Government of Saskatchewan Outcomes and Indicators for High School Math

Grade 9

Outcome: N9.1

Demonstrate (concretely, pictorially, and symbolically) understanding of powers with integral bases (excluding base 0) and whole number exponents including:

- **representing using powers**
- **evaluating powers**
- **powers with an exponent of zero**
- **solving situational questions.**

[C, CN, PS, R, T]

a. Demonstrate the difference between the exponent and base of a power by representing two powers with exponent and base interchanged (e.g., 2^3 and 3^2 or 10^3 and 3^{10}) using repeated multiplication or concrete models and describe the result.
b. Predict which of two powers represents the greater quantity, explain the reasoning, and verify using technology.
c. Analyze the role of brackets in powers by using repeated multiplication [e.g., $(-2)^4$, (-2^4), and -2^4] and generalize strategies for evaluating powers involving brackets.
d. Justify why a^0, $a \neq 0$, must equal to 1.
e. Predict whether the value of a given power will be positive or negative (e.g., what will the sign of -7^{15} be?).
f. Evaluate powers with integral bases (excluding base 0) and whole number exponents, with or without the use of technology.
g. Generalize, using repeated multiplication to represent powers, the exponent laws of powers with integral bases (excluding base 0) and whole number exponents:
$$(a^m)(a^n) = a^{m+n} \quad \frac{a^m}{a^n} = a^{m-n}, m > n \quad (a^m)^n = a^{mn} \quad (ab)^m = a^m b^m$$
h. Apply the exponent laws to expressions involving powers, and determine the quantity represented by the expression, with or without the use of technology.
i. Prove by contradiction that $a^m + a^n \neq a^{mn}$, $a^m - a^n \neq a^{m-n}$, and $a^m - a^n \neq a^{\frac{m}{n}}$.
j. Describe and apply strategies for evaluating sums or differences of powers.
k. Analyze a simplification of an expression involving powers for errors.

Outcome: N9.2

Demonstrate understanding of rational numbers including:

- **comparing and ordering**
- **relating to other types of numbers**
- **solving situational questions.**

[C, CN, PS, R, T, V]

a. Order a given set of rational numbers, in fraction and decimal form, by placing them on a number line and explaining the reasoning used (e.g., 3/5, - 0.666, 4,… , 0.5, -5/8).
b. Determine a rational number between two given rational numbers and describe the strategy used.
c. Create a representation depicting how whole numbers, fractions, decimals, integers, square roots, and rational numbers are related to each other.
d. Provide examples to explain how knowing about how to add, subtract, multiply, and divide integers and positive rational numbers informs knowing how to add, subtract, multiply, and divide rational numbers.
e. Provide examples to demonstrate how the order of operations can be extended to rational numbers.
f. Solve situational questions involving operations on rational numbers, with or without the use of technology.
g. Analyze a simplification of an expression involving rational numbers for errors.

Outcome: N9.3

Extend understanding of square roots to include the square root of positive rational numbers. [CN, ME, R, T, V]

a. Develop a generalization about what type of number results from the squaring of a rational number.
b. Describe strategies for determining if a rational number is a perfect square.
c. Determine the square root of a rational number that is a perfect square.
d. Determine the rational number for which a given rational number is its square root (e.g., 4/3 is the square root of what rational number?).
e. Explain and apply strategies involving benchmarks for determining an estimate of the square root of a rational number that is not a perfect square.
f. Determine, with the use of technology, an approximate value for the square root of a rational number that is not a perfect square.
g. Explain why the value shown by technology may only be an approximation of the square root of a rational number.
h. Describe a strategy that, if applied to writing a decimal number, would result in an irrational number (e.g., students describe a strategy in which they repeatedly write the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 but separate each group of these

digits by an increasing number of repeats of the digit 7 or 0.012345678970123456789770123456789770123…).

i. Determine a rational number whose square root would be between two given rational numbers and explain the reasoning used (e.g., a rational number whose square root is between 1/2 and 1/3 would be between 1/4 and 1/9 because those are 1/2 and 1/3 squared. I need to find a number between 1/4 and 1/9. I can do this by making the two fractions into fractions of the same type: 9/36 and 4/36. One number between these is 6/36 or 8/36).

Outcome: P9.1

Demonstrate understanding of linear relations including:

- **graphing**
- **analyzing**
- **interpolating and extrapolating**
- **solving situational questions.**

[C, CN, PS, R, T, V]

a. Observe and describe a situation relevant to self, family, or community that a given graph might represent and explain the meaning conveyed by the graph.
b. Sort a set of graphs into representations of linear and non-linear relations.
c. Sketch graphs for given linear relations, including horizontal and vertical lines, with and without the use of technology.
d. Generalize strategies for determining if a given linear relation will have a graph that is horizontal, vertical, increasing, or decreasing.
e. Extrapolate to determine a value for either variable in a linear relation beyond the shown graph.
f. Verify an extrapolated value from a graph by using substitution in the related linear relation.
g. Interpolate to determine a value for either variable in a linear relation within the shown graph.
h. Verify an interpolated value from a graph by using substitution in the related linear relation.
i. Solve situational questions by graphing linear relations and interpreting the resulting graphs.

Outcome: P9.2

Model and solve situational questions using linear equations of the form:

- **$ax = b$ $x/a = b, a \neq 0$ $ax + b = c$ $x/a + b = c, a \neq 0$ $ax = b + cx$ $a(x + b) = c$ $ax + b = cx + d$ $a(bx + c) = d(ex + f)$ $a/x = b, x \neq 0$**

where a, b, c, d, e, and f are rational numbers. [C, CN, PS, V]

a. Explain why the equation $a/x = b$, cannot have a solution of $x = 0$.
b. Write a linear expression representing a given pictorial, oral, or written pattern.
c. Write a linear equation to represent a particular situation.
d. Observe and describe a situation relevant to self, family, or community which could be represented by a linear equation.
e. Write a linear equation representing the pattern in a given table of values and verify the equation by substituting values from the table.
f. Model the solution of a linear equation using concrete or pictorial representations, and explain how to record the process symbolically.
g. Explain how the preservation of equality is involved in the solving of linear equations.
h. Verify, by substitution, whether or not a given rational number is a solution to a given linear equation.
i. Solve a linear equation symbolically.
j. Analyze the given solution for a linear equation that has resulted in an incorrect solution, and identify and explain the error(s) made.
k. Provide examples from the modern world in which linear equations are used and solved.

Outcome: P9.3

Demonstrate understanding of single variable linear inequalities with rational coefficients including:

- **solving inequalities**
- **verifying**
- **comparing**
- **graphing.**

[C, CN, PS, R, V]

a. Observe and describe situations relevant to self, family, or community, including First Nations and Métis communities, that involve inequalities and classify the inequality as being less than, greater than, less than or equal to, or greater than or equal to.
b. Verify whether or not a given rational number is part of the solution set for a linear inequality.
c. Generalize and apply rules for adding or subtracting a positive or negative number to determine the solution of an inequality.

d. Generalize and apply a rule for multiplying or dividing by a positive or negative number to determine the solution of an inequality.
e. Solve a linear inequality algebraically and explain the strategies used.
f. Compare and explain the process for solving a linear equation to the process for solving a linear inequality.
g. Explain how knowing the solution to a linear equality can be used to determine the solution of a related linear inequality, and provide an example.
h. Critique the statement: "For any linear equality, there are two related linear inequalities".
i. Graph the solution of a linear inequality on a number line.
j. Explain why there is more than one solution to a linear inequality.
k. Verify the solution of a given linear inequality using substitution for multiple elements, in the solution and outside of the solution.
l. Solve a situational question involving a single variable linear inequality and graph the solution.

Outcome: P9.4

Demonstrate understanding of polynomials (limited to polynomials of degree less than or equal to 2) including:

- **modeling**
- **generalizing strategies for addition, subtraction, multiplication, and division**
- **analyzing**
- **relating to context**
- **comparing for equivalency.**

[C, CN, R, V]

a. Model (concretely or pictorially) and describe the relationship between x and x^2.
b. Represent polynomials concretely or pictorially, and describe how the concrete or pictorial model reflects the symbolic form
c. Write a polynomial for a given concrete or pictorial representation.
d. Identify the variables, degree, number of terms, and coefficients, including the constant term, of a given simplified polynomial expression and explain the role or significance of each.
e. Identify the type of expression that is represented by a polynomial of degree 1.
f. Sort a set of polynomials into monomials, binomials, and trinomials.
g. Critique the statement "A binomial can never be a degree 2 polynomial".
h. Write equivalent forms of a polynomial expression by interchanging terms or by decomposing terms, and justify the equivalence.
i. Explain why terms with different variable exponents cannot be added or subtracted.
j. Generalize, from concrete and pictorial models, and apply strategies for adding and subtracting polynomials symbolically.
k. Verify whether or not the simplification of the addition or subtraction of two polynomials is correct and explain.
l. Describe the relationship between multiplication of a polynomial and a monomial, and determining the area of a rectangular region.
m. Generalize, from concrete and pictorial models, and apply strategies for multiplying a polynomial by a monomial.
n. Generalize, from concrete and pictorial models, and apply strategies for dividing a polynomial by a monomial.
o. Verify whether or not the simplification of the multiplication or division of a polynomial by a monomial is correct.

Outcome: SS9.1

Demonstrate understanding of circle properties including:

- **perpendicular line segments from the centre of a circle to a chord bisect the chord**
- **inscribed angles subtended by the same arc have the same measure**
- **the measure of a central angle is twice the measure of an inscribed angle subtending the same arc**
- **tangents to a circle are perpendicular to the radius ending at the point of tangency.**

[C, CN, PS, R, T, V]

a. Observe and describe situations relevant to self, family, or community that involve circles, chords, central angles, inscribed angles, radii, arcs, and/or points of tangency.
b. Construct a tangent line to a circle by applying the knowledge that a tangent line to the circle is perpendicular to a radius of the circle.
c. Generalize, from personal explorations, the relationship between the measures of inscribed angles subtended by the same arc.
d. Generalize, from personal explorations, the relationship between the measure of a central angle and the measure of inscribed angles subtended by the same arc.
e. Generalize, from personal explorations, the relationship between a perpendicular line segment from the centre of a circle to a chord and the chord.
f. Model how to find the diameter of a circle using an inscribed angle of 90° and explain why the strategy works.

g. Describe examples of where First Nations and Métis, past and present, lifestyles and worldviews demonstrate one or more of the circle properties (e.g., tipi and medicine wheel).
h. Solve a situational question involving the application of one or more of the circle properties.

Outcome: SS9.2

Extend understanding of area to surface area of right rectangular prisms, right cylinders, right triangular prisms, to composite 3-D objects. [CN, PS, R, V]

a. Describe 3-D composite objects from the natural and constructed world, including objects relevant to First Nations and Métis people (e.g., Mesoamerican pyramids).
b. Analyze a composite 3-D object to identify areas of overlap and explain the impact of these areas on determining the surface area of the composite 3-D object.
c. Critique the statement "To find the surface area of a composite 3-D object, add together the surface areas of the individual 3-D objects from which the composite 3-D object is comprised".
d. Determine the surface area of composite 3-D objects.
e. Solve situational questions involving the surface area of composite 3-D objects.
f. Give dimensions for a single 3-D object that will have the same surface area as a composite 3-D object.
g. Approximate the surface area of a 3-D object from the natural environment using composites of standard 3-D objects such as right rectangular prisms, right cylinders, and right triangular prisms.

Outcome: SS9.3

Demonstrate understanding of similarity of 2-D shapes. [C, CN, PS, R, V]

a. Observe and describe 2-D shapes, relevant to self, family, or community, that are similar.
b. Explain the difference between similarity and congruence of polygons.
c. Verify whether or not two polygons are similar.
d. Explain how ratios and proportionality are related to similarity of polygons.
e. Draw a polygon similar to a given polygon and explain the strategies used.
f. Solve situational questions involving the similarity of polygons.
g. Identify and describe situations relevant to self, family, or community that involve scale diagrams and explain the meaning of the scale factor involved.
h. Explain how scale diagrams are related to similarity, ratios, and proportionality.
i. Draw a diagram to scale that represents an enlargement or reduction of a given 2-D shape and explain the strategies used.
j. Explain how to determine the scale factor for a given 2-D shape and an enlargement or reduction of the shape.
k. Verify whether or not a given diagram is a scale diagram of a 2-D shape and, if it is, identify the scale factor for the diagram.
l. Solve situational questions involving scale diagrams and scale factors.

Outcome: SS9.4

Demonstrate understanding of line and rotation symmetry. [C, CN, PS, V]

a. Observe and describe examples of line and rotation symmetry in situations relevant to self, family, or community.
b. Classify different 2-D shapes or designs made of 2-D shapes, according to the number of lines of symmetry.
c. Complete a 2-D shape or design given part of a shape or design and one or more lines of symmetry.
d. Determine, with justification, if a given 2-D shape or design has rotation symmetry about the point at the centre of the shape or design and, if it does, state the order and angle of rotation.
e. Identify a line of symmetry, or the order and angle of rotation symmetry, in a given tessellation.
f. Describe examples of the use and significance of line and rotation symmetry in First Nations and Métis art.
g. Analyze different transformations of 2-D shapes on the Cartesian plane and describe the type of symmetry, if any, that results.
h. Determine whether or not two 2-D shapes on the Cartesian plane are related by either rotation or line symmetry and explain.
i. Create or provide an art work (such as a painting or dance) that demonstrates line and rotation symmetry, and identify the line(s) of symmetry and the order and angle of rotation.

Outcome: SP9.1

Demonstrate understanding of the effect of:

- **bias**
- **use of language**
- **ethics**
- **cost**
- **time and timing**
- **privacy**
- **cultural sensitivity and**
- **population or sample on data collection.**

[C, PS, R, T]

a. Analyze given case studies of data collection, including data pertaining to First Nations and Métis peoples, and identify potential problems related to bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity.
b. Provide examples to illustrate how bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity may influence the data collected.
c. Identify situations relevant to self, family, or community where a set of data was collected and classify each situation as involving a sample or the population.
d. Provide an example of a situation in which a population may be used to answer a question, and justify the choice.
e. Provide an example of a question where a limitation precludes the use of a population and describe the limitation (e.g., too costly, not enough time, limited resources).
f. Identify and critique given examples in which a generalization from a sample of a population, including from First Nations and Métis data, may or may not be valid for the population.
g. Explain different strategies for trying to minimize negative effects on data collection.
h. Explain the importance of protocols for respectful data collection and information sharing.

Outcome: SP9.2

Demonstrate an understanding of the collection, display, and analysis of data through a project. [C, PS, R, T, V]

a. Devise a project plan related to a situation relevant to self, family, or community, that involves:
    - formulating a question for investigation
    - choosing a data collection method that includes social considerations
    - electing a population or a sample, and justifying the choice
    - collecting the data
    - displaying the collected data in an appropriate manner
    - drawing conclusions to answer the question.
b. Create and apply a rubric to assess a project that includes the assessment of all requirements for the project.
c. Complete the project according to the plan, draw conclusions, and communicate findings to an audience.

Outcome: SP9.3

Demonstrate an understanding of the role of probability in society. [C, CN, R, T]

a. Observe examples of probabilities that impact or influence aspects of one's self, family, community, or environment and describe those impacts or influences.
b. Analyze the meaningfulness of a probability against the limitations of assumptions associated with that probability.
c. Provide examples of how a single probability could be used to support opposing positions.
d. Explain, using examples, how decisions based on probability may be a combination of theoretical probability, experimental probability, and subjective judgment.

Outcome: SP9.4

Research and present how First Nations and Métis peoples, past and present, envision, represent, and make use of probability and statistics.

a. Gather and document information regarding the significance and use of probability and statistics for at least one First Nation or Métis peoples from a variety of sources such as Elders and traditional knowledge keepers.
b. Compare the significance, representation, and use of probability and statistics for different First Nations and Métis peoples, and other cultures.
c. Communicate concretely, pictorially, orally, visually, physically, and/or in writing, what has been learned about the envisioning, representing, and use of probability and statistics by First Nations and Métis peoples and how these understandings parallel, differ from, and enhance one's own mathematical understandings about probability and statistics.

Grade 10

Workplace and Apprenticeship Mathematics 10

Outcome: WA10.1

Demonstrate understanding of the preservation of equality including solving problems that involve the manipulation and application of formulas related to:

- **perimeter**
- **area**
- **the Pythagorean Theorem**
- **primary trigonometric ratios**
- **income.**

Indicators:

(It is intended that this outcome be integrated with other outcomes throughout the course.)

a. Verify, using examples, and explain why different forms of the same formula are equivalent.
b. Verify if given forms of the same formula are equivalent and justify the conclusion.
c. Describe, using examples, how a given formula is used in a trade or an occupation.
d. Create, solve, and verify the reasonableness of solutions to situational questions relevant to self, family, or community that involve the use of a formula.
e. Analyze solutions to questions that involve formulae to verify the preservation of equality and correct, with explanation, if necessary.
f. Solve situational questions that involve the application of a formula that:
    - does not require manipulation
    - does require manipulation.

Outcome: WA10.2

Analyze puzzles and games that involve spatial reasoning using problem solving strategies.

