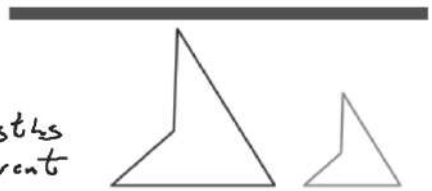
Chapter 3 Transformations

Side lengths stay same length

Angles stay the same measure

A rigid motion is a transformation that preserves length and angle measure. Is the transformation a rigid motion? Explain.



SOLUTION

↳ No Side lengths are different

Preimage $\triangle ABC$

Image $\triangle A'B'C'$

Start with

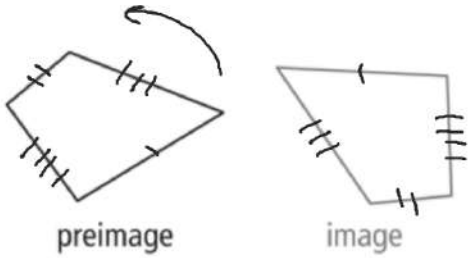
Finish

A' read "A prime"

A'' read "A Double Prime"

Try It!

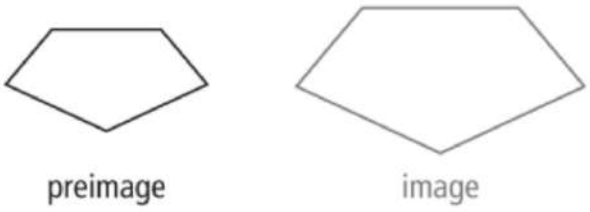
1. a. Is the transformation a rigid motion? Explain.



Yes sides same length

Angles same measure.

1. b. Is the transformation a rigid motion? Explain.



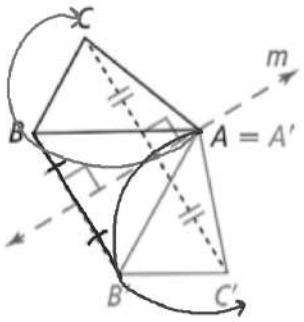
No side lengths are not the same.

Reflections

A reflection is a transformation that reflects each point in the preimage across a line of reflection.

A reflection has these properties:

- If a point A is on line m, then the point and its image are the same point (that is, $A' = A$).
- If a point B is not on line m, line m is the perpendicular bisector of $\overline{BB'}$.



The reflection of $\triangle ABC$ across line m can be written as $R_m(\triangle ABC) = \triangle A'B'C'$.

A reflection is a rigid motion so length and angle measures are preserved.

Orientation is not preserved

R_m

Reflection over line m

Quadrilateral $FGHJ$ has coordinates $F(0, 3)$, $G(2, 4)$, $H(4, 2)$, $J(-2, 0)$.

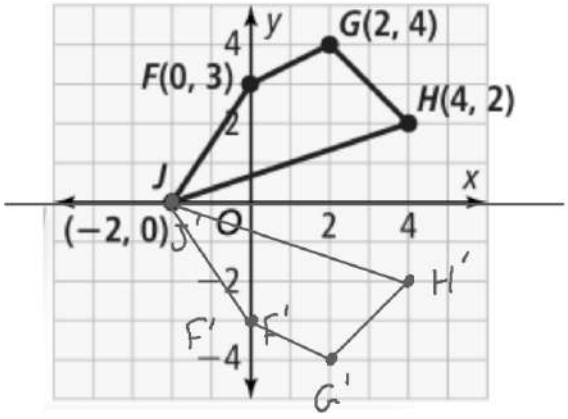
A. Graph and label $FGHJ$ and $R_{x\text{-axis}}(FGHJ)$. What is a general rule for reflecting a point across the x-axis?

$R_{x\text{-axis}}$ $J(-2,0)$ $F(0,3)$

Reflection over x-axis $J'(-2,0)$ $F'(0,-3)$

$G(2,4)$ $H(4,2)$

$G'(2,-4)$ $H'(4,-2)$



Reflect across x-axis

Change sign on y-value.

