

Calculus of Parametric Equations Answer key

(1) $x = t^4 + 1$ $y = t^3 + t$ $t = -1$
 $(2, -2)$

$$\frac{dy}{dx} = \frac{3t^2 + 1}{4t^3} = \frac{4}{-4} = -1$$

$$y + 2 = -1(x + 2)$$

(2) $x = \cos \theta$ $y = \sin \theta + \cos 2\theta$ $\theta = 0^\circ$

$$\frac{dy}{dx} = \frac{\cos \theta - 2 \sin \theta}{-\sin \theta} = \frac{1}{0}$$

$$* = (1, 1)$$

$$x = 1$$

~~$$y = 1$$~~

~~$$y = 1$$~~

(3) $x = e^t$ $y = (t-1)^2$ $(1, 1) \rightarrow t = 0$

$$\frac{dy}{dx} = \frac{2(t-1)}{e^t} = -2$$

$$y - 1 = -2(x - 1)$$

(4) $\frac{dy}{dx} = \frac{2t + 3t^2}{2t} = 1 + \frac{3}{2}t$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{3}{2}}{2t} = \frac{3}{4t}$$

$$\frac{d^2y}{dx^2} = \frac{3}{4t} > 0 \text{ concave up when } t > 0$$

$$(5) \quad x = t - e^t \quad y = t + e^{-t}$$

$$x = 1 - e^t \quad y = 1 - e^{-t}$$

$$\frac{dy}{dx} = \frac{1 - e^{-t}}{1 - e^t} = \frac{1 - \frac{1}{e^t}}{1 - e^t} = \frac{e^t - 1}{e^t(1 - e^t)} = \frac{e^t - 1}{e^t(1 - e^t)} = \frac{-1}{e^t} = -e^{-t}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{e^{-t}}{1 - e^t}}$$

Concave up when $\frac{e^{-t}}{1 - e^t} > 0$

$$\frac{e^{-t}}{1 - e^t} = 0 \rightarrow e^{-t} = 0 \rightarrow \frac{1}{e^t} = 0 \rightarrow \frac{1}{e^t} > 0$$

$$1 - e^t > 0 \\ e^t < 1$$

~~con~~ concave up
when $t < 0$

$$(6) \quad x = 2 \sin t \quad y = 3 \cos t$$

$$x' = 2 \cos t \quad y' = -3 \sin t$$

$$\frac{dy}{dx} = \frac{-3 \sin t}{2 \cos t} = -\frac{3}{2} \tan t$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{3}{2} \sec^2 t}{2 \cos t} = -\frac{3}{4} \sec^3 t = \frac{-3}{4 \cos^3 t}$$

Concave up when $-\frac{3}{4} \sec^3 t > 0$

$$\sec^3 t < 0$$

$\sec t < 0$ when $\cos t < 0$



$$\pi/2 \leq t \leq 3\pi/2$$

$$(7) \quad x = 10 - t^2$$

$$\frac{dx}{dt} = -2t$$

~~#~~
Vertical

$$-2t = 0$$

$$t = 0$$

$$(10, 0)$$

$$y = t^3 - 12t$$

$$\frac{dy}{dt} = 3t^2 - 12$$

Horizontal

$$3t^2 - 12 = 0$$

$$t = \pm 2$$

$$(6, 16) \quad (6, -16)$$

$$(8) \quad x = 2 \cos \theta$$

Vertical

$$-2 \sin \theta = 0$$

$$\theta = 0, 2\pi$$

$$\theta = \pi k$$

$$(2, 0) \quad (-2, 0)$$

$$y = \sin 2\theta$$

Horizontal

$$2 \cos 2\theta = 0$$

$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2} \rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