Indicators:

(It is intended that this outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction, and puzzles and games such as Tetris, Rubik's cube, Blokus, chess, checkers, Backgammon, Mastermind, Tic-Tac-Toe, Connect Four or Five, Battleship, and Cathedral World).

a. Determine, explain, and verify strategies to solve a puzzle or to win a game such as:
    - guess and check
    - look for a pattern
    - make a systematic list
    - draw or model
    - eliminate possibilities
    - solve a simpler problem
    - work backwards
    - develop alternative approaches.
b. Observe and analyze errors in a solution to a puzzle or in a strategy for winning a game and explain the reasoning.
c. Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Outcome: WA10.3

Demonstrate using concrete, and pictorial models, and symbolic representations, understanding of measurement systems including:

- **The Système International (SI)**
- **The British Imperial system**
- **The US customary system.**

Indicators:

It is intended that students explore, analyze for patterns, and develop understanding of many units in all three systems of measurements. The units used should be those that are appropriate to the workplace or apprenticeship context being considered. These units include:

- metres, grams, litres, and seconds along with appropriate prefixes such as kilo, hecto, deci, centi, and milli, as well as hectare, tonne, and degrees Celsius (SI system).
- inch, foot, board foot, yard, mile, acre, teaspoon, tablespoon, cup, pint, quart, gallon, bushel, ton, and degrees Fahrenheit (British and US systems where appropriate).

a. Research and present about a topic related to measurement systems such as:
    - how the SI system and the Imperial system were developed
    - the adoption of the SI system in Canada

- development of the US customary system and the relationship between the units of this system and those of the Imperial measurement system (e.g., units for length and area are the same; there are differences, however, for mass, weight, and capacity)
- early systems of measurement in Canada and contexts in which some are still used today (e.g., hand span and thumbs for measuring horses from the ground to its withers).

b. Describe situations relevant to self, family, or community in which the SI and/or Imperial systems are used.
c. Analyze the relationships between:
   - the base units of the metric system of measurement and the base ten number system
   - the prefixes used in the metric system and powers of ten
   - the related units for length, area, volume, capacity, mass, and temperature for each of the two systems of measurement.
d. Explain, using examples, how and why:
   - decimal numbers are usually used for SI units
   - fractions are usually used for Imperial units.
e. Provide an approximate measurement in:
   - SI units for a measure given in Imperial units (e.g., 1 inch is approximately 2.5 cm, 1 kg is a little more than 2 lbs)
   - Imperial units for a measurement given in SI units (e.g., 1 litre is approximately US gallon, 250 mL is approximately 1 cup).
f. Develop (using proportional reasoning), generalize, explain, and apply strategies (including formulas) to convert measurements from:
   - SI to Imperial units (e.g., Celsius to Fahrenheit, centimetres to inches, kilograms to pounds, litres to quarts)
   - Imperial to SI units (e.g., Fahrenheit to Celsius, inches to centimetres, pounds to kilograms, quarts to litres).

Outcome: WA10.4

Demonstrate, using concrete and pictorial models, and symbolic representations, understanding of linear measurement, including units in the SI and Imperial systems of measurement.

Indicators:

It is intended that contextual situations involve the four arithmetic operations on decimals or on fractions, and that students do not convert from one to the other when performing calculations unless required within the particular question.

a. Justify the choice of referents for units in both the SI system and the Imperial system (e.g, the width of the thumb for an inch and the width of the little finger for 1 centimetre).
b. Estimate, using personal referents for SI and Imperial units, linear measurements such as the dimensions of regular 3-D objects or 2-D shapes found in the classroom, at home, or in the community.
c. Explain why and how the same linear measurement can be expressed using:
   - equivalent SI units (e.g., 10 centimetres, 0.1 metres, and 100 millimetres)
   - equivalent Imperial units (e.g., 6 inches, $\frac{1}{2}$ foot, $\frac{6}{36}$ or $\frac{1}{6}$ yard).
d. Measure and record (using Canadian spelling and abbreviations for SI or Imperial units) using a variety of instruments such as trundle or surveyor's wheels, height gauges, metric and Imperial tapes and rulers, carpenter's squares, micrometers, fractional or electronic callipers, and pedometers:
   - distances at school, at home, and in the community
   - inside diameters, outside diameters, lengths, and widths of personally relevant 3-D objects.
e. Develop, generalize, explain, and apply strategies to convert units of linear measurements within the same system (e.g., feet to yards, or metres to millimetres).
f. Apply knowledge and skills with linear measurement to create and solve or verify the reasonableness of solutions to situational questions relevant to self, family, or community (including perimeter, circumference, and the length + width + height measurement used in shipping and air travel).
g. Determine the operation that should be used to solve a linear measurement problem and explain the reasoning (e.g., Write the equation that could be used to determine how many lengths of $\frac{6}{36}$ of a yard can be cut out of a log measuring $5\frac{1}{3}$ yards).
h. Develop, generalize, explain, and apply strategies to determine the midpoint of a linear measurement, such as the length, width, height, depth, diagonal length, or diameter of a 3-D object (e.g., A wall measures 5 yards and 6 $\frac{7}{8}$ inches. If a painting is to be hung centered on the wall, how far is the centre of the painting from each of the adjacent walls?).

i. Critique the statement "the distance between Regina and Saskatoon is 2 hours".

Outcome: WA10.5

Demonstrate using concrete and pictorial models, and symbolic representations, understanding of area of 2-D shapes and surface area of 3-D objects including units in SI and Imperial systems of measurement.

Indicators:

a. Describe situations relevant to self, family, or community in which SI and/or Imperial units for area measurement are used.
b. Justify the choice of referents for area measurements in both the SI and Imperial units (e.g., a dime or a small fingernail is about one cm^2 and the thumb nail is about 1 in^2).
c. Estimate, using strategies such as personal referents or grids, area and surface area measurements in SI or Imperial units including regular, composite, and irregular 2-D shapes and 3-D objects found in the classroom, at home, or in the community.
d. Develop, generalize, explain, and apply strategies (including measuring and applying formulae) for determining areas and surface areas of:
    - regular, composite, and irregular 2-D shapes, including circles.
    - 3-D objects, including right cylinders and right cones.
e. Create, solve, and verify the reasonableness of solutions to situational questions relevant to self, family, or community that involve area or surface area measurement of:
    - regular or irregular 2-D shapes including circles.
    - 3-D objects, including right cylinders and cones.
f. Develop, generalize, explain, and apply strategies to convert, within the same system of measurement, area measurements expressed in:
    - an SI unit squared to another SI unit squared
    - an Imperial unit squared to another Imperial unit squared.
g. Analyze, with and without the use of dynamic geometry software, the effect of changing the measurement of one or more dimensions on area and perimeter of rectangles and surface area of rectangular prisms.
h. Critique the statement "Area involves one face of a 2-D object while surface area is the sum of the areas of all the faces of a 3-D object".

Outcome: WA10.6

Apply understanding of the Pythagorean Theorem to solve problems.

Indicators:

a. Model, including the use of drawing, concrete materials, and technology, the meaning, role, and use of the Pythagorean Theorem, using examples and non-examples.
b. Observe and analyze a set of triangles to judge if the Pythagorean Theorem could be used to determine an unknown side length and explain the reasoning.
c. Describe historical and contemporary applications of the Pythagorean Theorem, including the 3:4:5 ratio (e.g., Explain the relationship between a circle of string that has 13 equidistant knots or beads forming 12 spaces of equal length on it to the Pythagorean Theorem).
d. Relate, using examples, ratios equivalent to 3:4:5 and other Pythagorean Triples to the Pythagorean Theorem.
e. Develop, generalize, apply, and explain strategies to verify if a corner of a 3-D object is square (90°) or if a parallelogram is a rectangle.
f. Observe and analyze the lengths and intersections of diagonals of various quadrilaterals and draw conclusions.
g. Determine if given triangles are right triangles and explain the reasoning.
h. Create , solve, and verify the reasonableness of solutions to problems relevant to self, family, or community, for which the Pythagorean Theorem can be used.

Outcome: WA10.7

Demonstrate understanding of similarity of convex polygons, including regular and irregular polygons. [C, CN, PS, V, T]

a. Analyze and generalize about the relationships between:
    - the corresponding sides of two or more polygons that have corresponding angles of equal measure
    - the corresponding angles of two or more polygons that have corresponding sides that are proportional.
b. Develop, generalize, explain, and apply strategies for determining if two or more convex polygons (regular or irregular) are similar.
c. Verify whether two or more given polygons are similar.
d. Draw polygons that are similar to a given polygon using measuring tools and technology, explain the strategies used, and justify why the polygons are similar.

e. Apply knowledge and skills related to similar polygons to solve situational questions relevant to self, family, or community that involve right triangles with a shared acute angle (e.g., determining the height of a tree).
f. Create and solve situational questions relevant to self, family, or community that involve similarity of polygons.
g. Critique the statement "All congruent polygons are similar and all similar polygons are congruent".

Outcome: WA10.8

Demonstrate an understanding of primary trigonometric ratios (sine, cosine, and tangent).

Indicators:

a. Observe a set of similar right triangles and analyze and draw conclusions about the ratios of the lengths, with respect to one acute angle of the:
    - side opposite to the side adjacent
    - side opposite to the hypotenuse
    - side adjacent to the hypotenuse.
b. Identify situations where the trigonometric ratios can be used for indirect measurement for angles and lengths.
c. Develop, generalize, explain, and apply formulae for the primary trigonometric ratios (cosine, tangent, and sine).
d. Analyze solutions to situational questions that involve primary trigonometric ratios to determine if they are reasonable and explain the reasoning.
e. Apply knowledge and skills related to the solving of right triangles using the primary trigonometric ratios to create and solve situational problems relevant to self, family, or community.

Outcome: WA10.9

Demonstrate understanding of angles including:

- **drawing and sketching**
- **replicating and constructing**
- **bisecting**
- **relating to parallel, perpendicular, and transversal lines**
- **solving problems.**

Indicators:

a. Justify the choice of personal referents for angles measuring 22.5°, 30°, 45°, 60°, 90°, and 180° and use them to estimate angle measurements (e.g., a corner of a sheet of paper is 90° so of a corner is 45°).
b. Sketch or draw angles of various measures, including acute, right, straight, obtuse, and reflex angles, and justify the choice of sketching or drawing in relation to the situation.
c. Explain, using examples, how to measure angles in different orientations using a variety of instruments such as a protractor, carpenter's square, or dynamic software.
d. Explain and illustrate how angles can be replicated (e.g., Mira, protractor, compass and straightedge, carpenter's square, and dynamic software).
e. Replicate angles in various orientations, with and without the use of technology.
f. Explain, using examples, the relationship between the bisecting of angles and axial symmetry.
g. Bisect angles in various orientations and explain the strategy used.
h. Observe and sort a set of pairs of lines as perpendicular, parallel, or neither, and justify.
i. Relate complementary and supplementary angles to parallel, perpendicular, and transversal lines.
j. Observe and identify in a situation relevant to self, family, or community adjacent angles that are complementary, supplementary, or neither, and explain the reasoning.
k. Generalize, develop, explain, and apply relationships between pairs of angles formed by parallel lines and a transversal, inclu
    - corresponding angles
    - vertically opposite angles
    - alternate interior angles
    - alternate exterior angles
    - interior angles on the same side of the transversal
    - exterior angles on the same side of the transversal.
l. Provide concrete and pictorial examples that show that there are no angle relationships (excluding vertically opposite angles) when two lines that are not parallel are crossed by a transversal.
m. Describe and apply strategies for determining if lines or planes are perpendicular or parallel in situations relevant to self, family, or community (e.g., are the walls perpendicular to the floor? Are the corners square? Are the seams on the duvet parallel? Are the joists parallel?).
n. Analyze personally relevant situations for the impact if lines or planes that are meant to be perpendicular or parallel are not (e.g., What would it be like if the stairs in a staircase were not parallel?).

o. Create and solve relevant situational questions that involve angles and/or parallel lines and transversals, including perpendicular transversals, and explain the reasoning.
p. Critique the statement "If two or more lines are perpendicular, then each is a transversal of the other and if two or more lines are transversals of each other, they are perpendicular".
q. Critique the statement "All lines are either parallel or transversals".
r. Analyze and describe the role of angles, parallel lines, perpendicular lines, and transversals in games and sports (e.g., chess, curling, pool, hockey, soccer, and basketball).

Outcome: WA10.10

Apply proportional reasoning to solve problems involving unit pricing and currency exchange.

Indicators:

a. Create and solve problems relevant to self, family, and community that involve best buy, and explain the solution in terms of the cost as well as other factors, such as quality and quantity.
b. Describe and analyze, using relevant examples taken from print and other media, different sales promotion techniques (e.g., deli meat at $2 per 100g seems less expensive than $20 per kilogram).
c. Determine the percent increase or decrease from an original price to a new price and explain the reasoning for the method
d. Develop (using proportional reasoning), explain, and apply strategies for:
    - comparing the unit price of two or more items
    - solving situational problems involving currency exchange
    - determining percent increase or decrease for a given situation.
e. Develop using proportional reasoning and mental mathematics strategies, explain, and apply strategies for estimating the cost of items or services in Canadian currency while in a foreign country or when making purchases via the Internet, and explain why this may be important.
f. Convert between Canadian currency and foreign currencies, such as Mexican or American currencies, using formulas, charts, or tables, and explain how to know if the solution is reasonable
g. Analyze solutions to situational questions that involve unit pricing or conversions of currency to determine if they are reasonable and explain the reasoning.
h. Research advantages and disadvantages for individuals, groups, businesses, and communities of having differences in currency rates between Canada and the United States.

Outcome: WA10.11

Demonstrate understanding of income including:

- **wages**
- **salary**
- **contracts**
- **commissions**
- **piecework**
- **self-employment**
- **gross pay**
- **net pay.**

Indicators:

a. Describe, using examples, various methods of earning income.
b. Research and record jobs that commonly use different methods of earning income, including hourly wage, wage and tips, salary, commission, contract, bonus, and shift premiums.
c. Describe the advantages and disadvantages for a variety of methods of earning income, such as hourly wage, tips, piecework, salary, commission, contract work, and self-employment.
d. Read and explain the information provided on pay stubs.
e. Determine in decimal form, from a time schedule, the total time worked in hours and minutes, including time and a half and/or double time and explain the strategies used.
f. Determine gross pay for different situations, including:
    - base hourly wage, with and without tips, from given or calculated hours worked
    - base hourly wage, plus overtime (time and a half, double time) from given or calculated hours worked
    - base wage, plus commission
    - single commission rate
    - graduated commission.
g. Determine the Canadian Pension Plan (CPP), Employment Insurance (EI), and income tax deductions for a given amount of gross pay.
h. Identify and describe deductions that may be relevant to self in the future which are used when calculating net pay (e.g., health plans, uniforms, union dues, charitable donations, payroll tax).

i. Investigate, using technology, "what if …" questions related to changes in income (e.g., "What if there is a change in the rate of pay?", "What if there is a change in the method of earning income?", "What if I can qualify for deductions?", "What if I work 80% time instead of full time?" , "What if I am sick for a long period of time?").
j. Create, research, and solve situational questions relevant to self, family, and community that involve income.
k. Critique the statement "When planning for a budget, it is important to calculate net pay rather than rely only on gross pay".

Foundations of Mathematics and Pre-Calculus 10

Outcome: FP10.1

Demonstrate understanding of factors of whole numbers by determining the:

- **prime factors**
- **greatest common factor**
- **least common multiple**
- **principal square root**
- **cube root.**

Indicators:

a. Develop, generalize, explain, and apply strategies for determining the greatest common factors or least common multiples.
b. Explain the relationship between factors and multiples.
c. Determine the prime factors of a whole number and explain the strategies used.
d. Analyze concretely, pictorially, or numerically and explain whether a whole number is a perfect square or a perfect cube.
e. Develop, generalize, explain, and apply strategies for determining the square root of a perfect square and the cube root of a perfect cube.
f. Investigate and report about the numbers 0 and 1 with respect to factors, multiples, square roots, and cube roots.
g. Solve problems that involve prime factors, greatest common factors, least common multiples, square roots, or cube roots.

Outcome: FP10.2

Demonstrate understanding of irrational numbers in both radical (including mixed radical) and exponent forms through:

- **representing**
- **identifying**
- **simplifying**
- **ordering**
- **relating to rational numbers**
- **applying exponent laws.**

Indicators:

a. Sort, with justification, a set of numbers into rational and irrational numbers.
b. Create and explain a pattern that describes the decimal form of an irrational number (e.g., write the digits from 0 to 9 in order, then put two of each digit – 0011223344 … – followed by three of each digit and so on).
c. Approximate the value of a given irrational number and explain the strategy used.
d. Order a set of Real numbers, including rational and irrational numbers, on a number line and explain the strategies used.
e. Express a radical as a mixed radical in simplest form (limited to numerical radicands).
f. Express a mixed radical as an entire radical (limited to numerical radicands).
g. Explain, using examples, how changing the value of the index of a radical impacts the value of the radical.
h. Represent, such as through the use of a graphic organizer, the relationships among the subsets of the Real numbers: natural, whole, integer, rational, and irrational.
i. Analyze patterns to generalize why $a^{-n} = \frac{1}{a^n}, a \neq 0$.
j. Analyze patterns to generalize why $a^{\frac{1}{n}} = \sqrt[n]{a}, n \neq 0, n \epsilon I$ and $a > 0$ when n is an even integer.
k. Extend and apply the exponent laws to powers with rational exponents (limited to expressions with rational and variable bases and integral and rational exponents):

- $(a^m)(a^n) = a^{m+n}$
- $a^m \div a^n = a^{m-n}, a \neq 0$
- $(a^m)^n = a^{mn}$

- $(ab)^m = a^m b^m$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$

l. Analyze simplifications of expressions involving radicals and/or powers for errors.
m. Express powers with rational exponents as radicals and vice versa.
n. Create a representation that conveys the relationship between powers, rational numbers, and irrational numbers.

Outcome: FP10.3

Demonstrate understanding of SI and imperial units of measurement including:

- **linear measurement**
- **surface area of spheres, and right cones, cylinders, prisms, and pyramids**
- **volume of spheres, and right cones, cylinders, prisms, and pyramids**
- **relationships between and within measurement systems.**

Indicators:

It is intended that students explore, analyze for patterns, and develop understanding of many units in both systems of measurement. These units include: metres, along with appropriate prefixes such as kilo, hecta, deci, centi, and milli, as well as hectare (SI system) inch, foot, yard, mile, acre (imperial system).

a. Provide personal referents for linear measurements, including millimetre, centimetre, metre, kilometre, inch, foot, yard, and mile and explain the choices.
b. Justify the choice of units and or referents for determining or estimating linear, surface area, or volume measurements in different contexts.
c. Explain the selection of measurement instruments (e.g., rulers, callipers, or tape measures) and the strategies used to determine linear measurements (e.g., circumference of a bottle, length of a curve, or perimeter of the base of an irregular 3-D object).
d. Critique the statement “the length of the wall is greater in yards than it is in metres”.
e. Compare the size of SI and imperial units of measurement (linear, surface area, and volume) using referents.
f. Develop, generalize, explain, and apply strategies and/or formulas for converting between units within the imperial or SI system of measurements, limited to linear, surface area, and volume units. (e.g., converting square feet to square yards or m3 to cm3).
g. Develop, generalize, explain and apply strategies and/or formulas for converting between:
    - SI and imperial units of linear, surface area, and volume measure
    - imperial and SI units of linear, surface area, and volume measure.
h. Verify, with explanation (such as unit analysis and/or mental mathematics and estimation), a conversion of units (within the SI or imperial systems of measurement or between them).
i. Analyze 3-D objects, their nets, and labelled diagrams to develop and generalize strategies and/or formulas for determining the surface area and volume of right cones, cylinders, prisms, and pyramids and composite objects.
j. Solve, using personal strategies and/or formulas, situational questions related to surface area, volume, and dimensions of right cones, cylinders, prisms, and pyramids, and composite 3-D objects.
k. Apply formulas to determine the surface area and/or volume of spheres.
l. Explain the relationship between the volumes of:
    - right cones and right cylinders with the same base and height
    - right pyramids and right prisms with the same base and height.
m. Analyze a treaty for its inclusion of measurements, such as in the surveying for land entitlement, and create and solve situational questions that are relevant to self, family, and community.

