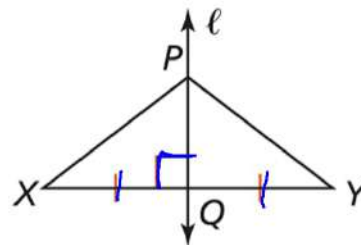# Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.
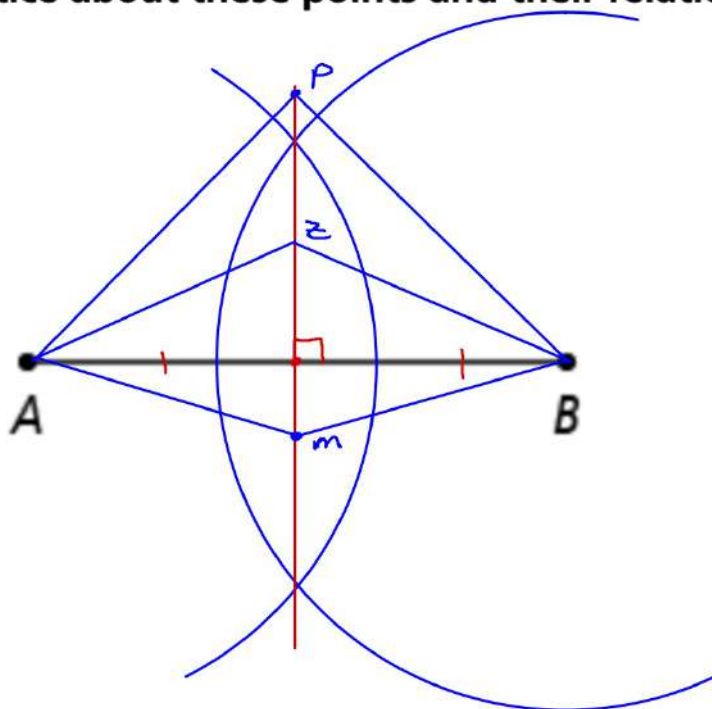
PROOF: SEE EXAMPLE 2.

If...

Then... $PX = PY$

How can you find points that are equidistant from the endpoints of $\overline{AB}$?

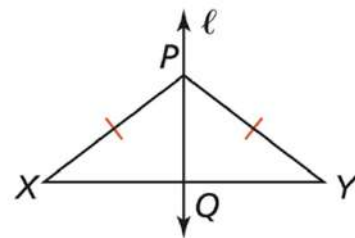**What do you notice about these points and their relationship with $\overline{AB}$?**

Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

If...

PROOF: SEE EXAMPLE 2 TRY IT.

Then... $XQ = YQ$ and $\overleftrightarrow{PQ} \perp \overline{XY}$
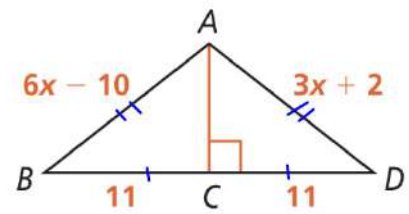
What is the value of *AD*?

SOLUTION

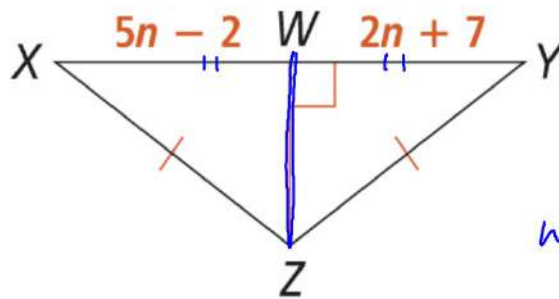$$6x - 10 = 3x + 2$$
$$3x - 10 = 2$$
$$3x = 12$$
$$x = 4$$

$$AD = 3x + 2$$
$$3(4) + 2$$
$$12 + 2$$
$$14$$

a. What is the value of *WY*?