$(x,y) \rightarrow (x,-y)$

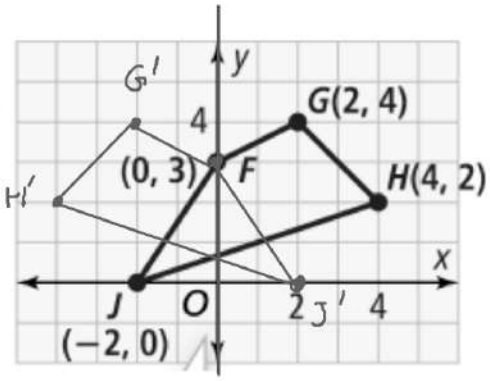
$R_{x\text{-axis}} \rightarrow (x,-y)$

Quadrilateral $FGHJ$ has coordinates $F(0, 3)$, $G(2, 4)$, $H(4, 2)$, $J(-2, 0)$.

B. Graph and label $FGHJ$ and $R_{y\text{-axis}}(FGHJ)$. What is a general rule for reflecting a point across the y-axis?

$F(0,3)$ $G(2,4)$ $H(4,2)$ $J(-2,0)$

$F'(0,3)$ $G'(-2,4)$ $H'(-4,2)$ $J'(2,0)$



Reflect over y-axis

Change sign on x-value

$(x,y) \rightarrow (-x,y)$

$R_{y\text{-axis}} \rightarrow (-x,y)$

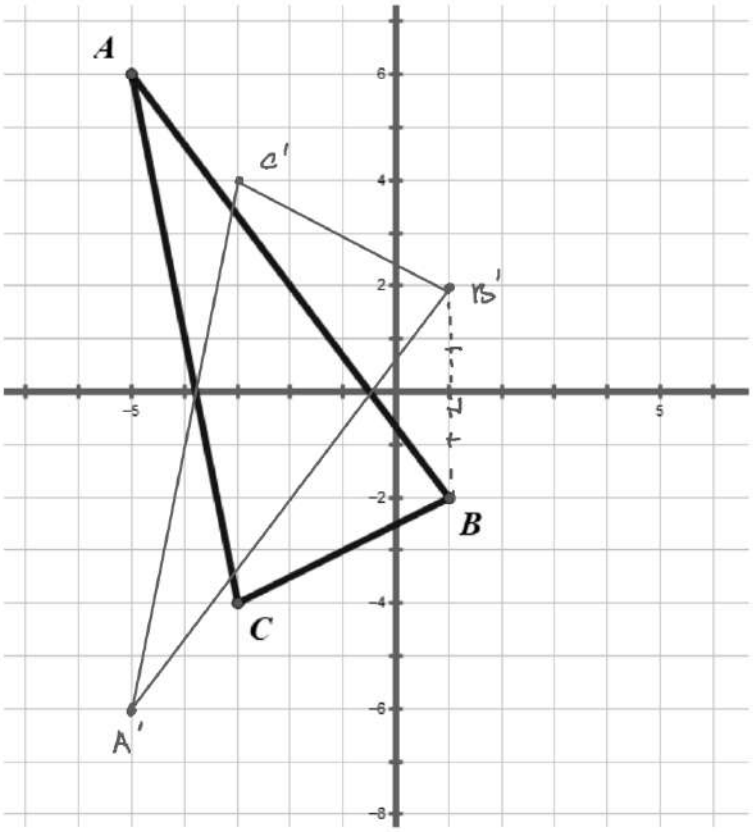
3. Triangle ABC has vertices $A(-5,\ 6)$, $B(1,\ -2)$, and $C(-3,\ -4)$. What are the coordinates of the vertices of $\triangle A'B'C'$ for each reflection?