Outcome: FP10.4

Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.
Indicators:

a. Develop, generalize, explain, and apply relationships between the ratios of side lengths and angle sizes in similar right triangles.
b. Demonstrate how to identify the hypotenuse of a right triangle and the adjacent and opposite sides to an acute angle in that right triangle.
c. Solve problems, with or without the use of technology, involving one or more right triangles by applying primary trigonometric ratios and/or the Pythagorean Theorem.
d. Create and solve problems that involve indirect and direct linear measurements by using the primary trigonometric ratios, the Pythagorean Theorem, and measurement instruments such as a clinometer or metre stick.

Outcome: FP10.5

Demonstrate understanding of the multiplication and factoring of polynomial expressions (concretely, pictorially, and symbolically) including:

- **multiplying of monomials, binomials, and trinomials**
- **common factors**
- **trinomial factoring**
- **relating multiplication and factoring of polynomials.**

Indicators:*(It is intended that the emphasis of this outcome be on binomial by binomial multiplication, with extension to polynomial by polynomial to establish a general pattern for multiplication.)*

a. Develop, generalize, explain, and apply a strategy of symbolic manipulation to determine the product of two binomials by analyzing concrete and pictorial models.
b. Explain the relationship between the multiplication of two binomial expressions and the area of a rectangular region.
c. Develop (concretely, pictorially, or symbolically), explain, and apply understanding of how multiplication of binomials is related to the multiplication of two-digit numbers (e.g., use algebra tiles and base ten blocks to compare and relate the products of (x + 1)(3x + 2) and (11)(32)).
d. Develop, generalize, explain, and apply a strategy for multiplying polynomials.
e. Analyze the multiplication of two polynomials for errors and explain the strategy used.
f. Explain why evaluating at a value for the variable in a product of polynomials in factored form should give the same solution as evaluating the expanded and simplified form of the polynomial product at the same value (e.g., explain why x2 + 5x + 6 should have the same value as (x + 3)(x + 2) when evaluated at x = -4).
g. Explain, using concrete or visual models, how the processes of factoring and multiplication are related.
h. Develop (using concrete materials, pictures, or visualization), generalize, explain, and apply strategies for factoring and verifying the factors of binomials, including numerical binomial expressions (e.g., 32 + 20 = 4(8 + 5)).
i. Sort a set of polynomials according to the type(s) of factoring that could be applied to them.
j. Explain and apply strategies for determining whether given factors are those of a given polynomial.
k. Develop, generalize, explain, and apply strategies for factoring a trinomial.
l. Critique the statement “any trinomial can be factored into two binomial factors”.
m. Explain how differences of squares can be factored using trinomial factoring strategies.
n. Explain why it is important to look for common factors first when factoring a trinomial.

Outcome: FP10.6

Expand and apply understanding of relations and functions including:

- **relating data, graphs, and situations**
- **analyzing and interpreting**
- **distinguishing between relations and functions.**

Indicators:

a. Provide and discuss examples of different types of relations relevant to one’s life, family, or community (e.g., person A is the mother of person B, or person A is a brother of person B.).

Note: For some First Nations and Métis, the way relations are defined might be at a more specific level. For example, for some Ojibway a word for “brother” does not exist, only “older brother” and “younger brother”.

b. Explain, by providing situational and graphical examples, the relationship between the categories of “relations” and “functions”.
c. Critique the statement “Relations and functions are the same thing”.
d. Graph, with or without technology, a set of data, and determine the restrictions on the domain and range.
e. Explain why data points should or should not be connected on the graph for a situation.
f. Provide and explain examples of situations that could be represented by a given graph.
g. Sketch a graph to represent a situation presented orally or in writing.
h. Determine, and express in a variety of ways, the domain and range of a graph, a set of ordered pairs, or a table of values.
i. Generalize, explain, and apply strategies for determining whether a set of ordered pairs or a graph represents a function.

Outcome: FP10.7

Demonstrate, with and without the use of technology, understanding of slope (concretely, pictorially, and symbolically) with respect to:

- **line segments and lines**
- **rate of change**
- **ratio of rise to run**
- **parallel lines**

- **perpendicular lines.**

Indicators:

a. Provide examples, relevant to self, family, or community, to explain the importance of slope.
b. Illustrate and explain, using examples relevant to self, family, or community, how slope is rate of change.
c. Determine the slope of a line segment by using the measurement or calculation of the rise and run.
d. Classify lines in a given set as having positive or negative slopes, and explain how the sign of the slope affects the interpretation or meaning of the slope.
e. Explain the meaning of zero or slopes with no Real value.
f. Explain why the slope of a straight line can be determined by using any two distinct points on that line.
g. Draw a line given its slope and a point on the line.
h. Determine another point on a line, given the slope and a point on the line.
i. Generalize, explain, and apply strategies for determining whether two lines are parallel or perpendicular.
j. Apply knowledge and skills related to slope to solve situational questions relevant to self, family, and community (e.g., determine the slopes of the poles in a tepee and the impact of changing the slopes on the dimensions and strength of the tepee).

Outcome: FP10.8

Demonstrate understanding of linear relations including:

- **representing in words, ordered pairs, tables of values, graphs, function notation, and equations**
- **determining characteristics including intercepts, slope, domain, and range**
- **relating different equation forms to each other and to graphs.**

Indicators:

a. Critique the statement “any straight line is the graph of a linear function”.
b. Explain, using examples, the impact of the domain of a linear function on the graph of the function (e.g., if the domain is not all Real numbers, then the graph will not show a solid line).
c. Analyze situations to identify, with justification, the independent and a dependent variable.
d. Analyze situations, graphs, tables of values, equations, or sets of ordered pairs to determine if the relationship described is linear.
e. Match corresponding types of representations of linear relations (e.g., situations, graphs, tables of values, equations, and sets of ordered pairs).
f. Develop, generalize, explain, and apply strategies for determining the intercepts (as values and ordered pairs) of a linear relation from its graph.
g. Determine the slope, domain, and range of the graph of a linear relation.
h. Sketch examples of linear relations to demonstrate the number of x or y intercepts possible for any line.
i. Match, with explanation, slopes and y-intercepts to graphs of linear relations.
j. Solve a situational question that involves the intercepts, slope, domain, or range of a linear relation.
k. Express the equation of a linear relation in different forms (including the slope-intercept or general form) and compare the graphs of the linear relations.
l. Generalize, explain, and apply strategies for drawing or sketching the graph of a linear relation in slope-intercept, general, or slope-point form, or function notation.
m. Graph, with and without technology, a linear relation given in slope-intercept, general, or slope-point form, and explain the strategy used to create the graph.
n. Analyze a set of linear relations for equivalent linear relations (e.g., $2x + 3y = 6$ is equivalent to $4x + 6y = 12$) and explain the reasoning.
o. Explain the relationship between linear functions written in function notation and written as equations with two variables, and how to change between the two forms.
p. Apply knowledge and skills related to function notation to solve situational questions.
q. Determine the related range value, given a domain value for a linear function (e.g., if $f(x) = 3x – 2$, determine $f(–1)$) and explain what the resulting value tells about the linear function.
r. Determine the related domain value, given a range value for a linear function (e.g., if $g(t) = 7 + t$, determine t so that $g(t) = 15$) and explain what the resulting value tells about the linear function.
s. Explain why a linear function would never have a term of x2 when in simplified form.

Outcome: FP10.9

Demonstrate understanding of the writing and application of equations of linear relations, given:

- **a graph of a relation**
- **a point that satisfies a relation and the slope of the relation**
- **two distinct points that satisfy a relation**
- **a point that satisfies the relation and the equation of a line parallel or perpendicular to the relation.**

Indicators:

a. Develop, generalize, explain, and apply strategies for writing an equation for a linear relation using data obtained from a graph.
b. Develop, generalize, explain, and apply strategies for writing an equation for a linear relation when given:
   - a point that satisfies the relation and the slope of the relation
   - two points that satisfy the relation
   - the coordinates of a point that satisfy the relation and the equation of a line parallel or perpendicular to the line.
c. Compare and critique the structure and purposes of different forms of linear relations, including $y = mx + b$, $A_x + B_y = C$, and $y - y_1 = m(x - x_1)$ (e.g., there is no way to write a vertical linear relation in the form $y = mx + b$).
d. Graph and write equations for linear data generated within an experiment or collected from a situation.
e. Apply knowledge and skills of linear relations and their equations to solve situational questions.

Outcome: FP10.10

Solve problems that involve systems of linear equations in two variables, graphically and algebraically.
Indicators:

a. Match, with justification, situations and systems of linear equations.
b. Sketch, describe, provide and explain situational examples of the different ways that the graphs of two linear equations (two variables) can intersect and explain the meaning of the points of intersection.
c. Develop, generalize, explain, and apply strategies for solving systems of equations graphically, with and without the use of technology and verify the solutions.
d. Develop, generalize, explain, and apply strategies, including verification of solutions, for solving systems of equations algebraically.
e. Critique the statement "two lines always intersect at exactly one point".
f. Apply knowledge and skills with systems of linear equations to solve situational questions.

Mathematics 11 (Modified)

Outcome: M11.1

Extend understanding of arithmetic operations to rational numbers to solve problems within the home, money, recreation, and travel themes.
Indicators:

a. Compare and order positive and negative numbers, using appropriate tools (e.g., change in temperature using a thermometer).
b. Apply arithmetic operations to whole numbers, integers, fractions, decimals, and percents.
c. Compare and convert among fractions**,** decimals and percents concretely, pictorially, and symbolically.
d. Determine rounding of decimals to the nearest unit, tenth and hundredth (e.g., calculations with money rounding to 2 decimal places).
e. Apply understanding of combined percents or percents of percents.
f. Justify the reasonableness of calculations and problem-solving strategies, using a variety of tools and/or strategies (e.g., estimation, mental mathematics, tables, graphs, calculators and/or computers).

Outcome: M11.2 **[WA 10.2]**

Demonstrate understanding of reasoning by analyzing puzzles and games.
Indicators:

a. Determine, explain and verify strategies to solve a puzzle or win a game such as:
   - guess and check
   - look for a pattern
   - make a systematic list
   - draw or model
   - eliminate possibilities
   - solve a simpler problem
   - work backwards
   - develop alternative approaches.

b. Analyze puzzles or games for patterns, describe the properties of a given pattern, and identify if a set of objects fits the pattern or not and explain why.
c. Observe and analyze errors in a solution to a puzzle or in a strategy for winning a game and explain the reasoning.
d. Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Sample games: *Tetris, Rubik's cube, Blokus, chess, checkers, Backgammon, Mastermind, Tic-Tac-Toe, Connect Four or Five, Battleship, Cathedral World, and Mancala.*

Outcome: M11.3 **[WA 10.3, WA 10.5, FP 10.3]**

Demonstrate understanding of data collection and analysis within the home, recreation, and travel themes.

Indicators:

a. Read and interpret graphs (e.g., line graph, broken-line graph, bar graph, histogram, circle graph) obtained from various sources (e.g., newspapers, magazines, Statistics Canada website) and communicate information represented in relation to real life situations (e.g., weight, weight training programs, box scores for sports, nutrition, sleep, physical activity, sporting events, and travel).
b. Design questionnaires (e.g., for a cafeteria to determine which juices to stock) or experiments (e.g., observing, counting, taking measurements) for gathering data.
c. Explain the difference between population and sample, describe the characteristics of a good sample, and explain why sampling is necessary (e.g., time, cost, or physical constraints).
d. Collect data from primary sources (e.g., surveys, questionnaires, experiments, interviews) or from secondary sources (e.g., Internet databases, newspapers, magazines).
e. Organize data from primary or secondary sources in a variety of ways (e.g. table, frequency table, stem and leaf plot).
f. Represent data using graphs (e.g., line graph, broken-line graph, bar graph, histogram, circle graph) using a variety of tools (e.g., dynamic statistical software, graphing calculator, spreadsheet) and justify the type of graph chosen.
g. Evaluate graphical representations of data through inferences, comparisons, and predictions and justify conclusions using convincing arguments.
h. Describe and analyze situations in which data has been collected (e.g., target heart rates, weather patterns and predictions, sports scores).

Outcome: M11.4 **(WA10.4)**

Demonstrate understanding of measurement in the Système International (metric) and Imperial System within the home and travel themes.

Indicators:

It is intended that students explore, analyze for patterns, and develop understanding of many units in the systems of measurements. The units used should be those that are appropriate to the context being considered. These units include:

- *metres, grams, litres, and seconds along with appropriate prefixes such as kilo, centi, and milli, and degrees Celsius (SI system).*
- *inch, foot, mile, teaspoon, tablespoon, cup, pint, quart, gallon, and degrees Fahrenheit (Imperial system).*

a. Determine and explain the lengths of common objects in the metric and imperial systems, using a variety of tools (e.g., measuring tape, metre or yard stick, measuring cups, graduated cylinders, trundle wheel).
b. Estimate lengths and distances in metric units and in imperial units by applying personal referents (e.g., the width of a finger is approximately 1 cm; the length of a piece of standard loose-leaf paper is about 1 ft; the capacity of a pop bottle is 2 L).
c. Develop, explain, and apply strategies to estimate quantities (e.g., books in a shelving unit, time to complete a job, people in a crowd).
d. Determine and explain the mathematics related to time including:
    - converting units of time measure
    - 24-hour clock
    - time zones
    - flight arrival and departure times
    - elapsed time.
e. Convert measures within and between systems (e.g., centimeters and metres, feet and inches, pounds and kilograms, degrees Celsius and degrees Fahrenheit) using a variety of tools (e.g., tables, calculators, online conversion tools).
f. Discuss and approximate measures between systems (e.g., 1 inch is approximately 2.5 cm, 1 kg is a little more than 2 lbs, 1 litre is approximately $\frac{1}{4}$ US gallon).
g. Describe the situations in which SI and/or Imperial units of measurement are used.
h. Analyze the relationships between the related units for length, area, temperature, and currency measures within and between systems.
i. Estimate, measure, and calculate perimeters (e.g., wall paper borders, fencing, baseboards).
j. Estimate, measure, and calculate areas of rectangles, triangles, circles, and of related composite shapes (e.g., wall space to be painted, floors to be covered, square footage of a living space, laying sod, patio slabs, floor tiles, wall paper).
k. Critique the statement "the distance between Regina and Saskatoon is 2 hours".

Outcome: M11.5 [WA 10.9]

Demonstrate understanding of angles to solve problems within the home theme.

Indicators:

a. Justify the choice of personal referents for angles measuring 22.5°, 30°, 45°, 60°, 90°, and 180° and use them to estimate angle measurements (e.g., a corner of a sheet of paper is 90° so ½ of a corner is 45°).
b. Explain, using home construction examples (e.g., mitre cuts, framing, window and door casings, trusses, tile installations, crown moulding), how to measure angles in different orientations using a variety of instruments (e.g., protractor, carpenter's square, and dynamic software).
c. Explain and illustrate how angles can be replicated and drawn (e.g., Mira, protractor, compass and straightedge, carpenters square and dynamic software).
d. Identify, classify, and sketch angles of various measures, including acute, right, straight, obtuse, and reflex angles.
e. Explain, using examples, the relationship between the bisecting of angles and axial symmetry.
f. Bisect angles in various orientations and explain the strategy used.
g. Identify adjacent angles that are complementary, supplementary, or neither, and explain the reasoning.
h. Solve situational problems involving complementary and supplementary angles.
i. Identify vertically opposite angles and solve situational problems.

Outcome: M11.6 **[WA 10.6]**

Demonstrate understanding of the Pythagorean Theorem to solve problems within the home theme.

Indicators:

a. Model, including the use of drawings, concrete materials, and technology, the meaning, role, and use of the Pythagorean Theorem, using examples and non-examples.
b. Apply the Pythagorean 3:4:5 ratios to determine if angles are square (right angles) in home construction contexts.
c. Apply the Pythagorean Theorem to solve for a missing side that has an irrational solution.
d. Estimate the values of irrational numbers using a table of perfect squares, multiplication chart, or a number line and show appropriate rounding of irrational numbers.
e. Observe and analyze the use of Pythagorean lengths of diagonals of various building structures (e.g., trusses, frames, door jambs, window casings).

Outcome: M11.7 [WA 10.10]

Demonstrate understanding of proportional reasoning within the home, money, recreation, and travel themes.

Indicators:

a. Explain and apply strategies to solve ratio and rate problems.
b. Recognize and represent equivalent rates and ratios.
c. Calculate and compare the unit rate of items and the unit cost of items (e.g., heart rates in various situations, walking speed, rate of pay, cost per linear foot).
d. Calculate and compare costs of items (e.g., lodgings, transportation, recreation fees, cellular mobile phone plans).
e. Estimate and calculate conversions between Canadian and American currency using proportional reasoning.
f. Identify and describe applications of proportional reasoning (e.g., applying fertilizers, mixing gasoline and oil for use in small engines, estimating cooking time needed per pound, determining the fiber content of different sizes of food servings, calculating overtime **pay).**

Outcome: M11.8 [WA 10.11]

Demonstrate understanding of income.