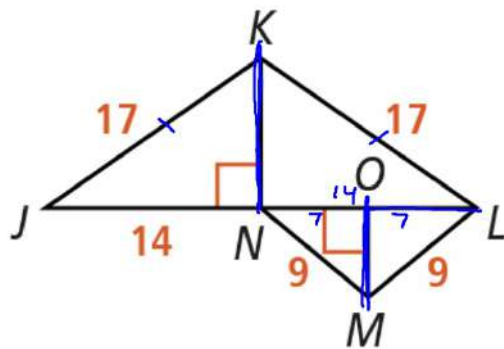X $\overset{5n-2}{\underset{\text{||}}{}}$ W $\overset{2n+7}{\underset{\text{||}}{}}$ Y

Z

$5n - 2 = 2n + 7$

$3n - 2 = 7$

$3n = 9$

$n = 3$

$WY = 2n + 7$

$2(3) + 7$
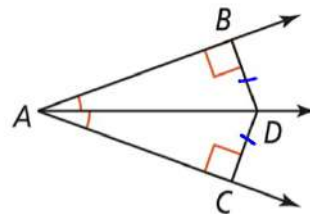
$6 + 7$

13

What is the value of *OL*?

Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

If...

PROOF: SEE EXERCISE 9.

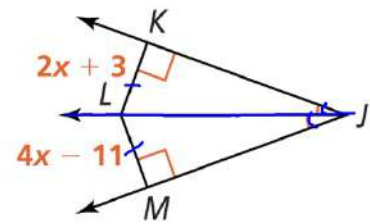**Then...** $BD = CD$

What is the value of *KL*?

SOLUTION
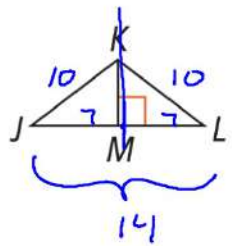
$2x + 3 = 4x - 11$

$3 = 2x - 11$

$14 = 2x$

$x = 7$

$KL = 2x + 3$

$2(7) + 3$

$14 + 3$

17

If *JL* = 14, *KL* = 10, and *ML* = 7, what is *JK*?

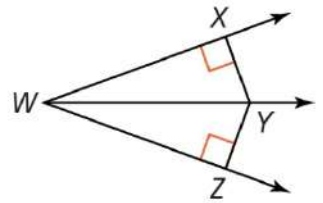Enter your answer

K

10   10

J   7   7   L

M

14

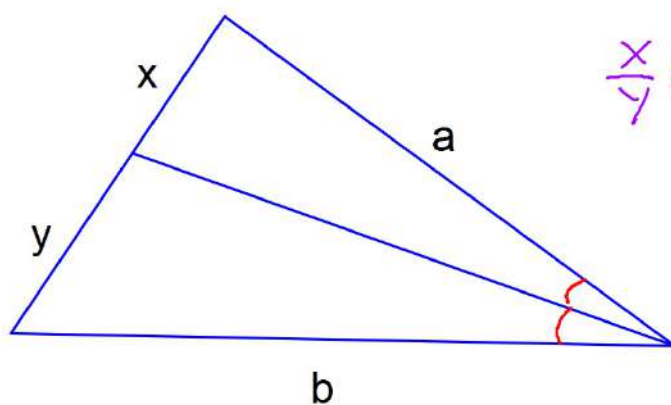If $\angle XWY \cong \angle ZWY$ and $XY = 4$, what is YZ?

Enter your answer
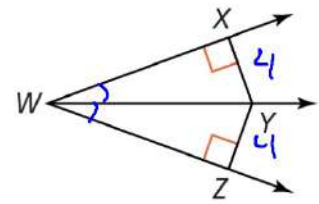
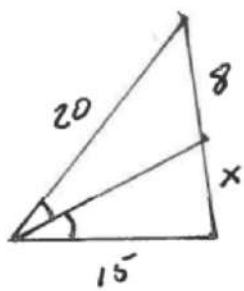**7.** If $XY = 6.5$, $ZY = 6.5$, and $m\angle ZWY = 18$, what is $m\angle XWZ$?

Enter answer here

The angle bisector cuts the opposite side in the same ratio as the two other sides.