~~$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$~~

$$(\sqrt{2}, 1) \quad (-\sqrt{2}, -1)$$

$$(9) \quad x = t - t^2 \quad y = \frac{4}{3} t^{3/2}$$

$$L = \int_1^2 \sqrt{(1-2t)^2 + (2t^{1/2})^2}$$

$$L = \int_1^2 \sqrt{1-4t+4t^2+4t}$$

$$L = \int_1^2 \sqrt{1+4t^2} \quad \text{fn Int}$$

$$L = 3.168$$

$$(10) \quad x = t + \cos t \quad y = t - \sin t$$

$$L = \int_0^{2\pi} \sqrt{(1-\sin t)^2 + (1-\cos t)^2}$$

$$= \int_0^{2\pi} \sqrt{1-2\sin t + \sin^2 t + 1-2\cos t + \cos^2 t}$$

$$= \int_0^{2\pi} \sqrt{3-2\sin t-2\cos t} \quad \text{fn Int}$$

$$L = 10.037$$

$$(11) \quad x = 1 + 3t^2 \quad y = 4 + 2t^3$$

$$L = \int_0^{10} \sqrt{(6t)^2 + (6t^2)^2}$$

$$= \int_0^{10} \sqrt{36t^2 + 36t^4}$$

$$= \int_0^{10} \sqrt{36t^2(1+t^2)} = 6 \int_0^{10} t \sqrt{1+t^2}$$

$$6 \left[\frac{1}{3} (1+t^2)^{3/2} \right]_0^{10}$$

$$\frac{3}{2} (1+t^2) \cdot 2t$$

$$6 \left[\frac{1}{3} (2)^{3/2} + \frac{1}{3} \right]$$

$$2(2^{3/2}) + 2$$

(12) $x = \frac{t}{1+t}$ $y = \ln(1+t)$ $0 \leq t \leq 1$

$$\frac{dx}{dt} = \frac{(1+t) - t}{(1+t)^2} \qquad \frac{dy}{dt} = \frac{1}{1+t}$$

$$= \frac{1}{(1+t)^2}$$

fn Int

$$L = 4.083$$

$$L = \int_0^2 \sqrt{\left(\frac{1}{(1+t)^4}\right)^2 + \left(\frac{1}{(1+t)^2}\right)^2}$$

$$= \int_0^2 \frac{1 + 1 + 2t + t^2}{(1+t)^4} = \int \frac{t^2 + 2t + 2}{(1+t)^4} = \sqrt{\frac{(t+1)^2}{(t+1)^4}} = \sqrt{\frac{1}{(t+1)^2}} = \frac{1}{t+1}$$

~~$$= \ln(t+1) \Big|_0^2 = \ln 3$$~~

~~ANSWER~~

~~$$= \int \frac{t(t+1) + 2}{(1+t)^4} =$$~~

~~$$\text{Let } u = t+1 \\ du = dt$$~~

$$(13) \quad r(t) = \langle t^2 - 1, t \rangle \text{ at } t=1$$

$$v(t) = \langle 2t, 1 \rangle$$

$$v(1) = \langle 2, 1 \rangle$$

$$a(t) = \langle 2, 1 \rangle \rightarrow a(1) = \langle 2, 1 \rangle$$

$$\text{speed} = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$(14) \quad r(t) = \langle 2 - t, 4t^{1/2} \rangle \text{ at } t=1$$

$$v(t) = \langle -1, 2t^{-1/2} \rangle$$

$$v(1) = \langle -1, 2 \rangle$$

$$a(t) = \langle 0, -t^{-3/2} \rangle$$

$$a(1) = \langle 0, -1 \rangle$$

$$\text{speed} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

$$(15) \quad \lim_{t \rightarrow 0^+} \langle \cos t, \sin t \rangle = \langle 1, 0 \rangle$$

$$(16) \quad \lim_{t \rightarrow 0^+} \left\langle \frac{e^t - 1}{t}, \frac{3}{1+t} \right\rangle = \langle 1, 3 \rangle$$

L'Hopital's
 $\frac{e^t}{1}$

$$(17) \quad r(t) = \langle 2\sin t, 2\cos t \rangle$$

$$\text{Total Distance} = \int_{-10}^{10} \sqrt{(2\cos t)^2 + (-2\sin t)^2} = 40$$

$$t=0 \quad (4, -1)$$

$$(18) \quad v(t) = \langle t^2, t \rangle$$

$$x(4) = 4 + \int_0^4 t^2 \quad y(4) = -1 + \int_0^4 t$$

$$= 4 + \left[\frac{1}{3} t^3 \right]_0^4 \quad = -1 + \left[\frac{1}{2} t^2 \right]_0^4$$

$$= -1 + 8$$

$$x(4) = 4 + \frac{1}{3}(4)^3$$

$$= 7$$

$$\left\langle 4 + \frac{64}{3}, 7 \right\rangle = \left\langle \frac{76}{3}, 7 \right\rangle$$

$$(19) \quad v(t) = \langle \sin t, \cos t \rangle \quad t=0 \quad (-3, 2)$$

$$x(4) = -3 + \int_0^4 \sin t \quad y(4) = 2 + \int_0^4 \cos t$$

$$= -3 + \cos t \Big|_0^4 \quad = 2 + \sin t \Big|_0^4$$

$$= -3 + (\cos 4 - \cos 0) \quad = 2 + (\sin 4 - \sin 0)$$

$$= -3 + \cos 4 - 1 \quad = 2 + \sin 4$$

$$= -4 + \cos 4$$

$$\langle -4 + \cos 4, 2 + \sin 4 \rangle$$