Indicators:

a. Gather, interpret, and describe various remuneration methods of earning income (e.g., hourly rate, overtime rate, job or project rate, commission, salary, gratuities) and remuneration schedules (e.g., weekly, biweekly, semimonthly, monthly).
b. Research and record jobs that commonly use different remuneration methods of earning income (e.g., hourly wage, wage and tips, salary, commission, contract, piecework, bonus, and shift premiums).
c. Describe the advantages and disadvantages for various remuneration methods of earning income (e.g., hourly wage, tips, piecework, salary, commission, contract work, and self-employment).
d. Solve problems and make decisions involving different remuneration methods and schedules.
e. Analyze and complete timesheets.
f. Explain and assess the information provided on pay stubs.
g. Determine gross pay for different situations, including base hourly wage, with and without tips, from given or calculated hours worked base hourly wage, plus overtime (time and a half, double time) from given or calculated hours worked base wage, plus commission, single commission rate and graduated commission.
h. Gather, interpret, and describe information about government payroll deductions (e.g., CPP, EI, income tax) and other payroll deductions (e.g., contributions to pension plans other than CPP; union dues; charitable donations; benefit-plan contributions).

i. Estimate and compare, using current data (e.g., federal tax tables), the percent of total earnings deducted through government payroll deductions for various benchmarks (e.g., $15 000, $20 000, $25 000).
j. Describe the relationship between gross pay, net pay, and payroll deductions (e.g., net pay is gross pay less government payroll deductions and any other payroll deductions), and estimate net pay in various situations.
k. Investigate, with or without technology, "what if …" questions related to changes in income (e.g., "What if there is a change in the rate of pay?", "What if there is a change in the method of earning income?", "What if I can qualify for deductions?", "What if I work 80% time instead of full time?", "What if I am sick for a long period of time?", "What if an athlete earned one million dollars last year, then how many hours would I have to work to earn that much money?").

Outcome: M11.9

Demonstrate understanding of responsible spending habits.

Indicators:

a. Identify and justify personal expenses (e.g., mobile phone, vehicle, electronics, recreation, travel, home renovations, and aesthetics).
b. Explain considerations made when prioritizing spending money (e.g., recurring expenses and unexpected opportunities).
c. Estimate the cost and justify affordability of a desired purchase.
d. Determine PST and GST on purchases and discuss exemptions.
e. Create a personal spending log over a set period of time and explain the advantages.
f. Compare and contrast the cost of purchasing items or services at various vendors.
g. Compare various sales incentives (e.g., "Group on", percent discounts, pre-sale gift with purchase, reward zone points, buy 1, and get 1 . . . (BOGO)) and discuss the value of estimating.
h. Investigate and analyze "what if …" questions related to personal spending (e.g., cash versus debit purchases involving the penny or rounding).
i. Research and report on the estimated costs involved in a large expense (e.g., a trip, home renovation, or an activity or sport).

Grade 11

Workplace and Apprenticeship Mathematics 20

Outcome: WA20.1

Expand and apply understanding of the preservation of equality including solving problems that involve the manipulation and application of formulae including volume and capacity, surface area, slope and rate of change, simple interest and finance charges.

Indicators:

(It is intended that this outcome be integrated with other outcomes throughout the course.)

a. Verify, using examples, and explain why different forms of the same formula are equivalent.
b. Verify whether given forms of the same formula are equivalent and justify the conclusion.
c. Describe, using examples, how a given formula is used in a trade or an occupation.
d. Create, solve, and verify the reasonableness of solutions to situational questions that involve the use of a formula relevant to self, family, or community.
e. Analyze solutions to questions that involve formulae to verify the preservation of equality and correct if necessary.
f. Solve with or without the use of technology situational questions that involve the application of a formula that:
   - do not require manipulation
   - do require manipulation.

Outcome: WA20.2

Demonstrate the ability to analyze puzzles and games that involve numerical reasoning using problem solving strategies.

Indicators:

(It is intended that this outcome be integrated throughout the course by using puzzles and games such as *cribbage, magic square, Yahtzee, Sudukos and Kakuro.)*

a. Determine, explain, and verify strategies to solve a puzzle or to win a game such as:
   - guess and check
   - look for a pattern
   - make a systematic list
   - draw or model
   - eliminate possibilities
   - formulate and simplify a problem that is similar to the original problem
   - work backwards
   - develop alternative approaches.

b. Observe and analyze errors in solutions to puzzles or in strategies for winning games to identify errors and explain the reasoning.
c. Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Outcome: WA20.3

Extend and apply understanding of surface area, volume and capacity using concrete and pictorial models and symbolic representations (SI or imperial units of measurement).

It is intended that units of measure should be those that are appropriate to the workplace or apprenticeship context being considered. These units include:

• metres, grams, litres, and seconds along with appropriate prefixes such as kilo, hecto, deci, centi, and milli, as well as hectare, tonne, and degrees Celsius (SI system).

• inch, foot, board foot, yard, mile, acre, teaspoon, tablespoon, cup, pint, quart, gallon, bushel, ton, and degrees Fahrenheit (British and US systems where appropriate).

Indicators:

a. Observe, analyze, generalize, and explain using examples including nets, the relationships between area, surface area and volume.
b. Observe, analyze, and compare volume and capacity using examples.
c. Critique the statement: "Volume and capacity represent the same attribute to measure and the same units of measure can be used for either volume or capacity".
d. Identify and describe situations in which given SI or imperial volume or capacity units would be used.
e. Justify and compare the choice of referents for surface area, volume and capacity measurements in both SI and imperial units.
f. Justify and apply strategies including use of personal referents to estimate the surface area and volume of 3-D objects, and the capacity of containers.
g. Solve situational questions that involve:
    - the volume of 3-D objects and composite 3-D objects in a variety of contexts
    - the capacity of containers.
h. Convert given volume, surface area and capacity measurements:
    - expressed in one SI unit to another SI unit (including units squared and units cubed)
    - expressed in one imperial unit to another imperial unit (including units squared and units cubed).
i. Determine the surface area and volume of prisms, cones, cylinders, pyramids, spheres and composite 3-D objects, using a variety of measuring tools such as rulers, tape measures, calipers and micrometers and explain the strategy used.
j. Determine the capacity of prisms, cones, pyramids, spheres and cylinders, using a variety of measuring tools and methods, such as graduated cylinders, measuring cups, measuring spoons and displacement and explain the strategy and formula used.
k. Analyze and generalize the relationship between the volumes of:
    - cones and cylinders with the same base and height
    - pyramids and prisms with the same base and height.
l. Analyze and illustrate, using examples, the effect of dimensional changes on area, surface area and volume.
m. Solve using a variety of strategies including the manipulation of formulae situational questions that involve:
    - the surface area of 3-D objects, including spheres
    - the volume of 3-D objects, including composite 3-D objects
    - the capacity of containers.

Outcome: WA20.4

Solve problems that involve at least two right triangles.

Indicators:

a. Analyze and sort a set of illustrations of triangles in a given context according to whether they are right triangles or not and justify the sort.
b. Apply personal strategies to determine with justification if solutions to problems that involve two or three right triangles are reasonable.
c. Sketch representations of 2-D shapes or 3-D objects from given contexts or situations.
d. Apply personal strategies including the primary trigonometric ratios to solve situational questions that involve:
    - angles of elevation or angles of depression and explain the reasoning.
    - more than two right triangles and explain the reasoning.

Outcome: WA20.5

Extend and apply understanding of 3-D objects including:

- **top, bottom and side views**
- **exploded views**
- **component parts**
- **scale diagrams.**

Indicators: *(It is intended that the concepts in this outcome be explored and developed through relevant contexts such as flat-packed furniture or sewing patterns.)*

a. Describe and sketch or draw with or without technology and using a variety of strategies including isometric paper :
   - 2-D representations of 3-D objects relevant to self, family and community
   - 3-D objects, given the top, front and side views
   - a one-point perspective view of given 3-D objects
   - the components of given exploded diagrams, and explain their relationship to the original 3-D objects
   - 2-D representations of 3-D objects, given their exploded view.

b. Draw to scale :
   - top, front and side views of given actual 3-D objects
   - the components of a 3-D object.

c. Construct models of 3-D objects, given the top, front and side views.
d. Analyze a set of views of a 3-D objects to determine if they represent a given object and explain the reasoning.
e. Identify and justify the point of perspective of given one-point perspective drawings of 3-D objects.

Outcome: WA20.6

Demonstrate understanding of personal budgets and their importance for financial planning.

Indicators:

a. Identify and justify income and expenses that could be included in a personal budget.
b. Explain considerations that must be made when developing a budget, e.g., prioritizing, recurring and unexpected expenses.
c. Create a personal budget based on given income and expense data or from personally collected data and justify the reasoning.
d. Modify a budget to achieve a set of personal goals.
e. Investigate and analyze, with or without technology, “what if …” questions related to personal budgets.
f. Explain using examples the advantages of creating personal budgets.

Outcome: WA20.7

Demonstrate understanding of compound interest.

Indicators:

a. Solve situational questions that involve simple interest, given three of the four values in the formula I=Prt and explain the reasoning.
b. Analyze and generalize the relationship between simple interest and compound interest.
c. Solve, using a formula, situational questions that involve compound interest.
d. Explain, using examples, the effect of changing different factors on compound interest such as different compounding periods, different interest rates, and starting at a younger age,
e. Estimate, using the Rule of 72, the time required for a given investment to double in value and explain the reasoning.

Outcome: WA20.8

Demonstrate understanding of financial institution services used to access and manage personal finances, including credit options.

a. Research and present orally with the aid of visuals (electronic or other), various types of banking services available from various financial institutions, such as online services, different types of accounts, telephone banking, mobile banking , ATM banking or credit cards.
b. Analyze given or personal situations to determine the type of account that best meets the needs of the criteria for each of the situations.
c. Research and explain various automated teller machine (ATM) service charges.
d. Describe the advantages and disadvantages of:
   - online banking.
   - debit card purchases
   - different types of credit options, including bank and store credit cards, personal loans, lines of credit, overdraft.
e. Describe ways that try to ensure the security of personal and financial information; e.g., passwords, encryption, protection of personal identification number (PIN) and other personal identity information.
f. Research, compare and report on credit card options from various companies and financial institutions.

g. Analyze credit options related to the use of credit, such as service charges, interest, payday loans and sales promotions, to make informed decisions and plans and explain the reasoning.
h. Describe strategies to use credit effectively, such as negotiating interest rates, planning payment timelines, reducing accumulated debt and timing purchases.
i. Solve situational questions that involve credit linked to sales promotions, credit cards or loans.
j. Critique the statement, "It is always better to have the lowest possible limit on a credit card."

Outcome: WA20.9

Demonstrate concretely, pictorially, and symbolically understanding of slope with respect to:

- **rise over run**
- **rate of change**
- **solving problems.**

with and without the use of technology.

a. Research and present contexts that involve slope including the mathematics involved, e.g. ramps, roofs, road grade, flow rates within a tube, skateboard parks, ski hills.
b. Analyze and generalize relationships between slopes in given contexts such as 3:1 and a 1:3 roof pitch or slopes for downhill skiing and snowboarding and explain implications of each slope including safety and functionality.
c. Describe conditions under which a slope will be either 0 or undefined and explain the reasoning.
d. Critique the statement, "It requires less effort to independently use a wheel chair to climb a ramp of a certain height that has a slope of 1:12 rather than a slope of 1:18.
e. Justify, using examples and illustrations:
    - slope as rise over run
    - slope as rate of change
f. Analyze slopes of objects, such as ramps or roofs, to determine if the slope is constant and explain the reasoning.
g. Analyze, generalize and explain using illustrations, the relationship between slope and angle of elevation e.g., for a ramp with a slope of 7:100, the angle of elevation is approximately 4 degrees or pitch of a roof, grade on a road, slope in pipes for plumbing, azimuth in the sky.
h. Solve situational questions that involve slope or rate of change, verify and explain why solutions are reasonable or not.

Outcome: WA20.10

Extend and apply proportional thinking to solve problems that involve unit analysis and scale.

a. Explain the process of unit analysis used to solve a problem (e.g., given km/h and time in hours, determine how many km; given revolutions per minute, determine the number of revolutions per second).
b. Solve situational questions, using unit analysis and explain the reasoning.
c. Explain, using examples, how unit analysis and proportional reasoning are related; e.g., to change km/h to km/min, multiply by 1h/60min because hours and minutes are proportional (constant relationship).
d. Solve, using personal strategies such as using proportions or tables, situational questions that involve conversions of units within and between SI and/or imperial systems of measurement, e.g., km to m or km/h to ft/sec.
e. Describe, using examples, contexts in which scale representations are used.
f. Determine, using proportional reasoning, the dimensions of objects given scale drawings or models.
g. Construct models of 3-D objects, given the scale.
h. Draw, with or without technology, a scale diagram of 3-D objects.
i. Solve situational questions that involve scale and explain the reasoning.
j. Explain the importance of scale in mathematical drawings and/or in situational applications.

Outcome: WA20.11

Extend and apply understanding of representing data using graphs including:

- **bar graphs**
- **histograms**
- **line graphs**
- **circle graphs.**

a. Pose questions that could be answered using histograms, construct the histogram and draw conclusions.
b. Analyze and compare sets of bar graphs and histograms.
c. Analyze sets of data in a variety of contexts to determine and create with or without technology possible graphs that could be used to represent the data and explain the advantages and disadvantages of each graph.
d. Critique the statement, "All histograms are bar graphs and all bar graphs are histograms".

e. Analyze graphs including bar graphs, histograms, line graphs and circle graphs to determine and describe trends.
f. Explain, using examples, :
   - how the same graph can be used to justify more than one conclusion
   - how different graphic representations of the same data set can be used to emphasize a point of view.
   - what happens if the graph is drawn to the wrong scale,
   - the graph selected is not reflective of the data set.
g. Solve, using a variety of strategies including interpolation and extrapolation, contextual problems that involve data analysis and graphs.

Foundations of Mathematics 20

Outcome: FM20.1

Demonstrate understanding of the mathematics involved in an historical event or an area of interest.

Indicators:

Develop a rubric or other scoring schema for the assessment of the research and presentation.

a. Collect primary or secondary data (quantitative or qualitative) related to the topic.
b. Assess the accuracy, reliability, and relevance of the primary or secondary data (quantitative/qualitative) collected by:
   - identifying examples of bias and points of view
   - identifying and describing the data collection methods
   - determining whether or not the data are relevant
   - determining whether or not the data are consistent with information obtained from other sources on the same topic.
c. Interpret data, using statistical methods if applicable.
d. Identify controversial issues, if any, and present multiple sides of the issues with supporting data.
e. Organize and create a presentation (oral, written, multimedia, etc.) of the research findings and conclusions.

Outcome: FM20.2

Demonstrate understanding of inductive and deductive reasoning including:

- **analyzing conjectures**
- **analyzing spatial puzzles and games**
- **providing conjectures**
- **solving problems.**

Indicators:

Note: It is intended that:

- proofs NOT be limited to the two column proof style
- analysis and conjectures related to spatial puzzles and games be incorporated throughout the course.

a. Make conjectures by observing patterns and identifying properties, and justify the reasoning.
b. Provide examples of how inductive reasoning might lead to false conclusions.
c. Critique the following statement “Decisions can be made and actions taken based upon inductive reasoning”.
d. Identify situations relevant to self, family, or community involving inductive and/or deductive reasoning.
e. Prove algebraic number relationships, such as divisibility rules, number properties, mental mathematics strategies, or algebraic number tricks using deductive reasoning.
f. Prove conjectures using deductive reasoning.
g. Analyze an argument for its validity.
h. Identify errors in proofs that lead to incorrect conclusions (e.g., a proof that ends with 2 = 1).
i. Solve situational questions that involve inductive or deductive reasoning.
j. Determine, explain, and verify strategies for solving puzzles or winning games, such as:
   - guess and check
   - analyze a pattern
   - make a systematic list
   - create a drawing or model
   - eliminate possibilities
   - solve simpler problems
   - work backward.
k. Create a variation of a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Outcome: FM20.3

Expand and demonstrate understanding of proportional reasoning related to:

- **rates**
- **scale diagrams**
- **scale factor**
- **area**
- **surface area**
- **volume.**

Indicators:

a. Identify and describe situations relevant to one's self, family, or community that involve proportional reasoning.
b. Create non-symbolic representations for rates, including pictures and graphs.
c. Describe situations in which a given rate might occur.
d. Explain the meaning of rates given in context, such as the arts, commerce, the environment, medicine, or recreation.
e. Solve situational questions that require the use of proportional reasoning, including those that involve the isolation of a variable.
f. Analyze situations in which unit rates can be determined and suggest reasons why the rates would or would not be used to make decisions in each situation (i.e., are other factors in the situation outweighing the importance of the mathematical calculations?).
g. Explain, using examples, the relationship between the slope of a graph and a rate.
h. Identify and explain the effect of factors within given situations that could influence a particular rate.
i. Solve situational questions involving rates, including unit rates.
j. Identify and describe situations relevant to one's self, family, or community that involve scale diagrams of 2-D shapes and 3-D objects and determine the scale factor for the situations.
k. Develop, generalize, explain, and apply strategies for solving situational questions based upon scale diagrams of 2-D shapes and 3-D objects, including the determining of scale factors and unknown dimensions.
l. Draw, with or without the use of technology, a scale diagram of a 2-D shape relevant to self, family, or community to a specified scale factor (enlargement or reduction).
m. Solve situational problems involving scale diagrams of 2-D shapes and 3-D objects.
n. Determine relationships between scale factor and area of 2-D shapes or surface area of 3-D objects; and scale factor, surface area, and volume of 3-D objects.
o. Develop, generalize, explain, and apply strategies for determining scale factors, areas, surface areas, or volumes given the scale factor or the ratio of areas, surface areas, or volumes of 2-D shapes and 3-D objects.
p. Explain, with justification, the effect of a change in scale factor on the area of a 2-D shape or the surface area or volume of a 3-D object.
q. Solve situational questions that involve scale factors, areas, surface areas, and volumes, including ones that require the manipulation of formulas.

Outcome: FM20.4

Demonstrate understanding of properties of angles and triangles including:

- **deriving proofs based on theorems and postulates about congruent triangles**
- **solving problems.**

Indicators:

Note: It is intended that students be allowed to explore and use different styles of proofs, including paragraph form and two-column proofs. The emphasis should be on the logic used and the communication of that logic and not on the format of the proofs themselves.

a. Identify and describe situations relevant to self, family, or community that involve parallel lines cut by transversals.
b. Develop, generalize, explain, apply, and prove relationships between pairs of angles formed by transversals and parallel lines, with and without the use of technology.
c. Prove and apply the relationship relating the sum of the angles in a triangle.
d. Generalize, using inductive reasoning, a rule for the relationship between the sum of the interior angles and the number of sides (n) in a polygon, with or without technology.
e. Apply knowledge of angles formed by parallel lines and transversals to identify and correct errors in a given proof.
f. Explore and verify whether or not the angles formed by non-parallel lines and transversals create the same angle relationships as those created by parallel lines and transversals.
g. Solve situational problems that involve:
    - angles, parallel lines, and transversals
    - angles, non-parallel lines, and transversals
    - angles in triangles
    - angles in polygons.
h. Develop, generalize, explain, and apply strategies for constructing parallel lines.

