

Fundamental Theorem of Algebra

a polynomial of degree n has n roots.

→ BUT you have to count multiplicity of roots
(for example, double roots count as 2)

→ AND ALSO you have to count complex (non-real) roots

degree the highest exponent on x

(if you write the polynomial in factored form)

root / zero / x -intercept ← all the same thing!

a value of x that makes the polynomial 0.

factors

$(x - \#)(x - \#) \dots$ where the $\#$ s are the roots

complex roots = roots that are non-real complex numbers

note: non-real complex roots come in complex conjugate pairs.

so if $a + bi$ is a root, then $a - bi$ is also a root.

one error in
each half

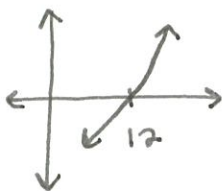
Multiplicity of roots

Single root

example:

$$\text{root} = 12$$

$$\text{factor} = (x - 12)$$

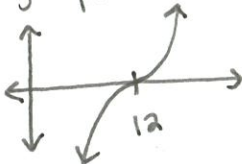


double root

$$\text{root} = 12$$

$$\text{factors: } (x - 12)^2$$
$$\text{or } (x - 12)(x - 12)$$

graph:



quadruple root

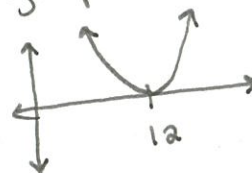
looks like double root
but flatter / boxier

triple root

$$\text{root} = 12$$

$$\text{factors: } (x - 12)^3$$
$$\text{or } (x - 12)(x - 12)(x - 12)$$

graph:

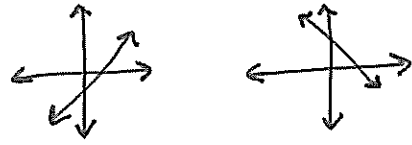


quintuple root

looks like triple root
but flatter / boxier

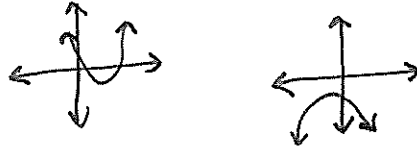
Graphs of polynomials of different degrees

degree 1 (linear): graph is a line

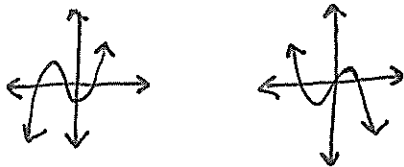


one error
on this page

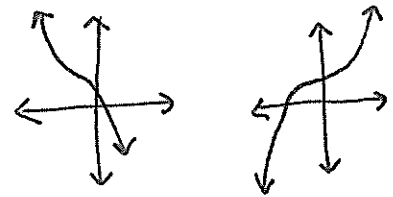
degree 2 (quadratic): graph is a parabola



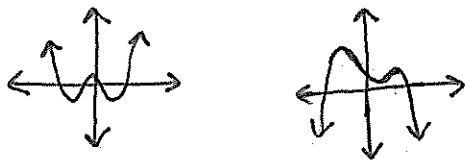
degree 3 (cubic):



could also be
less bumpy like:

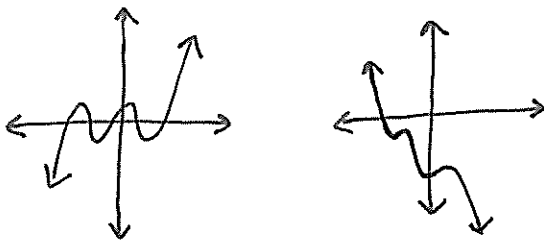


degree 4 (quartic):



could also be less bumpy
and look like a parabola,
but flatter

degree 5 (quintic):



could also be less bumpy
and look like a cubic

the higher the degree, the less bumpy the graph.