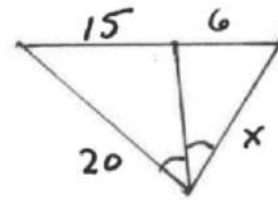$$\frac{x}{y} = \frac{a}{b} \quad or \quad \frac{x}{a} = \frac{y}{b}$$

$$\frac{x}{y} = \frac{a}{b}$$

Top-left figure:

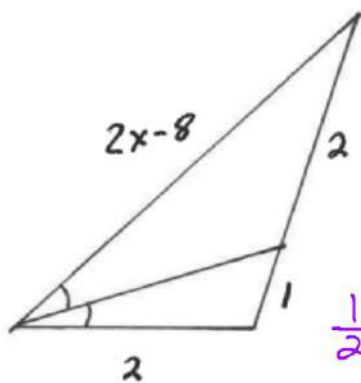$$\frac{8}{x} = \frac{20}{15}$$

$$20x = 120$$

$$x = 6$$

Triangle labeled: 20, 8, x, 15

Top-right figure:

$$\frac{15}{6} = \frac{20}{x}$$

$$15x = 120$$

$$x = 8$$

Triangle labeled: 15, 6, 20, x

Bottom-left figure:

$$\frac{1}{2} = \frac{2}{2x-8}$$

$$2x - 8 = 4$$

$$2x = 12$$

$$x = 6$$

Triangle labeled: 2x-8, 2, 1, 2

Bottom-right figure:

Triangle labeled: 4, 10, x+2, 15
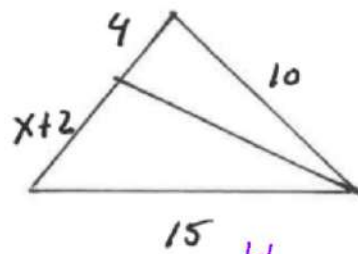
$$\frac{4}{x+2} = \frac{10}{15}$$

$$10(x+2) = 60$$

$$10x + 20 = 60$$

$$10x = 40$$

$$x = 4$$

Top left figure

$2x-11$ 2 4
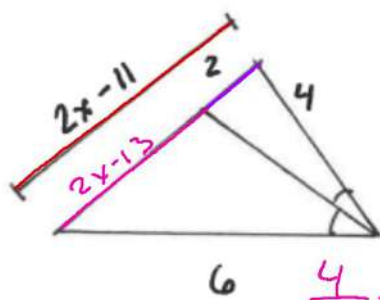
$2x-13$

6

$\dfrac{4}{6} = \dfrac{2}{2x-13}$

$4(2x-13) = 12$

$8x - 52 = 12$

$8x = 64$

$x = 8$

Top right figure

9 x 5

$9-x$
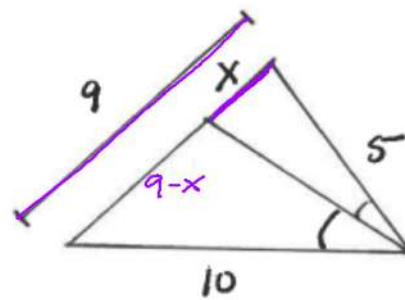
10

$\dfrac{5}{10} = \dfrac{x}{9-x}$

$5(9-x) = 10x$

$45 - 5x = 10x$

$45 = 15x$

$x = 3$

Bottom left figure

16 x 12

$16-x$
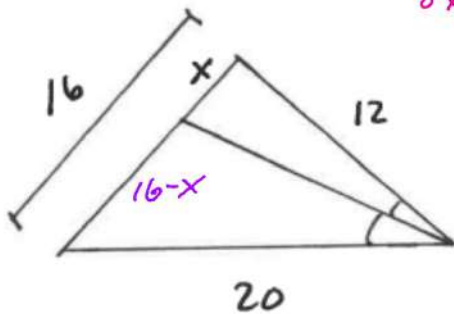
20

$\dfrac{12}{20} = \dfrac{x}{16-x}$

$20x = 12(16-x)$

$20x = 192 - 12x$

$32x = 192$

$x = 6$