Outcome: FM20.5

Demonstrate understanding of the cosine law and sine law (including the ambiguous case).

Indicators:

Note: This outcome is very similar to P20.5 from Pre-calculus 20. The difference is that in Foundations of Mathematics 20, students are expected to explain steps in a given proof for either the cosine law or the sine law, but not to generate the proof on their own.

a. Identify and describe situations relevant to self, family, or community that involve triangles without a right angle.
b. Develop, generalize, explain, and apply strategies for determining angles or side lengths of triangles without a right angle.
c. Draw diagrams to represent situations in which the cosine law or sine law could be used to solve a question.
d. Explain the steps in a given proof of the sine law or cosine law.
e. Illustrate and explain how one, two, or no triangles could be possible for a given set of measurements for two side lengths and the non-included angle in a proposed triangle.
f. Develop, generalize, explain, and apply strategies for determining the number of solutions possible to a situation involving the ambiguous case.
g. Solve situational questions involving triangles without a right angle.

Outcome: FM20.6

Demonstrate an understanding of normal distribution, including standard deviation and z-scores.

Indicators:

a. Identify situations relevant to self, family, or community in which standard deviation and the normal distribution are used and explain the meaning and relevance of each.
b. Explain the meaning and purpose of the properties of a normal curve, including mean, median, mode, standard deviation, symmetry, and area under the curve.
c. Calculate, using technology, the population standard deviation of a data set.
d. Critique the statement “Every set of data will correspond to a normal distribution”.
e. Analyze a data set to determine if it approximates a normal distribution.
f. Compare the properties of two or more normally distributed data sets and explain what the comparison tells you about the situations that the sets represent.
g. Explain, using examples that represent multiple perspectives, the application of standard deviation for making decisions in situations such as warranties, insurance, or opinion polls.
h. Solve situational questions that involve the interpretation of standard deviations to make decisions.
i. Determine, with or without technology, and explain the meaning of the z-score for a given value in a normally distributed data set.
j. Pose and solve situational questions relevant to self, family, or community that involve normal distributions and z-scores.

Outcome: FM20.7

Demonstrate understanding of the interpretation of statistical data, including:

- **confidence intervals**
- **confidence levels**
- **margin of error.**

Indicators:

Note: It is intended that the focus of this outcome be on interpretation of data rather than on statistical calculations.

a. Identify and explain the significance of the confidence interval, margin of error, or confidence level stated with respect to statistical data relevant to self, family, or community.
b. Explain how confidence levels, margins of error, and confidence intervals can be impacted by the size of the random sample used.
c. Make inferences and decisions with justification about a population from sample data using confidence intervals.
d. Provide and critique examples from print or electronic media in which confidence intervals and confidence levels are used to support a particular position.
e. Support a position or decision relevant to self, family, or community by analyzing statistical data, as well as considering other factors.

Outcome: FM20.8

Demonstrate understanding of systems of linear inequalities in two variables.

Indicators:

a. Identify situations relevant to self, family, or community which could be described using a system of linear inequalities in two variables.

b. Develop, generalize, explain, and apply strategies for graphing and solving systems of linear inequalities, including justification of the choice of solid or broken lines.
c. Develop, generalize, explain, and apply strategies for verifying solutions to systems of linear inequalities, including the use of test points.
d. Explain, using examples, the meaning of the shaded region in the graphical solution of a system of linear inequalities.
e. Write a system of linear inequalities for a given graph.
f. Match optimization questions and the graphs of sets of linear inequalities.
g. Apply knowledge of graphing of systems of linear inequalities and linear programming to solve optimization questions.

Outcome: FM20.9

Demonstrate an understanding of the characteristics of quadratic functions of the form $y = a(x - p)^2 + q$, including:

- **vertex**
- **intercepts**
- **domain and range**
- **axis of symmetry.**

Indicators:

Note: It is intended that the completion of the square not be required. The use of the terms zeros, roots, and intercepts should reflect the definitions of these terms in the provided glossary.

a. Identify situations and objects relevant to self, family, or community which could be described using a quadratic function.
b. Develop, generalize, explain, and apply strategies for determining the intercepts of the graph of a quadratic function, including factoring, graphing (with or without the use of technology), and use of the quadratic formula.
c. Conjecture and verify a relationship among the roots of an equation, the zeros of the corresponding function, and the x-intercepts of the graph of the function.
d. Explain, using examples, why the graph of a quadratic function may have zero, one, or two x-intercepts.
e. Write a quadratic equation in factored form given the zeros of a corresponding quadratic function or the x-intercepts of a corresponding quadratic function.
f. Develop, generalize, explain, and apply strategies (with or without the use of technology) to determine the coordinates of the vertex of the graph of a quadratic function.
g. Develop, generalize, explain, and apply a strategy for determining the equation of the axis of symmetry of the graph of a quadratic function when given the x-intercepts of the graph.
h. Develop, generalize, explain, and apply strategies for determining the coordinates of the vertex of the graph of a quadratic function and for determining if the vertex is a maximum or a minimum.
i. Generalize about and explain the effects on the graph of a quadratic function when the values for a, p, and q are changed.
j. Develop, generalize, explain, and apply strategies for determining the domain and range of a quadratic function.
k. Explain what the domain and range of a quadratic function tell about the situation that the quadratic function models.
l. Develop, generalize, explain, and apply strategies for sketching the graph of a quadratic function.
m. Solve situational questions involving the characteristics and graphs of quadratic functions.
n. Critique the statement "Any function that can be written in the form $y = a(x - p)^2 + q$ will have a parabolic graph."

Pre-Calculus 20

Outcome: PC20.1

Demonstrate understanding of the absolute value of real numbers and equations and functions involving the absolute value of linear and quadratic functions.

Indicators:

a. Provide examples relevant to one's life, family, or community that illustrate different situations in which quantities referenced are positive, negative, or an absolute value and justify.
b. Determine the distance of two real numbers of the form $\pm a$, $a \in R$, from 0 on a number line, and relate this to the absolute value of a ($|a|$).
c. Determine the absolute value of a real number.
d. Order, with justification, a set of real numbers that includes the absolute value of one or more of the quantities.
e. Explain, with the use of examples, how absolute value fits into the order of operations used on expressions involving real numbers.
f. Determine the value of numerical expressions involving absolute value(s).
g. Simplify expressions involving absolute value(s).
h. Analyze, describe, and explain the relationship between the graph of $y = f(x)$ and $y = |f(x)|$.

i. Create a table of values for y = |f(x)| given y = f(x).
j. Sketch the graph of y = |f(x)| given y = f(x) and explain the reasoning.
k. Develop and apply strategies for determining the intercepts, domain, and range of y = |f(x)| given the equation of the function or its graph.
l. Explain what the range of the function y = |f(x)| reveals about the graph of the function.
m. Develop, generalize, explain, and apply strategies for graphically determining (with and without the use of technology) the solution set of an equation involving absolute values of algebraic expressions.
n. Develop, generalize, explain, and apply strategies for algebraically determining the solution set of an equation involving absolute values of algebraic expressions.
o. Analyze and generalize conclusions about absolute value inequalities of the form |f(x)| < 0.
p. Identify and correct errors in a solution to an absolute value equation.
q. Solve situational questions involving absolute value functions or equations.
r. Analyze and generalize the relationship between | x | and $\sqrt{x^2}$ and between | f(x) | and $\sqrt{(f(x))^2}$.

Outcome: PC20.2

Expand and demonstrate understanding of radicals with numerical and variable radicands including:

- **computations**
- **solving equations (limited to square roots and one or two radicals).**

Indicators:

a. Develop, generalize, explain, and apply strategies for expressing an entire radical (with numerical or variable radicand) as a mixed radical.
b. Develop, generalize, explain, and apply strategies for expressing a mixed radical (with numerical or variable radicand) as an entire radical.
c. Order a set of real numbers which includes radical expressions with numerical radicands.
d. Develop, generalize, explain, and apply strategies for simplifying radical expressions (with numerical and/or variable radicands).
e. Develop, generalize, explain, and apply strategies for rationalizing the denominator of rational expressions with monomial or binomial denominators.
f. Describe the relationship between rationalizing a binomial denominator of a rational expression and the product of the factors of a difference of squares expression.
g. Verify and explain, using examples, that $(-x)^2 = x^2$, $\sqrt{x^2} = |x|$, and $\sqrt{x^2} = \pm x$.
h. Solve situational questions that involve radical expressions.
i. Develop, explain, and apply strategies for determining the values of a variable for which a given radical expression is defined.
j. Develop, explain, and apply strategies for determining non-permissible values (restrictions on values) for the variable in a radical equation.
k. Develop, explain, and apply algebraic strategies for determining and verifying the roots of a radical equation.
l. Explain why some roots determined in solving a radical equation are extraneous.
m. Model and solve situational questions that involve radical equations.

Outcome: PC20.3

Expand and demonstrate understanding of rational expressions and equations (up to and including degree 2 numerators and denominators) including:

- **equivalent forms of expressions**
- **operations on expressions**
- **solving equations that can be simplified to linear or quadratic equations.**

Indicators:

a. Develop, verify, explain, and apply strategies for determining equivalent rational expressions.
b. Compare the determining of equivalent rational expressions to determining equivalent rational numbers.
c. Verify, with explanation, whether or not a given value is permissible for a given rational expression.
d. Develop, explain, and apply strategies for determining the non-permissible values of a rational expression.
e. Develop, explain, and apply strategies for simplifying rational expressions.
f. Explain why the non-permissible values of a simplified rational expression must be stated as those of the original rational expression.
g. Apply understanding of rational expressions to locate and correct errors in the simplification of a rational expression.
h. Develop, verify, explain, and apply strategies for adding, subtracting, multiplying, and dividing rational expressions, including the determination of non-permissible values.
i. Compare the performing of operations on rational expressions to performing the same operations on rational numbers.

j. Develop, explain, and apply strategies for simplifying rational expressions that involve two or more operations on the rational expressions.
k. Develop, explain, and apply algebraic strategies for determining the solution, including non-permissible values, of equations involving rational expressions.
l. Explain why a value obtained in solving a rational equation may not be a solution of the equation.
m. Model and solve situational questions involving rational expressions.

Outcome: PC20.4

Expand and demonstrate understanding of the primary trigonometric ratios including the use of reference angles ($0° \leq \theta \leq 360°$) and the determination of exact values for trigonometric ratios.

Indicators:

a. Provide examples relevant to one's self, family, or community that illustrate the need to define a standard position for angles.
b. Sketch an angle in standard position given the measure of the angle.
c. Determine and justify, with or without sketching, the quadrant in which an angle in standard position terminates.
d. Determine the reference angle for an angle in standard position.
e. Analyze, describe, and generalize the relationship between the reference angles for angles (in standard positions) that are reflections of each other across both the x- and y- axes (e.g., 30° and 150°, or -60° and 60°).
f. Sketch an angle in standard position given a point P(x, y) on the terminal arm of the angle.
g. Develop, generalize, explain, and apply strategies for determining a point on the terminal arm of the angle in each quadrant that has the same reference angle as the angle with P(x, y) on its terminal arm.
h. Develop, explain, and apply strategies for determining the distance between the origin and a point P(x, y) on the terminal arm of an angle.
i. Develop, generalize, explain, and apply strategies for determining the value of $\sin\theta$, $\cos\theta$, and $\tan\theta$ when given a point P(x, y) on the terminal arm of θ.
j. Develop, generalize, explain, and apply strategies for determining $\sin\theta$, $\cos\theta$, and $\tan\theta$ for quadrantal angles.
k. Develop, generalize, explain, and apply strategies for determining the sign (without calculation or the use of technology) of $\sin\theta$, $\cos\theta$, or $\tan\theta$ for a given value of θ.
l. Develop, explain, and apply strategies for solving, for all values of θ, equations of the form $\sin\theta = a$ or $\cos\theta = a$, where $-1 \leq a \leq 1$, and equations of the form $\tan\theta = a$, where a is a real number.
m. Analyze 30°- 60°- 90° and 45°- 45°- 90° triangles to generalize about the relationship between pairs of sides in such triangles in relation to the angles.
n. Develop, generalize, explain, and apply strategies for determining the exact value of the sine, cosine, or tangent (without the use of technology) of an angle with a reference angle of 30°, 45°, or 60°.
o. Describe and generalize the relationships and patterns in and among the values of the sine, cosine, and tangent ratios for angles from 0° to 360°.
p. Create and solve a situational question relevant to one's self, family, or community which involves a trigonometric ratio.
q. Identify angles for which the tangent ratio does not exist and explain why.

Outcome: PC20.5

Demonstrate understanding of the cosine law and sine law, including the ambiguous case.

Indicators:

a. Provide a diagram or picture to illustrate a situation relevant to one's self, family, or community that involves a triangle without a right angle.
b. Develop, explain, and apply strategies for solving a non-right angle triangle using the primary trigonometric ratios.
c. Derive and explain a proof of the sine law or cosine law.
d. Provide an example of a situation relevant to one's self, family, or community that involves the need to consider the ambiguous case and provide a diagram or picture to illustrate the situation and explain why the ambiguous case needs to be considered.
e. Apply the sine law and/or cosine law to solve situational questions.
f. Critique the statement "For every possible pair of angles (whose sum is less than 180°) and line segment, a triangle can be constructed".
g. Critique the statement "The sine law and the cosine law only apply to non-right triangles".

Outcome: PC20.6

Expand and demonstrate understanding of factoring polynomial expressions including those of the form:

- $\mathbf{a^2x^2 - b^2y^2,\ a \neq 0,\ b \neq 0}$
- $\mathbf{a(f(x))^2 - b(f(x)) + c,\ a \neq 0}$
- $\mathbf{a^2(f(x))^2 - b^2(g(y))^2,\ a \neq 0,\ b \neq 0}$

where a, b, and c are rational numbers.

Indicators:

a. Develop, generalize, explain, and apply strategies for factoring polynomial expressions of the form:
   - $a^2x^2 - b^2y^2$, $a \neq 0$, $b \neq 0$, a and b are real numbers
   - $ca^2x^2 - cb^2y^2$, $a \neq 0$, $b \neq 0$, a, b, and c are real numbers
   - $a(f(x))^2 - b(f(x)) + c$, $a \neq 0$, a, b, and c are real numbers
   - $da(f(x))^2 - db(f(x)) + dc$, $a \neq 0$, a, b, c, and d are real numbers
   - $a^2(f(x))^2 - b^2(g(y))^2$, $a \neq 0$, $b \neq 0$, a and b are real numbers
   - $da^2(f(x))^2 - db^2(g(y))^2$, $a \neq 0$, $b \neq 0$, a, b, and d are real numbers

b. Verify, with explanation, whether or not a given binomial is a factor for a given polynomial.

Outcome: PC20.7

Demonstrate understanding of quadratic functions of the form $y=ax^2+bx+c$ and of their graphs, including:

- **vertex**
- **domain and range**
- **direction of opening**
- **axis of symmetry**
- **x- and y-intercepts.**

Indicators:

a. Generalize a rule from sets of graphs, using inductive reasoning, and explain about how different values of a (including 1, 0, and -1) transform the graph of $y = ax^2$.

b. Generalize a rule from sets of graphs, using inductive reasoning, and explain about how different values of q (including 0) transform the graph of $y = x^2 + q$.

c. Generalize a rule from sets of graphs, using inductive reasoning, and explain how different values of p (including 0) transform the graph of $y = (x - p)^2$.

d. Develop, generalize, explain, and apply strategies for determining the coordinates of the vertex, the domain and range, the axis of symmetry, x- and y- intercepts, and direction of opening of the graph of the function $f(x) = a(x-p)^2 + q$ without the use of technology.

e. Develop, explain, and apply strategies for graphing functions of the form $f(x) = a(x - p)^2 + q$ by applying transformations related to the values of a, p, and q.

f. Develop, explain, and apply strategies (that do not require graphing or the use of technology) for determining whether a quadratic function will have zero, one, or two x-intercepts.

g. Develop, explain, and apply strategies for writing a quadratic function in the form of $y = a(x - p)^2 + q$ that represents a given graph or set of characteristics of a graph.

h. Develop, generalize, explain, verify, and apply a strategy (including completing the square) for writing a quadratic function in the form $y = ax^2 + bx + c$ in the form $y = a(x - p)^2 + q$.

i. Using knowledge about completing the square, identify and correct errors in a given example of completing the square.

j. Develop, generalize, explain, and apply strategies for determining the coordinates of the vertex, the domain and range, the axis of symmetry, x- and y- intercepts, and direction of opening of the graph of a function in the form $y = ax^2 + bx + c$.

k. Sketch the graph of a quadratic function given in the form $y = ax^2 + bx + c$.

l. Write a quadratic function that models a given situation and explain any assumptions made.

m. Analyze quadratic functions (with or without the use of technology) to answer situational questions.

Outcome: PC20.8

Demonstrate understanding of quadratic equations including the solution of:

- **single variable equations**
- **systems of linear-quadratic and quadratic-quadratic equations in two variables.**

Indicators:

Note: It is intended that the quadratic equations be limited to those that correspond to quadratic functions.

a. Explain, using examples, the relationship among the roots of a quadratic equation, the zeros of the corresponding quadratic function and the x-intercepts of the graph of the quadratic function.

b. Derive the quadratic formula, using deductive reasoning.

c. Apply strategies for solving quadratic equations of the form $ax^2 + bx + c = 0$ including:
   - determining square roots
   - factoring
   - completing the square
   - applying the quadratic formula

- graphing its corresponding function, with and without the use of technology.

d. Explain different strategies for verifying the solution to a quadratic equation.
e. Explain, using examples, how the discriminant may be used to determine whether a quadratic equation has two, one, or no real roots; and relate this knowledge to the number of zeros that the corresponding quadratic function will have.
f. Apply knowledge of quadratic equations and functions to identify and correct any errors within a solution to a quadratic equation.
g. Solve situational questions involving the writing and solving of quadratic equations.
h. Match systems of linear-quadratic and quadratic-quadratic functions to situations.
i. Develop, generalize, explain, and apply strategies for solving systems of linear-quadratic and quadratic-quadratic functions, including:
   - graphically
   - algebraically
   - with the use of technology.
j. Explain the meaning of the intersection point of a system of linear-quadratic or quadratic-quadratic equations in terms of the situation being modeled.
k. Illustrate and explain how a system of linear-quadratic or quadratic-quadratic equations may have zero, one, two, or an infinite number of solutions.
l. Solve situational questions by using systems of linear-quadratic or quadratic-quadratic equations.

