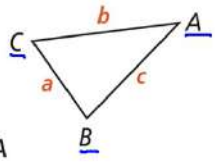
Section 5.4

THEOREM 5-9

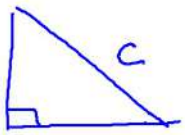
If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.

PROOF: SEE EXERCISE 13.

If... $b > a$

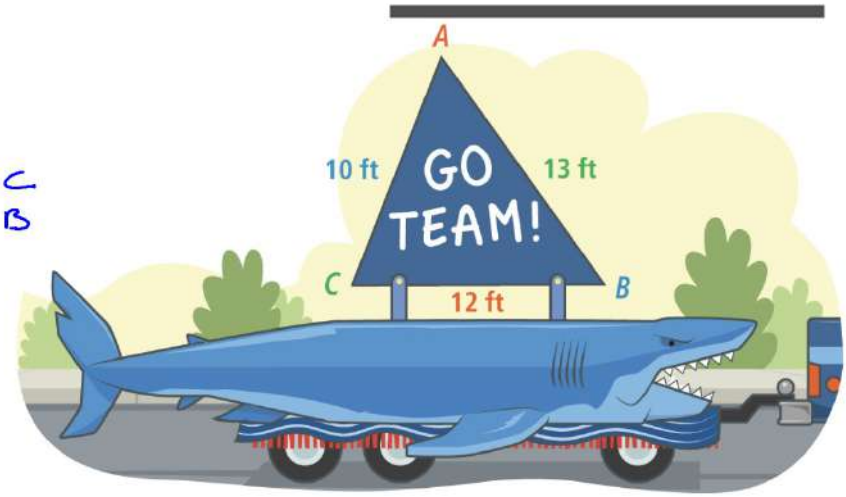


Then... $m\angle B > m\angle A$



To support a triangular piece of a float, a brace is placed at the largest angle and a guide wire is placed at the smallest angle.
Which angle is the largest? → ∠C
Which angle is the smallest? → ∠B

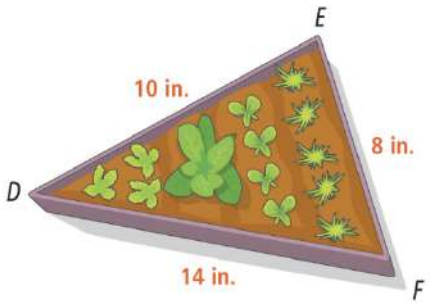
SOLUTION



Lucas sketched a diagram for a garden box.

List the angles from least to greatest.

$\angle D, \angle F, \angle E$



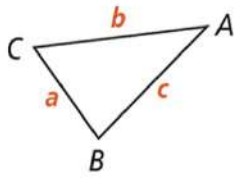
Converse of Theorem 5-9

If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.

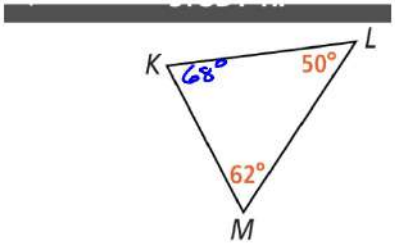
PROOF: SEE EXAMPLE 3.

If... $m\angle B > m\angle A$

Then... $b > a$

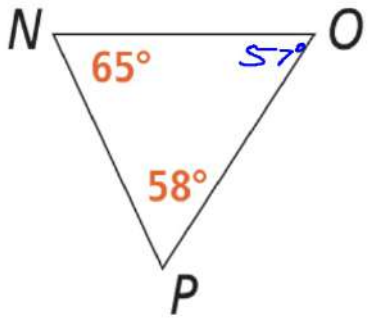


Which side of $\triangle KLM$ is the longest? $\overline{LM}$



SOLUTION

List the sides of triangles NOP from least to greatest.



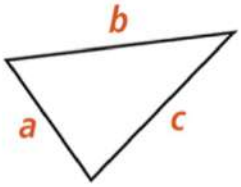
$\overline{NP}$, $\overline{NO}$, $\overline{OP}$

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

PROOF: SEE EXERCISE 14.

If...



