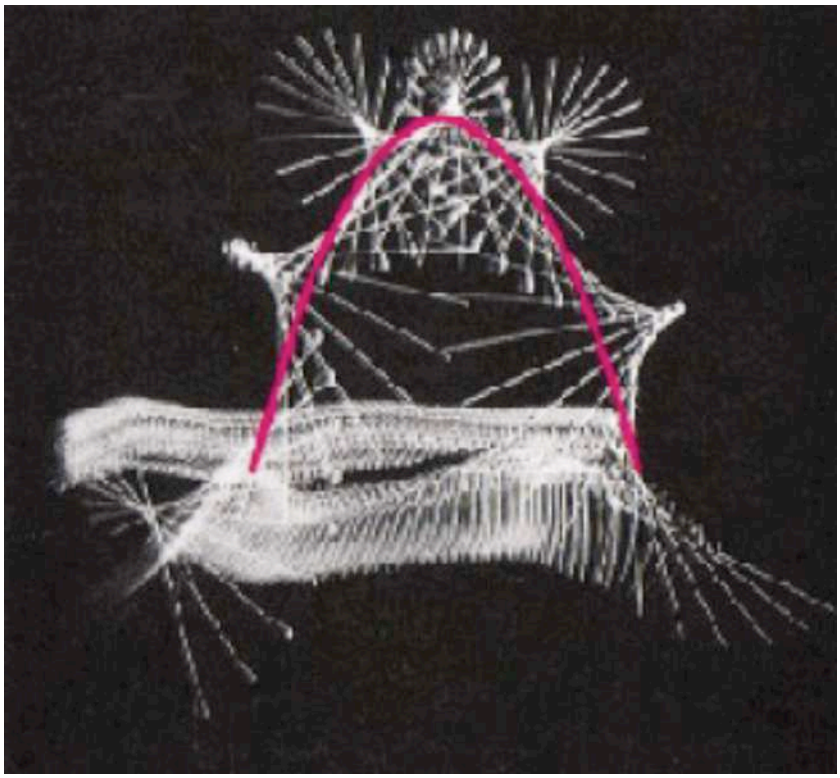
AP PHYSICS C

Motion of the Center of Mass

Background

Consider the multiflash photograph of a baton thrown in the air. Although the motion of the baton is complicated, the motion of the center of mass is simple.



While the baton is in the air, the center of mass follows a parabolic path, the same path that would be followed by a point particle. The acceleration of the center of mass of a system equals the net external forces acting on the system divided by the total mass of the system. For the baton thrown in the air, the acceleration of the center of mass is **g**, downward.

To find the acceleration of the center of mass, we can find the velocity by differentiating with respect to time.

$$M\frac{dr_{cm}}{dt} = m_1\frac{dr_1}{dt} + m_2\frac{dr_2}{dt} + \ldots = \sum_i m_i\frac{dr_i}{dt}$$

or

$$Mv_{cm} = m_1v_1 + m_2v_2 + \ldots = \sum_i m_iv_i$$

We differentiate again to obtain the acceleration of the center of mass:

$$Ma_{cm} = m_1a_1 + m_2a_2 + \ldots = \sum_i m_ia_i$$

According to Newton's Second Law, we can replace m_ia_i with $\mathbf{F}_i$, the net force acting on the ith particle. Forces acting on the particle fall into two categories: internal forces (due to the interactions with other particles) and external forces (due to agents outside the system):

$$F_i = m_ia_i = F_{i,\text{int}} + F_{i,ext}$$

Substituting this:

$$Ma_{cm} = \sum_i F_{i,\text{int}} + \sum_i F_{i,ext}$$

According to Newton's Third Law, for each internal force acting on one particle, there is an equal and opposite force acting on the other particle. The internal forces thus occur in apirs of equal and opposite forces. When we sum over all the particles in the system, the internal forces cancel, $\sum \mathbf{F}_{i,\text{int}} = 0$, leaving only external forces. Thus,