Outcome: PC20.9

Expand and demonstrate understanding of inequalities including:

- **one-variable quadratic inequalities**
- **two-variable linear and quadratic inequalities.**

Indicators:

a. Develop, generalize, explain, and apply strategies for determining the solution region for two-variable linear or two-variable quadratic inequalities.
b. Explain, using examples, how test points can be used to determine the solution region that satisfies a two-variable inequality.
c. Explain, using examples, when a solid or broken line should be used in the graphic solution of a two-variable inequality.
d. Explain what the solution region for a two-variable inequality means.
e. Solve a situational question that involves a two-variable inequality.
f. Develop, generalize, explain, and apply strategies, such as case analysis, graphing, roots and test points, or sign analysis, to solve one-variable quadratic inequalities.
g. Model and solve a situational question that involves a one-variable quadratic inequality.
h. Interpret the solution to a situational question that involves a one-variable quadratic inequality.

Outcome: PC20.10

Demonstrate understanding of arithmetic and geometric (finite and infinite) sequences and series.

Indicators:

a. Identify assumptions made in determining that a sequence or series is either arithmetic or geometric.
b. Provide an example of a sequence that follows an identifiable pattern, but that is neither arithmetic nor geometric.
c. Provide an example of an arithmetic or geometric sequence that is relevant to one's self, family, or community.
d. Generate arithmetic or geometric sequences from provided information.
e. Develop, generalize, explain, and apply a rule and other strategies for determining the values of t_1, a, d, n, or t_n in situational questions that involve arithmetic sequences.
f. Develop, generalize, explain, and apply a rule and other strategies for determining the values of t_1, a, d, n, or S_n in situational questions that involve arithmetic series.
g. Solve situational questions that involve arithmetic sequences and series.
h. Develop, generalize, explain, and apply a rule and other strategies for determining the values of t_1, a, r, n, or t_n in situational questions that involve geometric sequences.
i. Develop, generalize, explain, and apply a rule and other strategies for determining the values of t_1, a, r, n, or S_n in situational questions that involve geometric series.
j. Develop, generalize, and explain a rule and strategies for determining the sum of an infinite geometric series and apply this knowledge to the solving of situational questions.
k. Analyze a geometric series to determine if it is convergent or divergent and explain the reasoning.

Outcome: PC20.11

Demonstrate understanding of reciprocal functions of:

- **linear functions**
- **quadratic functions.**

Indicators:

a. Describe the relationship between a function and its reciprocal.
b. Apply knowledge of rational expressions to determine non-permissible values for reciprocal functions.
c. Analyze and describe the relationship between vertical asymptotes and non-permissible values.
d. Develop, explain, and apply strategies for graphing (with or without the use of technology) $y = \frac{1}{f(x)}$ given either the graph or equation for y = f(x) where f(x) is a polynomial of degree ≤ 2.
e. Develop, explain, and apply strategies for graphing (with or without the use of technology) y = f(x) given either the graph or equation for $y = \frac{1}{f(x)}$ where f(x) is a polynomial of degree ≤2.
f. . Sketch the graph of a function in the form $y = \frac{1}{f(x)}$
g. Analyze reciprocal functions to describe the end behaviour of the functions.

Mathematics 21 (Modified)

Outcome: M21.1

Extend and apply understanding of the preservation of equality by solving problems that involve the manipulation and application of formulae within home, money, recreation, and travel themes. [WA10.1 and WA20.1]

Indicators:

a. Verify whether given forms of the same formula are equivalent and justify the conclusion.
b. Describe, using examples, how a given formula is used in a home, money, recreation, and travel context.
c. Create, solve, and verify the reasonableness of solutions to questions that involve the use of a formula.
d. Analyze solutions to questions that involve formulae to verify the preservation of equality, correct if necessary, and explain the reasoning.
e. Solve, with or without the use of technology, questions that involve the application of a formula that:
   - does not require manipulation
   - does require manipulation.

Sample Formulae: *volume and capacity, surface area, slope and rate of change, primary trigonometric ratios, finance charges, and income.*

Outcome: M21.2

Demonstrate understanding of numerical reasoning and problem solving strategies by analyzing puzzles and games. [FM20.2 and WA20.2]

Indicators:

a. Make conjectures by observing patterns and identifying properties, and justify the reasoning.
b. Observe and analyze errors in solutions to puzzles or in strategies for winning games to identify and correct errors, if necessary, and explain the reasoning.
c. Solve questions that involve numerical reasoning.
d. Create a variation of a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Sample Games*: Cribbage, Magic Square, Yahtzee, Sudokos, Kakuro, Kaponk, Guesstamations, and Qwirkle.*

Outcome: M21.3

Extend and apply understanding of measures of central tendency to analyze data. [WA30.9]

Indicators:

a. Examine the distribution of a set of data, using smallest and largest value, frequency, value in the middle and patterns.
b. Explain, using examples, the advantages and disadvantages of each measure of central tendency.
c. Explain the appropriate use of measures of central tendency, including mean, mode, and median.
d. Calculate and interpret measures of central tendency, including mean, mode and median, to solve problems (e.g. batting averages, target heart rate, average temperature).
e. Compare two or more sets of data, using measures of central tendency.

Outcome: M21.4

Demonstrate understanding of slope.

Indicators:

a) Research and present contexts that involve slope including the mathematics involved (e.g., ramps, roofs, road grade, skateboard parks, ski hills, treadmill).

b) Critique the statement, "It requires less effort to independently use a wheelchair to climb a ramp of a certain height that has a slope of 1:12 rather than a slope of 1:18."

c) Justify, using examples and illustrations slope as rise over run.

d) Analyze slopes of objects, such as ramps or roofs, to determine if the slope is constant and explain the reasoning.

e) Analyze, generalize, and explain, using illustrations, the relationship between slope and angle of elevation (e.g., for a ramp (or pitch of a roof, grade of a road, slope in pipes for plumbing, azimuth in the sky) that has a slope of 7:100, the angle of elevation is approximately 4 degrees).

Outcome: M21.5

Demonstrate understanding of angles created by parallel, perpendicular, and transversal lines and solve problems within the home theme. [WA10.9]

Indicators:

a. Observe and sort pairs of lines as perpendicular, parallel, or neither, and justify the reasoning (e.g., tiling, ceiling tiles, flooring, framing, cutting a window frame).

b. Generalize, develop, explain, and apply relationships between pairs of angles formed by parallel lines and a transversal, including:
   - corresponding angles
   - vertically opposite angles
   - alternate interior angles
   - alternate exterior angles
   - interior angles on the same side of the transversal
   - exterior angles on the same side of the transversal.

c. Provide concrete and pictorial examples that show that there are no angle relationships (excluding vertically opposite angles) when two lines that are not parallel are crossed by a transversal.

Outcome: M21.6

Demonstrate understanding of primary trigonometric ratios (sine, cosine, and tangent) [WA10.8]

Indicators:

a. Describe the properties of a triangle.

b. Determine the missing angle in a triangle.

c. Observe a set of similar right triangles and analyze and draw conclusions about the ratios of the lengths, with respect to one acute angle of the:
   - side opposite to the side adjacent
   - side opposite to the hypotenuse
   - side adjacent to the hypotenuse.

a) Apply formulae for the primary trigonometric ratios (cosine, tangent, and sine).

b) Describe, using examples, how a trigonometric formula is used in the home context.

c) Analyze solutions to questions that involve primary trigonometric ratios to determine if they are reasonable and explain the reasoning.

Outcome: M21.7

Demonstrate and extend understanding of similarity and proportional reasoning related to scale factors, scale drawing, scale models, surface area, and volume. [FM20.3]

Indicators:

a. Explain how ratios and proportionality are related to similarity of shapes.

b. Explain how scale factor is related to similarity, ratios, and proportionality.

c. Draw enlargements and reductions to scale.

d. Determine the scale factor from scale drawings.

e. Describe the relationship between scale factors, scale drawings, and maps.

f. Interpret directions and analyze locations using scale factors and scale drawings of maps.

g. Determine distances represented on maps (e.g. provincial road map, local street map, Web-based maps), using given scales.

h. Explain the effect of a change in scale factor on the area of a 2-D shape or the surface area or volume of a 3-D object.

i. Draw a scale diagram of a 2-D shape to a specified scale factor (enlargement or reduction) and examine and describe the strategies used.
j. Draw a scale drawing of a familiar setting (e.g., classroom, bedroom, playground).
k. Manipulate concrete 3-D objects to identify, describe, and sketch top, front, and side views.
l. Analyze a set of views of 3-D objects to determine if they represent a given object and explain the reasoning.
m. Construct models of 3-D objects, given the top, bottom, and side views.
n. Design and construct a 3-D scale model of an object or space (e.g., bedroom, hockey rink, ice fishing shack, dog house).
o. Describe the relationship between the area of the base and the volume of a 3-D object.
p. Solve situational questions involving the volumes and surface areas of rectangular prisms, triangular prisms, and cylinders, and of related composite figure (e.g., refrigerator or freezer, soil, cement, gravel, grain bins, sheds).

Outcome: M21.8

Demonstrate understanding of budgets. [WA20.6]

Indicators:

a. Identify fixed and variable expenses that could be included in a personal budget.
b. Explain considerations that must be made when developing a budget (e.g., prioritizing, recurring and unexpected expenses).
c. Research the costs of expenses (e.g., bus pass, rent, phone, electricity, power, groceries) to create and justify a personal budget.
d. Analyze and modify a budget to achieve a set of personal goals.
e. Investigate and analyze, with or without technology, "what if …" questions related to personal budgets.
f. Explain the advantages and challenges of creating personal budgets.
g. Record and monitor purchases to determine personal expenditures.
h. Investigate, plan, design, and prepare a budget to solve home, recreation, or travel problems using appropriate technologies (e.g., design or decorating websites, design or drawing software, spreadsheet).
i. Create a monthly transportation budget that involves the fixed costs (e.g., licence fee, insurance) and variable costs (e.g., maintenance, fuel) of owning and operating a vehicle.

Outcome: M21.9

Demonstrate understanding of financial institution services. [WA20.7 and WA20.8]

Indicators:

a. Research various types of banking services available from various financial institutions, such as online services, different types of accounts, telephone banking, mobile banking, ATM banking, or cheques.
b. Consider the services that banking institutes and financial advisors offer to assist in personal budgeting.
c. Analyze the type of account that best meets the criteria for the provided examples and personal situations.
d. Research and explain various charges acquired when using chequing accounts, ATMs, and savings accounts.
e. Describe the advantages and disadvantages of online banking, debit card purchases, chequing accounts, and savings accounts.
f. Discuss the use of cheques and determine how to write them.
g. Describe methods taken to ensure the security of personal and financial information (e.g., passwords, encryption, protection of personal identification number (PIN) and other personal identity information) and their effectiveness.
h. Research and discuss various investment options, such as savings accounts, Canada Savings Bonds, Guaranteed Investment Certificates, term investments, RRSPs, and RESPs.
i. Determine simple interest, given three of the four values in the formula $I=Prt$ and explain the reasoning.
j. Determine compound interest using a formula.
k. Compare and contrast simple interest and compound interest.
l. Explain, using examples, the effect of changing different factors on compound interest (e.g., different amortization periods, interest rates, compounding periods, and terms).
m. Estimate, using the Rule of 72, the time required for a given investment to double in value and explain the reasoning.

Outcome: M21.10

Demonstrate understanding of financial decision making including analysis of renting, leasing, and buying on credit. [FM30.1 and WA30.6]

Indicators:

a. Define credit and determine its appropriate use.
b. Research and discuss various borrowing options, such as credit cards, loans, line of credit, and mortgage.
c. Develop an understanding of a credit rating.

d. Gather and interpret information about credit ratings, and describe the factors used to determine credit ratings and the consequences of a good or bad rating.
e. Research a variety of credit cards to compare the advantages, disadvantages, and promotions of various credit cards (e.g., financial institutions, store credit cards).
f. Discuss the effects of carrying an outstanding balance on a credit card at current interest rates.
g. Identify and compare an installment charge account (e.g., The Brick) or a thirty-day account (e.g., contractor's charge accounts).
h. Determine, using technology, the total cost of a compound interest loan from various institutions (e.g., banks, payday loans) under a variety of conditions (e.g., different amortization periods, interest rates, compounding periods, and terms).
i. Compare renting, leasing, and buying of large cost items and generate reasons for considering each choice.
j. Determine the costs related to renting and buying housing.
k. Research and present various options for purchasing or leasing a vehicle (oral, written, multimedia, etc.).
l. Justify a decision related to buying, leasing, or leasing to buy a vehicle, based on factors such as personal finances, intended use, maintenance, warranties, mileage, and insurance.
m. Solve, with or without technology, questions that involve the purchase, lease, or lease to purchase of a vehicle.
n. Collect and interpret information about the procedures and costs involved in insuring a vehicle (e.g., car, motorcycle, snowmobile) and the factors affecting insurance rates (e.g., gender, age, driving record, model of vehicle, use of vehicle), and compare the insurance costs for different vehicles.**analysis of renting, leasing, and buying** on credit.

Outcome: M21.11

Demonstrate understanding of the mathematics involved in an area of interest. [FM20.1]

Indicators:

a. Investigate and summarize tourist information around a location of interest (e.g., hours of operation, entry costs, transportation options, travel reviews, safety concerns).
b. Organize and create a presentation on the chosen location (e.g., pros and cons for visiting, travel brochure, video, guidebook).
c. Identify and describe situations, experiences, or locations around the area of interest that are relevant to self, family, or community.
d. Compare social justice issues that are present in the location of choice to those present in your community or another community.
e. Identify and explain cultural activities and/or views of mathematics related to the location of interest.
f. Identify and analyze cultural items related to the mathematics at the location of interest.
g. Identify controversial issues or historical events that are or have occurred at the location of interest.
h. Analyze the influences that historically significant events have had on the current field of mathematics.

Grade 12

Workplace and Apprenticeship Mathematics 30

Outcome: WA30.1

Analyze puzzles and games that involve logical reasoning using problem-solving strategies.

Indicators:

Note: This outcome is intended to be integrated throughout the course by using logical puzzles and games such as Chess, Sudoku, Mastermind, Nim, Reversi.

a. Determine, explain, and verify strategies to solve a puzzle or to win a game such as:
   - guess and check
   - look for a pattern
   - make a systematic list
   - draw or model
   - eliminate possibilities
   - formulate and simplify a problem that is similar to the original problem
   - work backwards
   - develop alternative approaches.

b. Observe and analyze errors in solutions to puzzles or in strategies for winning games, and explain the reasoning.
c. Create a variation on a puzzle or a game, and describe a strategy for solving the altered puzzle or winning the game.

Outcome: WA30.2

Demonstrate concretely, pictorially, and symbolically an understanding of limitations of measuring instruments including:

- **precision**
- **accuracy**
- **uncertainty**
- **tolerance.**

Indicators:

a. Explain, using concrete models and pictorial representations, the difference between precision and accuracy.
b. Analyze given contexts to generalize and explain why:
   - a certain degree of precision is required
   - a certain degree of accuracy is required.
c. Compare the degree of accuracy of two or more given instruments used to measure the same attribute.
d. Relate the degree (margin) of accuracy to the uncertainty of a given measure.
e. Analyze and justify the degree of precision and accuracy required in contextual problems.
f. Analyze given contexts to calculate maximum and minimum values, using a given degree (range) of tolerance.
g. Compare and describe, using examples, the limitations of measuring instruments used in a specific trade or industry, (e.g., tape measure versus Vernier caliper).
h. Create and solve situational questions that involve precision, accuracy, or tolerance, and explain the reasoning and the strategy used to arrive at the solution.

Outcome: WA30.3

Solve problems that involve the sine law and cosine law, excluding the ambiguous case.

Indicators:

a. Identify and describe the use of the sine law and cosine law in construction, industrial, commercial, and artistic applications.
b. Solve situational questions that involve the sine law or cosine law.

Outcome: WA30.4

Extend and apply understanding of the properties of triangles, quadrilaterals, and regular polygons to solve problems.

Indicators:

a. Analyze, generalize, and explain properties of polygons using illustrations, including:
   - triangles (isosceles, equilateral, scalene, and right triangles)
   - quadrilaterals in terms of angle measures, side lengths, diagonal lengths, and angles formed by the intersection of diagonals
   - regular polygons.
b. Explain, using examples, why a given property does or does not apply to certain polygons (e.g., the diagonals of a square are perpendicular, but the diagonals of a rectangle are not even though squares are rectangles).
c. Identify and explain applications of the properties of polygons in construction, industry, commerce, domestic, and artistic contexts.
d. Create and solve situational questions that involve the application of the properties of polygons.

Outcome: WA30.5

Extend and apply understanding of transformations on 2-D shapes and 3-D objects including:

- **translations**
- **rotations**
- **reflections**
- **dilations.**

Indicators:

a. Analyze original 2-D shapes and 3-D objects and their images to identify and justify the single transformation that was performed.
b. Draw the image of 2-D shapes given:
   - a single transformation including a translation, rotation, reflection, and justify why it is a translation, rotation, or reflection.
   - a combination of successive transformations and explain the reasoning.
c. Create designs using translations, rotations, and reflections in all four quadrants of a coordinate grid.
d. Analyze and describe designs that involve translations, rotations, and reflections in all four quadrants of a coordinate grid, and explain the reasoning.
e. Research and present, orally, in writing, or using multimedia, applications of transformations using examples and illustrations in construction, industrial, commercial, domestic, and artistic contexts.
f. Analyze and generalize the relationship between reflections and lines or planes of symmetry.
g. Explain how and why the concept of similarity can be used to determine if an image is a dilation of a given shape, and provide examples.
h. Determine whether or not given images are dilations of given shapes and explain the reasoning.
i. Draw, with or without technology, a dilation image for a given 2-D shape and 3-D object, and explain how the original 2-D shape or 3-D object and its image are proportional.

j. Solve contextual problems that involve transformations and explain the reasoning.

