

★ 2 errors to find ★

proving trig identities

- Is it an identity? If so, then it's true for all possible angles. Try plugging in an angle to see if it's true.

Protip: Avoid special angles like  $90^\circ$ ,  $45^\circ$ , etc.

Protip: If you're not sure, try another angle!

- If it's not an identity, disprove it by showing that there is an angle it doesn't work for. Plug in an angle that doesn't work and show that the equation is false.

Example: Prove that  $\sin(2\theta) = 2\sin(\theta)$  is not an identity.

$$\text{if } \theta = 78^\circ, \text{ then } \sin(2\theta) = \sin(2 \cdot 78^\circ) \approx 0.4067$$

$$\text{but } 2\sin(\theta) = 2 \cdot \sin(78^\circ) \approx 1.956$$

the equation does not hold for  $\theta = 78^\circ$ .

- If it is an identity, prove it like this:

① Start with one side of the equation.

Protip: it's usually easier to start with the less complicated side.

This is not the only way to prove things!

② Do legal algebra moves to change your expression into equivalent expressions, ending up at the other side of the equation. Make sure you can explain why each step works! Some options:

• start by converting everything to  $\sin$  &  $\cos$  ←

• algebra / simplifying moves

• use trig identities that were proven previously

• you can replace a 1 with  $\sin^2\theta + \cos^2\theta$

• multiply by a clever form of 1

highly recommended!!

## Example of trig proof

Proof of:  $\sec \theta (\sec \theta - \cos \theta) = \tan^2 \theta$

Proof:  $\sec \theta (\sec \theta - \cos \theta)$  ← start with complex side

$= \frac{1}{\cos \theta} \left( \frac{1}{\cos \theta} - \cos \theta \right)$  ← replace with sin & cos

$= \frac{1}{\cos \theta} \left( \frac{1}{\cos \theta} - \frac{\cos \theta}{1} \cdot \boxed{\phantom{1}} \right)$

multiply by clever form of 1 to get common denominator

$= \frac{1}{\cos \theta} \left( \frac{1}{\cos \theta} - \frac{\cos^2 \theta}{\cos \theta} \right)$

$= \frac{1}{\cos \theta} \left( \frac{1 - \cos^2 \theta}{\cos \theta} \right)$  ← subtract fractions

$= \frac{1 - \cos^2 \theta}{\cos^2 \theta}$  ← multiply fractions

$= \frac{\sin^2 \theta + \cos^2 \theta - \cos^2 \theta}{\cos^2 \theta}$  ← replace 1 with  $\sin^2 + \cos^2$

$= \frac{\boxed{\phantom{\sin^2 \theta}}}{\cos^2 \theta}$

combine like terms

$= \left( \frac{\sin \theta}{\cos \theta} \right)^2$  ← property of exponents

$= \tan^2 \theta$  ← because  $\tan = \frac{\sin}{\cos}$

□

★ Fill in the blanks! ★

Leave space for examples of solving trig equations!

(or get an additional sheet of pastel paper)