$\left(\frac{1}{1}\right)^1+\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^3+\left(\frac{1}{4}\right)^4\cdots \leftarrow \frac{1}{1^1}+\frac{1}{2^2}+\frac{1}{3^3}+\frac{1}{4^4}+\cdots$

What you'll Learn About
Root Test

The bases are approaching a number smaller than 1 so the series converges

$A)\ \sum_{n=1}^{\infty}\frac{(-1)^n}{10^n}$ converges Absolutely

$r=-\frac{1}{10}$

$|r|<1$

$B)\ \sum_{n=1}^{\infty}\frac{(1)}{n^n}$ Converges

$\lim_{n\to\infty}\left|\sqrt[n]{\frac{1}{n^n}}\right|=$

$\lim_{n\to\infty}\left(\frac{1}{n^n}\right)^{1/n}=\frac{1^{1/n}}{(n^n)^{1/n}}=\frac{1}{n}=0<1$

$C)\ \sum_{n=1}^{\infty}\left(\frac{n}{3n+10}\right)^n$

$\lim_{n\to\infty}\left[\left(\frac{n}{3n+10}\right)^n\right]^{1/n}=\frac{1}{3}<1$

converges

$D)\ \sum_{n=1}^{\infty}\left(\frac{5n}{3n+10}\right)^{2n}$

$\lim_{n\to\infty}\left[\left(\frac{5n}{3n+10}\right)^{2n}\right]^{1/n}=\left(\frac{5}{3}\right)^2>1$

diverges

$E)\ \sum_{n=1}^{\infty}\left(\frac{-2n}{n+10}\right)^n$ diverges

$\lim_{n\to\infty}\left|\left(\frac{-2n}{n+10}\right)^n\right|^{1/n}=2>1$

What you'll Learn About
Alternating Series Test

A) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

Converges $p = 2 > 1$

$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$

$S_{100} = 1.6349$

$S_{1000} = 1.6439$

$S_{10000} = 1.6448$

B) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

Converges absolutely because the series alternates and the terms decrease in absolute value to 0

$-1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16}$

$S_{100} = -.8224$

$S_{1000} = -.8224$

$S_{10000} = -.8224$

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

C) $\sum_{n=1}^{\infty} \frac{1}{n} =$

Harmonic Series
Diverges

$S_{100} = 5.187$

$S_{1000} = 7.4854$

$S_{10000} = 8.17836$

$-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \cdots$

D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

Alternating Harmonic Series

Conditionally Converges

$\rightarrow$ alternating series

$S_{100} = -.6881$

$S_{1000} = -.6926$

$S_{10000} = -.6928$

Conditional Convergence because the series is alternating and the terms decrease in absolute value to 0

$$E)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}} =$$

Absolute Convergence?

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ diverges

compare to $\sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic

$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2+1}}}{\frac{1}{n}} = \frac{n}{\sqrt{n^2+1}} = 1$

① $\lim_{n\to\infty} \left| \frac{(-1)^n}{\sqrt{n^2+1}} \right| = 0$

Absolute value of the terms approach 0

② $\left| \frac{(-1)^{n+1}}{\sqrt{(n+1)^2+1}} \right| < \left| \frac{(-1)^n}{\sqrt{n^2+1}} \right|$

next term smaller than the previous

Conditional Convergence

$$F)\ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1/3}} =$$

① $\lim_{n\to\infty} \left| \frac{(-1)^{n-1}}{n^{1/3}} \right| = 0$

② $\left| \frac{(-1)^{n+1-1}}{(n+1)^{1/3}} \right| < \left| \frac{(-1)^{n-1}}{n^{1/3}} \right|$

Absolute?

$\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ diverges

$p = \frac{1}{3} < 1$

Conditional convergence NO Diverges

$$G)\ \sum_{n=1}^{\infty} \frac{(-1)^n n^4}{n^3+1} =$$

① $\lim_{n\to\infty} \left| \frac{(-1)^n n^4}{n^3+1} \right| \neq 0$

Conditional Convergence	$H)\ \sum_{n=1}^{\infty}\frac{\cos(n\pi)}{2^n}=\sum_{n=1}^{\infty}\frac{(-1)^n}{2^n}$ Absolute Convergence $\sum_{n=1}^{\infty}\frac{1}{2^n}$ $r=\frac{1}{2}\quad \lvert r\rvert<1$ Converges ① $\lim_{n\to\infty}\left\lvert\frac{(-1)^n}{2^n}\right\rvert=0$ ② $\left\lvert\frac{(-1)^{n+1}}{2^{n+1}}\right\rvert<\left\lvert\frac{(-1)^n}{2^n}\right\rvert$ $I)\ \sum_{n=1}^{\infty}\frac{(-1)^n n}{\sqrt{n^2+1}}=$ J) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^4+2}=$ $K)\ \sum_{n=1}^{\infty}\frac{(-1)^n}{(1.1)^n}$

	8) $\sum_{n=1}^{\infty}\frac{(-1)^n e^{1/n}}{n}=$ 11) $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n^{3/4}}=$ 12) $\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi}{2}\right)}{n!}=$ 15) $\sum_{n=1}^{\infty}\frac{(-1)^n n^n}{n!}=$