Outcome: WA30.6

Demonstrate understanding of options for acquiring a vehicle including:

- **purchasing without credit**
- **purchasing with credit**
- **leasing**
- **leasing to purchase.**

Indicators:

a. Research and present various options for purchasing or leasing a vehicle (oral, written, multimedia, etc.).
b. Justify a decision related to buying, leasing, or leasing to buy a vehicle, based on factors such as personal finances, intended use, maintenance, warranties, mileage, and insurance.
c. Solve, with or without technology, situational questions that involve the purchase, lease, or lease to purchase of a vehicle.

Outcome: WA30.7

Explore and critique the viability of small business options with respect to:

- **expenses**
- **sales**
- **profit or loss.**

Indicators:

a. Analyze small businesses such as a hot dog stand to identify and describe expenses, and explain factors, such as seasonal variations and hours of operation that might impact their profitability.
b. Research and describe feasible small business options for a given community.
c. Analyze a small business to generate options that might improve its profitability, and report to an audience.
d. Determine the break-even point for small businesses and explain the reasoning.

Outcome: WA30.8

Extend and apply understanding of linear relations including:

- **patterns and trends**
- **graphs**
- **tables of values**
- **equations**
- **interpolation and extrapolation**
- **problem solving.**

Indicators:

a. Analyze graphs, tables of values, number patterns, and/or equations to generalize characteristics of linear relations.
b. Analyze relations in sets of graphs, tables of values, number patterns, and/or equations to sort according to whether the relations are linear or nonlinear.
c. Represent and explain the linear relation in given contexts, including direct or partial variations, using equations, tables of values, and/or sketches of graphs.
d. Analyze contexts and their graphs to explain why the points on the graphs should or should not be connected.
e. Create, with or without technology, a graph to represent a data set, including scatterplots.
f. Analyze graphs of data sets, including scatterplots, to generalize and describe the trends.
g. Analyze and sort a set of scatterplots according to the trends represented (linear, nonlinear, or no trend).
h. Critique statements such as "Trends allow us to predict exactly what will happen in the near future."
i. Solve situational questions that require interpolation or extrapolation of information.
j. Relate slope and rate of change to linear relations.
k. Match given contexts with their corresponding graphs and explain the reasoning.
l. Create and solve situational problems that involve the application of a formula for a linear relation.

Outcome: WA30.9

Extend and apply understanding of measures of central tendency to solve problems including:

- **mean**
- **median**
- **mode**
- **weighted mean**
- **trimmed mean.**

Indicators:

a. Explain, using examples, the advantages and disadvantages of each measure of central tendency.
b. Determine the mean, median, and mode for sets of data and explain the reasoning.
c. Analyze calculations of measures of central tendency to identify and correct errors if necessary.
d. Critique statements such as "It is not possible to have a set of data which displays a mean, a median, and a mode of the same value."
e. Identify the outlier(s) in a set of data, explain why they are outliers, and discuss their effect on the mean, median, and mode of that data set.
f. Calculate the trimmed mean for sets of data and justify the removal of the outliers.
g. Explain, using examples such as course marks, why some data in a set would be given a greater weighting in determining the mean.
h. Calculate the mean of a set of numbers after allowing the data to have different weightings (weighted mean) and explain the reasoning.
i. Explain, using examples from print and other media, how and why measures of central tendency and outliers are used to provide different interpretations of data.
j. Create and solve situational questions that involve measures of central tendency.

Outcome: WA30.10

Demonstrate understanding of percentiles.

Indicators:

a. Explain, using examples, percentile ranks in a context.
b. Explain how and why decisions can be made based on percentile rank.
c. Compare, using examples, percent and percentile rank.
d. Analyze and generalize the relationship between median and percentile.
e. Solve situational questions that involve percentiles and percentile charts.
f. Critique statements such as "Predictions based on percentile ranks are always 100% accurate."

Outcome: WA30.11

Extend and apply understanding of probability. [C, CN, PS, R]

a. Research and present orally, in writing, or using multimedia, applications of probability (e.g., medication, warranties, insurance, lotteries, weather prediction, 100-year flood, failure of a design, failure of a product, vehicle recalls, approximation of area).
b. Calculate the probability of an event based on a data set, (e.g., determine the probability of a randomly chosen light bulb being defective).
c. Express given probabilities as fractions, decimals, percentages, and using words.
d. Analyze, generalize, and compare odds and probability including part-whole and part-part relationships.
e. Determine the probability of an event, given the odds for or against.
f. Explain, using examples, how decisions may be based on a combination of theoretical probability calculations, results of experimental probability, and subjective judgments.
g. Solve situational questions that involve probability.
h. Critique statements such as, "It is not possible to express odds as fractions".

Foundations of Mathematics 30

Outcome: FM30.1

Demonstrate understanding of financial decision making including analysis of:

- **renting, leasing, and buying**
- **credit**
- **compound interest**
- **investment portfolios.**

Indicators:

a. Compare the advantages and disadvantages of simple interest and compound interest.
b. Identify and describe situations that involve compound interest.
c. Graph and compare the total interest paid or earned over different compounding periods for the same annual interest rate, principal, and time.
d. Develop, generalize, explain, and apply strategies for determining the total interest to be paid on a loan given the principal, interest rate, and number of compounding periods for the loan.
e. Determine, using technology, the total cost of a loan under a variety of conditions (e.g., different amortization periods, interest rates, compounding periods, and terms).
f. Solve contextual problems that involve compound interest.

g. Analyze, using technology, different credit options that involve compound interest, including bank and store credit cards and special promotions, and provide justifications for the credit option.
h. Identify and describe examples of assets that appreciate or depreciate relevant to one's self, family, and community.
i. Compare renting, leasing, and buying of large cost items and generate reasons for considering each choice.
j. Solve situational questions related to the costs of renting, leasing, and buying (including questions that require formula manipulation).
k. Solve, using technology, situational questions that involve cost-and-benefit analysis.
l. Analyze the strengths and weaknesses of two or more investment portfolios, and make recommendations for selection based upon this analysis.
m. Determine, using technology, the total value of an investment when there are regular contributions to the principal.
n. Graph and compare the total value of an investment with and without regular contributions.
o. Apply the Rule of 72 to solve investment problems and explain the limitations of the rule.
p. Investigate and report possible investment strategies that could be used to achieve a financial goal.
q. Compare the advantages and disadvantages of long-term and short-term investment options.
r. Investigate and compare small investments over a long term and larger investments over a shorter term.

Outcome: FM30.2

Demonstrate understanding of inductive and deductive reasoning including:

- **analysis of conditional statements**
- **analysis of puzzles and games involving numerical and logical reasoning**
- **making and justifying decisions**
- **solving problems.**

Indicators:

a. Develop, generalize, verify, explain, and apply strategies to solve a puzzle or win a game such as:
   - guess and check
   - look for a pattern
   - make a systematic list
   - draw or model
   - eliminate possibilities
   - simplify the original problem
   - work backwards to develop alternative approaches.
b. Identify and correct errors in a solution to a puzzle or in a strategy to win a game.
c. Create a variation on a puzzle or game and describe a strategy for solving the puzzle or winning the game.
d. Analyze an "if-then" statement, make a conclusion, and explain the reasoning.
e. Make and justify decisions related to "what-if?" questions, in contexts such as probability, finance, sports, games, or puzzles, with or without technology.
f. Write the converse, inverse, and contrapositive of an "if-then" statement, determine if each new statement is true, and if it is false, provide a counterexample.
g. Critique statements such as "If an 'if-then' statement is known to be true, then its converse, inverse, and contrapositive also will be true".
h. Identify and describe situations relevant to one's self, family, and community in which a biconditional (if and only if) statement can be made.
i. Solve situational questions, using a graphic organizer such as a truth table or Venn diagram, that involve logical arguments based upon biconditional, converse, inverse, or contrapositive statements.

Outcome: FM30.3

Demonstrate understanding of set theory and its applications.

Indicators:

a. Provide and describe examples, relevant to one's self, family, and community, of empty set, disjoint sets, subsets, and universal sets.
b. Create graphic organizers such as Venn diagrams to display relationships within collected data or sets of numbers.
c. Name a specific region in a Venn diagram using the Boolean operators (or, and, not) or set notation, and explain in words what that region represents with respect to a specific situation.
d. Develop, generalize, and apply strategies for determining the elements in the complement, the intersection, or the union of sets.
e. Identify situations in which set theory is used and explain the role of set theory in each situation. (e.g., specific Internet searches, database queries, data analysis, games, and puzzles)
f. Solve situational questions that involve sets, including analysis of solutions for errors, using set notation where appropriate.

Outcome: FM30.4

Extend understanding of odds and probability.

Indicators:

a. Provide and explain the meaning of statements of probability and odds relevant to one's self, family, and community (e.g., statements of probability found in media, science, medicine, sports, sociology, and psychology).
b. Explain, using examples, the relationship between odds (part-part) and probability (part-whole).
c. Express odds as a probability and vice versa.
d. Determine the probability of, or the odds for and against, an outcome in a situation.
e. Explain, using examples, how decisions may be based on probability or odds and on subjective judgments.
f. Solve contextual problems that involve odds and probability.
g. Identify, describe, and justify examples of correct and incorrect use of the words "odds" or "probability" in daily language or in the media.
h. Critique the statement, "If the odds are close, then the probability of the two outcomes also is close".

Outcome: FM30.5

Extend understanding of the probability of two events, including events that are:

- **mutually exclusive**
- **non-mutually exclusive**
- **dependent**
- **independent.**

Indicators:

a. Provide examples of events relevant to one's self, family, and community that are mutually exclusive or non-mutually exclusive and explain the reasoning.
b. Analyze two events to determine if they are complementary.
c. Represent, using set notation or graphic organizers, mutually exclusive (including complementary) and non-mutually exclusive events.
d. Create and solve contextual problems that involve the probability of mutually exclusive events.
e. Create and solve contextual problems that involve the probability of non-mutually exclusive events.
f. Provide examples of events relevant to one's self, family, and community that are dependent or independent and explain the reasoning.
g. Determine the probability of an event, given the occurrence of a previous event.
h. Determine the probability of two dependent or two independent events.
i. Solve situational questions that involve determining the probability of dependent and independent events.

Outcome: FM30.6

Demonstrate understanding of combinatorics including:

- **the fundamental counting principle**
- **permutations (excluding circular permutations)**
- **combinations.**

Indicators:

a. Represent and solve counting problems using a graphic organizer.
b. Develop, generalize, explain, and apply the fundamental counting principle.
c. Identify and justify assumptions made in solving a counting problem.
d. Create and solve situational questions involving the fundamental counting principle.
e. Develop, generalize, explain, and apply strategies for determining the number of arrangements of *n* elements taken *n* at a time.
f. Explain, using examples, how factorials are related to the determination of permutations and combinations.
g. Determine, with or without technology, the value of a factorial.
h. Solve equations that involve factorials.
i. Develop, generalize, explain, and apply strategies for determining the number of permutations of *n* elements taken *r* at a time.
j. Develop, generalize, explain, and apply strategies for determining the number of permutations of *n* elements taken *n* at a time where some of the elements are not distinguishable.
k. Solve situational questions involving probability and permutations.
l. Explain, using examples, why order is or is not important when counting arrangements.
m. Identify examples relevant to one's self, family, and community where the number of possible arrangements would be of interest to explain why the order within any particular arrangement does or does not matter.
n. Develop, generalize, explain, and apply strategies for determining the number of combinations of *n* elements taken *r* at a time.

o. Critique statements such as "If a question about determining the number of possible arrangements gives the names of the people involved, then it is a permutation question".

Outcome: FM30.7

Demonstrate understanding of the representation and analysis of data using:

- **polynomial functions of degree ≤ 3**
- **logarithmic functions**
- **exponential functions**
- **sinusoidal functions.**

Indicators:

a. Analyze the graphs of polynomial functions and report on the characteristics of those graphs.
b. Graph data and determine, with the use of technology, the polynomial function that best approximates the data.
c. Develop, generalize, explain, and apply strategies for determining the characteristics of polynomial functions from their equations.
d. Identify the degree and sign of the leading coefficient for a polynomial function that would best approximate a set of data.
e. Analyze the graphs of exponential and logarithmic functions and report on the characteristics of those graphs.
f. Graph data and determine, with the use of technology, the exponential or logarithmic function that best approximates the data.
g. Develop, generalize, explain, and apply strategies for determining the characteristics of exponential and logarithmic functions from their equations.
h. Analyze the graphs of sinusoidal functions and report on the characteristics of those graphs.
i. Graph data and determine, with the use of technology, the sinusoidal function that best approximates the data.
j. Develop, generalize, explain, and apply strategies for determining the characteristics of sinusoidal functions from their equations.
k. Match equations of polynomial, logarithmic, exponential, and sinusoidal functions to their corresponding graphs.
l. Interpret graphs of polynomial, logarithmic, exponential, and sinusoidal functions to describe the situations that each function models and explain the reasoning.
m. Solve, using technology, situational questions that involve data that is best represented by graphs of polynomial, exponential, logarithmic, or sinusoidal functions and explain the reasoning.

Outcome: FM30.8

Research and give a presentation of a current event or an area of interest that requires data collection and analysis.

Indicators:

a. Develop a rubric or other scoring schema to assess the research and presentation.
b. Collect primary or secondary data (quantitative or qualitative) related to the topic.
c. Assess the accuracy, reliability, and relevance of the collected primary or secondary data (quantitative/qualitative) by:
   - identifying examples of bias and points of view
   - identifying and describing the data collection methods
   - determining whether or not the data is relevant
   - determining whether or not the data is consistent with information obtained from other sources on the same topic.
d. Interpret data, using statistical methods if applicable.
e. Identify controversial issues and present multiple sides of the issue with supporting data.
f. Organize and create a presentation (oral, written, multimedia, etc.) of the research findings and conclusions.

Pre-Calculus 30

Outcome: PC30.1

Extend understanding of angles to angles in standard position, expressed in degrees and radians.

Indicators:

a. Sketch angles in standard position including positive and negative degrees.
b. Investigate and describe the relationship between different systems of angle measurements, with emphasis on radians and degrees.
c. Sketch, in standard position, an angle measuring 1 radian.
d. Sketch, in standard position, any angle measuring $k\pi$ radians where $k \in Q$.
e. Develop and apply strategies for converting between angle measures in degrees and radians (exact value or decimal approximation).

f. Develop and apply strategies for determining all angles that are coterminal to an angle within a specified domain (in degrees and radians).
g. Develop, explain, and apply strategies for determining the general form for all angles that are coterminal to a given angle (in degrees and radians).
h. Explain the relationship between the radian measure of an angle in standard position and the length of the arc cut on a circle of radius *r*, and solve situational questions based on that relationship.

Outcome: PC30.2

Demonstrate understanding of the unit circle and its relationship to the six trigonometric ratios for any angle in standard position.

Indicators:

a. Derive the equation of a circle with centre (0,0) and radius *r*.
b. Derive the equation of the unit circle from the application of the Pythagorean theorem or the distance formula.
c. Develop and generalize the six trigonometric ratios in terms of *x, y,* and *r*, using a point that is the intersection of the terminal arm of an angle with the unit circle.
d. Develop, generalize, and apply strategies for determining the six trigonometric ratios for any angle given a point on the terminal arm of the angle.
e. Determine, with technology, the approximate value of the trigonometric ratios for any angle (in radians or degrees).
f. Develop, generalize, explain, and apply strategies, including using the unit circle or a reference triangle, for determining the exact trigonometric ratios for angles whose measures are multiples of 0°, 30°, 45°, 60°, 90° (when expressed in degrees), 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, or $\frac{\pi}{2}$ (when expressed in radians).
g. Explain and apply strategies (with or without the use of technology) to determine the measures, in degrees or radians, of the angles in a specified domain that have a particular trigonometric ratio value.
h. Explain and apply strategies to determine the exact values of the other trigonometric ratios, given the value of one trigonometric ratio in a specified domain.
i. Sketch a diagram to represent the context of a problem that involves trigonometric ratios.
j. Solve situational questions using trigonometric ratios.

Outcome: PC30.3

Demonstrate understanding of the graphs of the primary trigonometric functions.

Indicators:

a. Sketch, with or without technology, the graph of *y*=sin *x*, *y*=cos *x*, and *y*=tan *x*.
b. Determine and summarize the characteristics (amplitude, asymptotes, domain, period, range, and zeros) of the graphs of *y* = sin *x*, *y* = cos *x*, or *y* = tan *x*.
c. Develop, generalize, and explain strategies for determining the transformational impact of changing the coefficients a, b, c, and d in $y = a\sin b(x - c) + d$ and $y = a\cos b(x - c) + d$ on the graph of *y* = sin *x* and *y* = cos *x* respectively, including amplitude, asymptotes, domain, period, phase shift, range, and zeros.
d. Develop and apply strategies to sketch, without technology, graphs of the form $y = a\sin b(x - c) + d$ or $y = a\cos b(x - c) + d$.
e. Write equations for given graphs of sine or cosine functions.
f. Identify, with justification, a trigonometric function that models a situational question.
g. Explain how the characteristics of the graph of a trigonometric function relate to the conditions in a situational question.
h. Solve situational questions by analyzing the graph of trigonometric functions.

Outcome: PC30.4

Demonstrate understanding of first- and second-degree trigonometric equations.

Indicators:

a. Verify, with or without technology, whether or not a value is a solution to a particular trigonometric equation.
b. Develop and apply strategies for determining algebraically the exact form of the solution to a trigonometric equation.
c. Determine, using technology, the approximate solution in degrees and radians of a trigonometric equation in a restricted domain.
d. Explain the relationship between the general solution of trigonometric equations to the zeros of the related trigonometric functions limited to sine and cosine functions.
e. Determine, using technology, the general solutions for trigonometric equations.
f. Analyze solutions for given trigonometric equations to identify errors, and correct if necessary.

Outcome: PC30.5

Demonstrate understanding of trigonometric identities including:

- **reciprocal identities**
- **quotient identities**
- **Pythagorean identities**
- **sum or difference identities (restricted to sine, cosine, and tangent)**
- **double-angle identities (restricted to sine, cosine, and tangent)**

Indicators:

a. Explain the difference between a trigonometric identity and a trigonometric equation.
b. Verify numerically (using degrees or radians) whether or not a trigonometric statement is a trigonometric identity.
c. Critique statements such as "If three different values verify a trigonometric identity, then the identity is valid".
d. Determine, with the use of graphing technology, the potential validity of a trigonometric identity.
e. Determine the non-permissible values of a trigonometric identity.
f. Develop, explain, and apply strategies for proving trigonometric identities algebraically.
g. Explain and apply strategies for determining the exact value of a trigonometric ratio by using sum, difference, and double-angle identities.