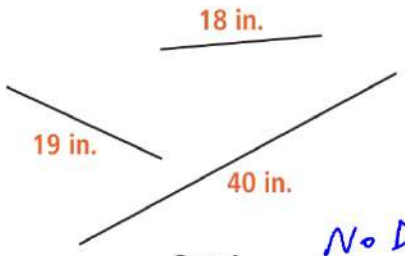
Then... $a + b > c$

$a + c > b$

$b + c > a$

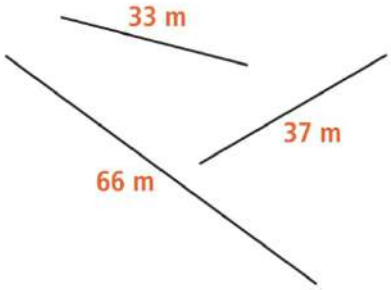
All 3 have to be true

A. Which of the following sets of segments could be the sides of a triangle?



Set 1

No △



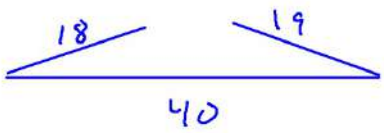
Set 2

SOLUTION

$18+19>40$ ✗

$40+19>18$ ✓

$40+18>19$ ✓

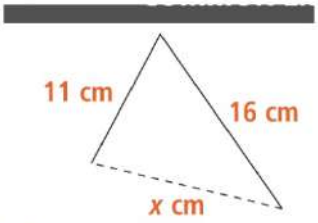


$33+37>66$ ✓

$66+33>37$ ✓

$66+37>33$ ✓

B. A triangle has sides that measure 11 cm and 16 cm. What are the possible lengths of the third side?



SOLUTION

$5 < x < 27$

$11 + x > 16$
$-11 \quad -11$
$x > 5$

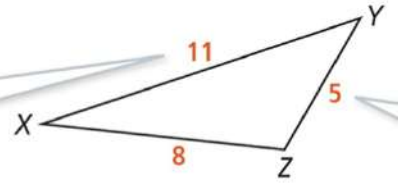
$16 + x > 11$
$x > -5$

$11 + 16 > x$
$27 > x$
$x < 27$

Inequalities in One Triangle

THEOREMS 5-9 AND 5-10

The longest side is opposite the largest angle.



The shortest side is opposite the smallest angle.

$ZY < XZ < XY$

$m\angle X < m\angle Y < m\angle Z$

THEOREM 5-11 Triangle Inequality Theorem

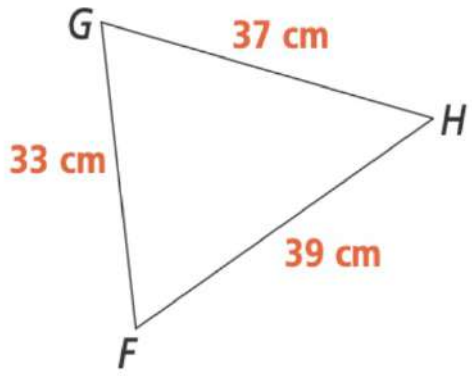
The sum of the lengths of any two sides is greater than the length of the third side.

$5 + 8 > 11$

$5 + 11 > 8$

$8 + 11 > 5$

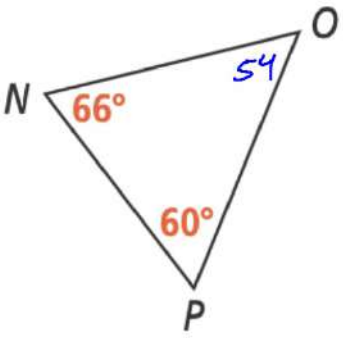
Identify the angles of $\triangle FGH$. SEE EXAMPLE 2.



18. Which angle is the smallest?

19. Which angle is the largest?

Identify the sides of $\triangle NOP$. SEE EXAMPLES 3 AND 4.



20. Which side is the longest? $\overline{OP}$

21. Which side is the shortest? $\overline{NP}$

Determine whether the side lengths could form a triangle. SEE EXAMPLE 5.

22. 13, 15, 9

$13+15>9$
$13+9>15$ Yes
$15+9>13$

23. 8, 15, 7

$8+15>7$
$15+7>8$ NO
$8+7>15$

24. 35, 20, 11

$35+20>11$
$20+11>35$ NO

25. 65, 32, 40

$65+32>40$
$32+40>65$ Yes
$65+40>32$

Given two sides of a triangle, determine the range of possible lengths of the third side. SEE EXAMPLE 5.

26. 10 in. and 12 in.

$10+12>x$ $22>x$ $2<x<22$

$10+x>12$ $x>2$

$12+x>10$ $x>-2$

27. 5 ft and 10 ft

$5+10>x$ $15>x$ $5<x<15$

$5+x>10$ $x>5$

$10+x>5$ $x>-5$

28. 200 m and 300 m

$200+300>x$ $500>x$ $100<x<500$

$300+x>200$ $x>100$

$200+>300$

29. 90 km and 150 km

$60<x<240$