a. $R_{x\text{-axis}}$ $(x,y) \rightarrow (x,-y)$

$A'(-5,-6)$

$B'(1,2)$

$C'(-3,4)$



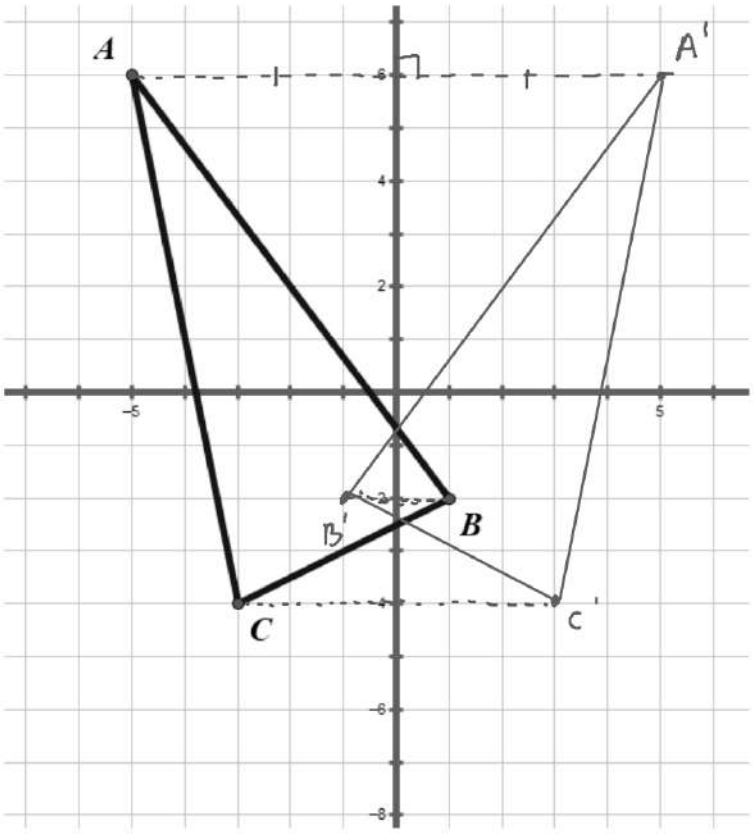
3. Triangle ABC has vertices $A(-5,\ 6)$, $B(1,\ -2)$, and $C(-3,\ -4)$. What are the coordinates of the vertices of $\triangle A'B'C'$ for each reflection?

$R_{y\text{-axis}}$ $(x,y) \rightarrow (-x,y)$

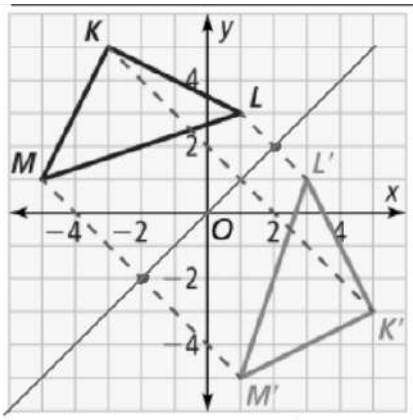
$A'(5,6)$

$B'(-1,-2)$

$C'(3,-4)$



What reflection rule maps $\triangle KLM$ to its image?



Step 1
Write the coordinates of the preimage and the image.

$K(-3, 5)$ → $K'(5, -3)$ ↷

$L(1, 3)$ → $L'(3, 1)$ ↷ ($x_1\ y_1$ → $x_2\ y_2$)

$M(-5, 1)$ → $M'(1, -5)$ ↷ ($x_1\ y_1$ → $x_2\ y_2$)

Step 2 Find mid point of $\overline{LL'}$ and $\overline{MM'}$

midpt formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

$\overline{LL'} = \left(\frac{1+3}{2}, \frac{3+1}{2}\right) \quad \left(\frac{-5+1}{2}, \frac{1+(-5)}{2}\right)$

$(2, 2) \quad (-2, -2)$

Step 3 Find the slope using midpts

$(2,2)$ (x_1, y_1) $(-2,-2)$ (x_2, x_2)

Slope Formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-2-2}{-2-2} = \frac{-4}{-4} = 1$

Step 4 Use point-slope form to find the equation of the line of reflection.

pt-slope form $y - y_1 = m(x - x_1)$

$m = 1 \quad (2, 2)$ ($x_1\ y_1$)

$y - 2 = 1(x - 2)$

$y - 2 = x - 2$

$+2 \quad +2$

$y = x$

4. What is a reflection rule that maps each triangle to its image?

a. $C(3, 8), D(5, 12), E(4, 6)$ and $C'(-8, -3), D'(-12, -5), E'(-6, -4)$

Step 2 Find midpt

$\overline{CC'}$ $\left(\frac{3+(-8)}{2}, \frac{8+(-3)}{2}\right)$ $(-2.5, 2.5)$

$\overline{EE'}$ $\left(\frac{4+(-6)}{2}, \frac{6+(-4)}{2}\right)$ $(-1, 1)$

Step 3 Slope using midpts

$m = \frac{1-2.5}{-1-(-2.5)}$