$$F_{net,ext} = \sum_i F_{i,ext} = Ma_{cm}$$

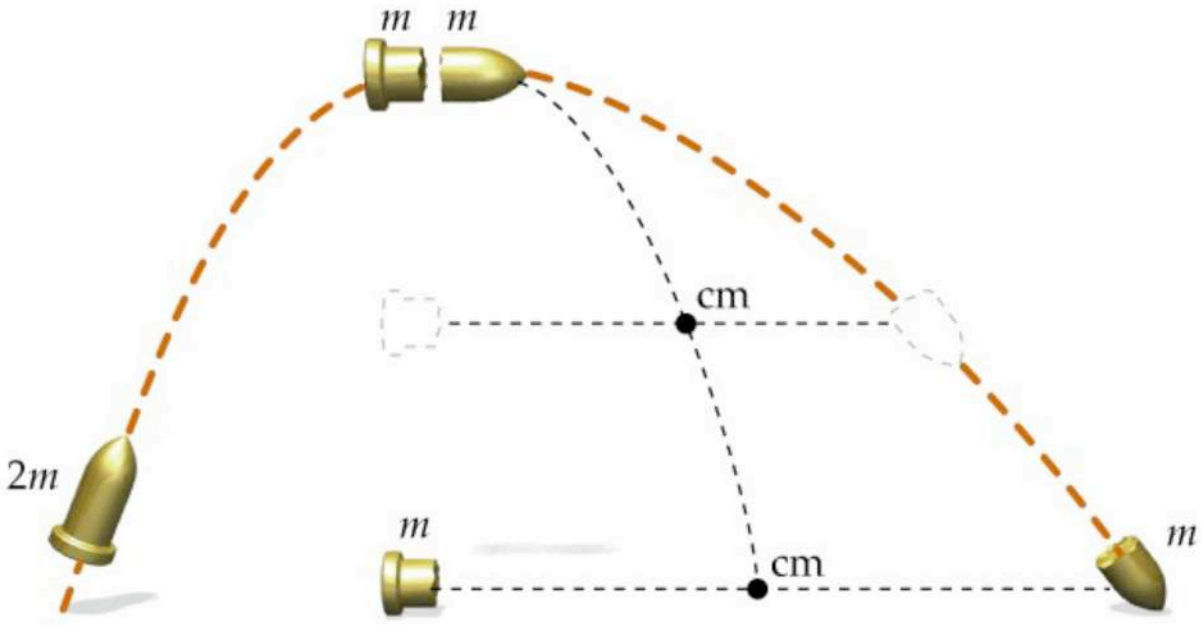
Or, the net external force acting on the system equals the total mass M of the system times the acceleration of the center of mass $\mathbf{a}_{cm}$.

Thus:

The center of mass of a system moves like a particle of mass $M = \sum m_i$, under the influence of the net external force acting on the system

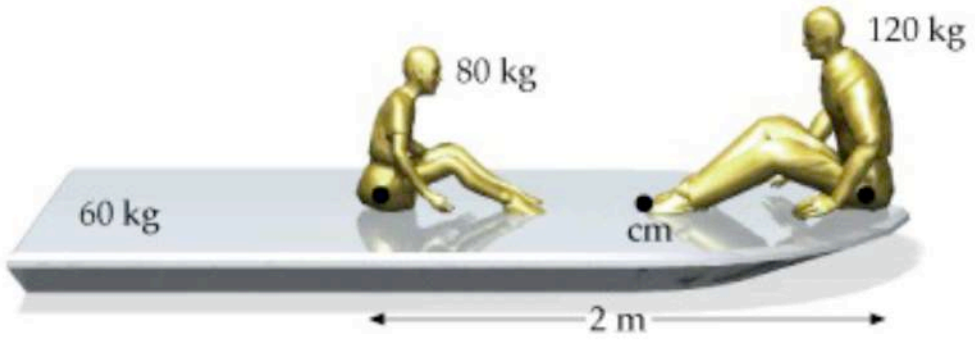
Example 1:

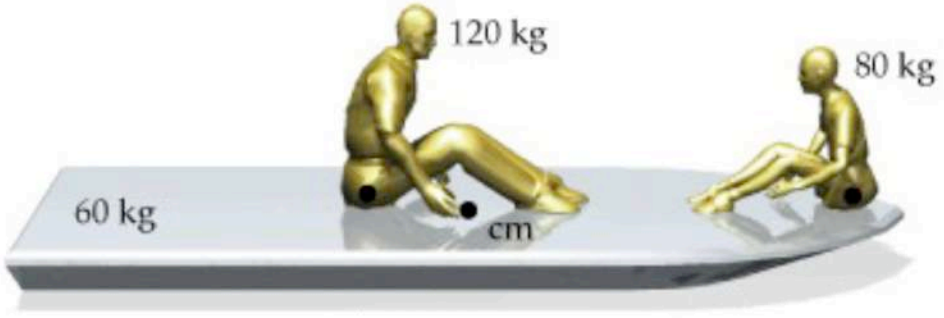
A projectile is firec into the air over level ground with an initial velocity of 24.5 m/sec at 36.9 degrees to the horizontal. At its highest point, it explodes into two fragments of equal mass. One fragment falls straight down to the ground. Where does the other fragment land?



Example 2:

You (mass 80 kg) and Bubba (mass 120 kg) are in a rowboat (mass 60 kg) on a calm lake. You are at the center of the boat, rowing, and he is at the back, 2 m from the center. You get tired and stop rowing. Bubba offers to row, and after the boat comes to rest, you change places. How far does the boat move? (Neglect any horizontal force exerted by the water).





Example 3:

A wedge of mass m_2 sits at rest on a scale as shown below. A small block of mass m_1 slides down the frictionless incline of the wedge. Find the scale reading while the block slides.