Outcome: PC30.6

Demonstrate an understanding of operations on, and compositions of, functions.

Indicators:

a. Sketch the graph of a function that is the sum, difference, product, or quotient of two functions whose graphs are given.
b. Write the equation of a function that results from the sum, difference, product, or quotient of two or more functions.
c. Develop, generalize, explain, and apply strategies for determining the domain and range of a function that is the sum, difference, product, or quotient of two other functions.
d. Write a function as the sum, difference, product, or quotient (or some combination thereof) of two or more functions.
e. Develop, generalize, explain, and apply strategies for determining the composition of two functions:
   - $f(f(x))$
   - $f(g(x))$
   - $g(f(x))$
f. Develop, generalize, explain, and apply strategies for evaluating a composition of functions at a particular point.
g. Develop, generalize, explain, and apply strategies for sketching the graph of composite functions in the form:
   - $f(f(x))$
   - $f(g(x))$
   - $g(f(x))$

where the equations or graphs of *f*(x) and *g*(x) are given.

h. Write a function as a composition of two or more functions.
i. Write a function by combining two or more functions through operations on, and compositions of, functions.

Outcome: PC30.7

Extend understanding of transformations to include functions (given in equation or graph form) in general, including horizontal and vertical translations, and horizontal and vertical stretches.

Indicators:

a. Compare and analyze various graphs of transformations of the function $y = f(x)$, and generalize about the effect of the placement of different coefficients on the original graph of $y = f(x)$.
b. Develop, generalize, explain, and apply strategies for sketching transformations of the graph of $y = f(x)$ to give the graph of $y - k = af(b(x - h))$.
c. Write the equation of a function that has undergone specified vertical translations, horizontal translations, vertical stretches, and/or horizontal translations of the function $y = f(x)$ for which the equation is given.

Outcome: PC30.8

Demonstrate understanding of functions, relations, inverses and their related equations resulting from reflections through the:

- **x-axis**

- **y-axis**
- **line** $y = x$.

Indicators:

a. Generalize and apply the relationship between the coordinates of an ordered pair and the coordinates of the corresponding ordered pair that results from a reflection through the x-axis, the y-axis, or the line $y = x$.

b. Develop and apply strategies for sketching the reflection of a function $y = f(x)$ through the x-axis, the y-axis, or the line $y = x$ when the graph of $f(x)$ is given but the equation is not.

c. Develop and apply strategies for sketching the graphs of $y = -f(x)$, $y = f(-x)$, and $x = -f(y)$ when the graph of $f(x)$ is given and the equation is not.

d. Develop and apply strategies for writing the equation of a function that is the reflection of the function $f(x)$ through the x-axis, y-axis, or line $y = x$.

e. Develop and apply strategies for sketching the inverse of a relation, including reflection across the line $y = x$ and the transformation $(x, y) \Rightarrow (y, x)$.

f. Sketch the graph of the inverse relation, given the graph of the relation.

g. Develop, generalize, explain, and apply strategies for determining if one or both of a relation and its inverse are functions.

h. Determine what restrictions must be placed on the domain of a function for its inverse to be a function.

i. Critique statements such as "If a relation is not a function, then its inverse also will not be a function".

j. Determine the equation and sketch the graph of the inverse relation, given the equation of a linear or quadratic relation.

k. Explain the relationship between the domains and ranges of a relation and its inverse.

l. Develop and apply numeric, algebraic, and graphic strategies to determine if two relations are inverses of each other.

Outcome: PC30.9

Demonstrate an understanding of logarithms including:

- **evaluating logarithms**
- **relating logarithms to exponents**
- **deriving laws of logarithms**
- **solving equations**
- **graphing.**

Indicators:

a. Explain the relationship between powers, exponentials, logarithms, and radicals.

b. Express a logarithmic expression as an exponential expression and vice versa.

c. Determine, without technology, the exact value of a logarithm such as $\log_2 8$.

d. Explain how to estimate the value of a logarithm using benchmarks (e.g., since $\log_2 8 = 3$ and $\log_2 16 = 4$, $\log_2 9$ is approximately equal to 3.1).

e. Derive and explain the laws of logarithms.

f. Apply the laws of logarithms to determine equivalent expressions for given logarithmic statements.

g. Determine, using technology, the approximate value of a logarithmic expression (e.g., $\log_2 9$).

h. Solve exponential equations in which the bases are powers of one another.

i. Solve exponential equations in which the bases are not powers of one another.

j. Develop, generalize, explain, and apply strategies for solving logarithmic equations and verify the solutions.

k. Explain why a value obtained in solving a logarithmic equation may be extraneous.

l. Solve situational questions that involve exponential growth or decay, such as loans, mortgages, and investments.

m. Solve situational questions involving logarithmic scales, such as the Richter scale and pH scale.

n. Analyze graphs of exponential functions of the form $y = a^x, x > 0$ and report about the relationships between the value of a and the domain, range, horizontal asymptote, and intercepts.

o. Sketch, with or without the use of technology, the graphs of exponential functions of the form $y = a^x, x > 0$.

p. Explain the role of the horizontal asymptote for exponential functions.

q. Develop, generalize, explain, and apply strategies for sketching transformations of the graph of $y = a^x, x > 0$.

r. Analyze graphs of logarithmic functions of the form $y = \log_b x, b > 1$ and report about the relationships between the value of b and the domain, range, vertical asymptote, and intercepts.
s. Sketch, with or without technology, the graphs of logarithmic functions of the form $y = \log_b x, b > 1$.
t. Explain the role of the vertical asymptote for logarithm functions.
u. Develop, generalize, explain, and apply strategies for sketching transformations of the graph of $y = \log_b x, b > 1$.
v. Demonstrate graphically that $y = \log_b x, b > 1$ and $y = b^x, b > 0$ are inverses of each other.

Outcome: PC30.10

Demonstrate understanding of polynomials and polynomial functions of degree greater than 2 (limited to polynomials of degree ≤ 5 with integral coefficients).

Indicators:

a. Develop, generalize, explain, and apply long division for dividing polynomials by binomials of the form $x - a, a \in I$.
b. Compare long division of polynomial expressions by binomial expressions to synthetic division, and explain why synthetic division works.
c. Divide a polynomial expression by a binomial expression of the form $x - a, a \in I$ using synthetic division.
d. Explain the relationship between the linear factors of a polynomial expression and the zeros of the corresponding polynomial function.
e. Generalize, through inductive reasoning, the relationship between the remainder when a polynomial expression is divided by $x - a, a \in I$ and the value of the polynomial expression at $x = a$ (The Remainder Theorem).
f. Explain and apply the factor theorem to express a polynomial expression as a product of factors.
g. Categorize, with justification, a set of functions into polynomial functions and non-polynomial functions.
h. Analyze graphs of polynomial functions to determine the impact of changing the values of the constant term and leading coefficient in the equation of a polynomial function with respect to the graph of the function.
i. Generalize and apply strategies for graphing polynomial functions of an odd or even degree.
j. Explain the relationship between:
    - the zeros of a polynomial function
    - the roots of the corresponding polynomial equation
    - the x-intercepts of the graph of the polynomial function.
k. Explain and apply strategies for determining the behaviour of the graph of a polynomial function at zeros with different multiplicities.
l. Sketch, with or without the use of technology, the graph of a polynomial function.
m. Solve situational questions by modelling the situations with polynomial functions and analyzing the graphs of the functions.

Outcome: PC30.11

Demonstrate understanding of radical and rational functions with restrictions on the domain.

Indicators:

a. Sketch the graph of the function $y = \sqrt{x}$ using a table of values, and state the domain and range of the function.
b. Develop, generalize, explain, and apply transformations to the function $y = \sqrt{x}$ to sketch the graph of $y - k = a\sqrt{b(x - h)}$.
c. Sketch the graph of the function $y = \sqrt{f(x)}$ given the graph of the function $y = f(x)$, and compare the domains and ranges of the two functions.
d. Describe the relationship between the roots of a radical equation and the x-intercepts of the graph of the corresponding radical function.
e. Determine, graphically, the approximate solutions to radical equations.
f. Sketch rational functions, with and without the use of technology.
g. Explain the behaviour (shape and location) of the graphs of rational functions for values of the dependent variable close to the location of a vertical asymptote.
h. Analyze the equation of a rational function to determine where the graph of the rational function has an asymptote or a hole, and explain why.
i. Match a set of equations for rational and radical functions to their corresponding graphs.

j. Describe the relationship between the roots of a rational equation and the x-intercepts of the graph of the corresponding rational function.
k. Determine graphically an approximate solution to a rational equation.
l. Critique statements such as "Any value that makes the denominator of a rational function equal to zero will result in a vertical asymptote on the graph of the rational function".

Outcome: PC30.12

Demonstrate understanding of permutations, including the fundamental counting principle.

Indicators:

a. Develop and apply strategies, such as lists or tree diagrams, to determine the total number of choices or arrangements possible in a situation.
b. Explain why the total number of possible choices is found by multiplying rather than adding the number of ways that individual choices can be made.
c. Provide examples of situations relevant to self, family, and community where the fundamental counting principle can be applied to determine the number of possible choices or arrangements.
d. Create and solve situational questions that involve the application of the fundamental counting principle.
e. Count, using graphic organizers, the number of ways to arrange the elements of a set in a row.
f. Develop, generalize, explain, and apply strategies, including the use of factorial notation, to determine the number of permutations possible if *n* different elements are taken *n* or *r* at a time.
g. Explain why *n* must be greater than or equal to *r* in the notation ${}_nP_r$.
h. Solve equations that involve ${}_nP_r$ notation such as ${}_nP_2 = 30$.
i. Develop, generalize, explain, and apply strategies for determining the number of permutations possible when two or more elements in the set are identical (non-distinguishable).

Outcome: PC30.13

Demonstrate understanding of combinations of elements, including the application to the binomial theorem.

Indicators:

a. Explain, with examples, how to distinguish between situations that involve permutations and those that involve combinations.
b. Develop, generalize, explain, and apply strategies for determining the number of ways that a subset of *k* can be selected from a set of *n* different elements.
c. Develop, generalize, explain, and apply strategies to determine combinations of *n* different elements taken *r* at a time in situational questions.
d. Explain why *n* must be greater than or equal to *r* in the notation ${}_nC_r$ or $\binom{n}{r}$.
e. Prove or explain using examples ${}_nC_r = {}_nC_{n-r}$ or $\binom{n}{r} = \binom{n}{n-r}$.
f. Solve equations involving combinations (e.g., ${}_nC_2 = 15$ or $\binom{n}{2} = 15$).
g. Explore and describe patterns found within Pascal's triangle, including the relationship between consecutive rows.
h. Explore and describe the relationship between the coefficients of the terms in $(x+y)^n$, and the combinations.
i. Develop, generalize, explain, and apply strategies for expanding $(x+y)^n, n \leq 4$.
j. Develop, generalize, explain, and apply strategies for determining specific terms within a particular expansion of $(x+y)^n$ given $n \in \mathbb{N}$.

Calculus 30

Outcome: C30.1

Extend understanding of functions including:

- **algebraic functions (polynomial, rational, power)**
- **transcendental functions (exponential, logarithmic, trigonometric)**
- **piecewise functions, including absolute value.**

Indicators:

a. Identify functions as algebraic, transcendental, and piecewise from their graphs and from their equations.
b. Identify functions as being even, odd, increasing, decreasing, one-to-one, and many-to-one from their graphs and from their equations.
c. Critique the statement, “The graph of an absolute value function will be located entirely above the x-axis.”
d. Develop, generalize, explain, and apply strategies for determining the domain of a function from its equation and from its graphical representation.

e. Develop, generalize, explain, and apply strategies for determining the range of a function from its equation and from its graphical representation.
f. Identify and express the domain and range of a function using set and interval notation.
g. Develop, explain, and apply strategies for determining characteristics including symmetry, direction, and end behaviour of functions from their equations and from their graphs.
h. Analyze rational functions to determine conditions where x intercepts, vertical asymptotes, and holes exist by identifying the values of the domain that produce values which are zero, undefined, or indeterminate.
i. Critique the statement, "If the denominator of a rational function equals zero at x = a, then the rational function has a vertical asymptote at x = a."

Outcome: C30.2

Extend understanding of factoring, absolute value, and solving inequalities to include:

- **rational expressions**
- **double inequalities**
- **absolute value inequalities.**

Indicators:

a. Extend and apply factoring by greatest common factor (GCF) to include negative and rational exponents.
b. Extend and apply factoring over the set of rational numbers to the set of real numbers.
c. Critique the statement, "All polynomials can be factored."
d. Develop, explain, and apply strategies for factoring sum of cubes, difference of cubes, and x^n-y^n (where n is a positive integer).
e. Develop, explain, and apply strategies for solving inequalities containing rational expressions.
f. Develop, explain, and apply strategies for solving double inequalities.
g. Develop, explain, and apply strategies for solving rational equations which include absolute value expressions.
h. Develop, explain, and apply strategies for solving absolute value inequalities containing rational expressions.

Outcome: C30.3

Demonstrate understanding of limits and continuity.

Indicators:

a. Develop and explain the meaning of a limit.
b. Explain the difference between the limit of a function and the value of a function.
c. Critique the statement, "If a function has a limit of L as x approaches a, then as x-values get close to a, the y-values of the function will get progressively closer to L."
d. Determine the value of a limit and express the value using limit notation when given:
    - a graph
    - an algebraic expression.
e. Analyze the graph of a function to determine if it is continuous.
f. Analyze the equation of a function to determine if it continuous.
g. Develop, explain, and apply strategies for determining if a function is continuous at a given point.
h. Develop, explain, and apply strategies for determining the type of discontinuity (removable, jump, or infinite) when given:
    - a graph
    - an algebraic expression.
i. Develop, explain, and apply strategies (e.g., direct substitution, factoring, simplifying, rationalizing) to determine limits, at real numbers and infinity, of functions including absolute value, root, and piecewise.
j. Identify the conditions under which a limit does not exist.

Outcome: C30.4

Demonstrate understanding of differentiation based on slope as a rate of change.

Indicators:

a. Identify and explain situations in which slope is used to describe a rate of change.
b. Interpret and explain the difference between average rate of change and instantaneous rate of change.
c. Solve situational problems involving average rates of change and instantaneous rates of change.
d. Develop, explain, and apply strategies for determining the slope of the tangent line at a particular point by finding the slopes of secant lines.
e. Develop, explain, and apply the following definition of a derivative:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

f. Develop, explain, and apply rules of differentiation:
    - power

- product
- quotient
- chain.

g. Apply two or more differentiation rules to a function.
h. Critique the statement, "It is possible to differentiate any function with the rules that we have studied."
i. Identify the value(s) of x where a function is not differentiable.
j. Critique the statement, "If a function is continuous, then it is differentiable."
k. Develop, explain, and apply the process of implicit differentiation.
l. Determine the equation of the tangent line and normal line at a specific point on a function.
m. Express derivatives using a variety of notations such as f'(x), y', d/dx, and dy/dx.
n. Critique the statement, "The f'(x) notation for the derivative is superior to the dy/dx notation."

Outcome: C30.5

Extend understanding of curve sketching by applying differentiation and limits.

Indicators:

a. Develop, explain, and apply strategies for finding higher order derivatives and their notations.
b. Develop, explain, and apply strategies for using the first derivative to determine:
   - critical points
   - increasing and decreasing intervals.
c. Develop, explain, and apply strategies for using the second derivative to determine:
   - points of inflection
   - concavity intervals.
d. Describe the difference between relative and absolute extrema.
e. Apply the first and second derivatives to determine relative and absolute extrema.
f. Analyze graphical representations of f(x) to identify critical point(s), increasing and decreasing intervals, point(s) of inflection, and concavity intervals.
g. Identify characteristics of f(x) when given the graph of the first derivative and/or second derivative.
h. Identify characteristics of f(x) when given descriptions of the first derivative and/or second derivative.
i. Apply understanding of limits to determine vertical and horizontal asymptotes.
j. Sketch the graph of a function with and without the use of technology.
k. Critique the statement, "An absolute maximum or minimum value occurs at x=a, if and only if f'(a) = 0."

Outcome: C30.6

Demonstrate understanding of the application of derivatives to solve problems including:

- **optimization**
- **rates of change**
- **related rates.**

Indicators:

a. Solve situational problems involving optimization as they apply to topics such as area, volume, and cost.
b. Solve situational problems involving rates of change as they apply to topics such as business, motion, and science.
c. Solve situational problems involving related rates by the application of implicit differentiation.
d. Critique the statement, "If the rate of change of the radius of a circle doubles, then the rate of change of the area will also double."

Outcome: C30.7

Demonstrate understanding of transcendental function derivatives and their applications.

Indicators:

a. Develop an understanding of *e* using limits.
b. Utilize the squeeze theorem to evaluate limits involving trigonometric functions.
c. Develop, explain, and apply strategies to determine derivatives of the following transcendental functions:
   - exponential and logarithmic functions using any base
   - sine and cosine functions.
d. Apply first and second derivatives to sketch graphs of:
   - exponential and logarithmic functions using any base
   - sine and cosine functions
   - composite transcendental functions.
e. Critique the statement, "A function and its derivative are always different."
f. Solve situational problems involving the derivatives of transcendental functions.

Outcome: C30.8

Demonstrate understanding of indefinite and definite integration:

- **by sight**
- **by substitution**
- **as used in the Fundamental Theorem of Calculus.**

Indicators:

a. Distinguish between indefinite and definite integration.
b. Critique the statement, "If a function can be differentiated, then it can be integrated."
c. Determine indefinite integrals by sight.
d. Determine indefinite integrals by substitution.
e. Apply the Fundamental Theorem of Calculus to evaluate definite integrals by sight and by substitution.
f. Solve situational questions involving integration.
g. Critique the statement, "The integral of f'(x)dx equals f(x)."
h. Develop, explain, and apply strategies for determining the area bounded by:
    - a curve and the x-axis over [a,b]
    - two curves.
i. Critique the statement, "To integrate any power, we apply in reverse the power rule for differentiation."