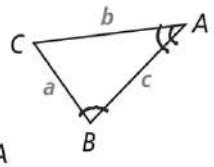
THEOREM 5-9

If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.

PROOF: SEE EXERCISE 13.

If... $b > a$

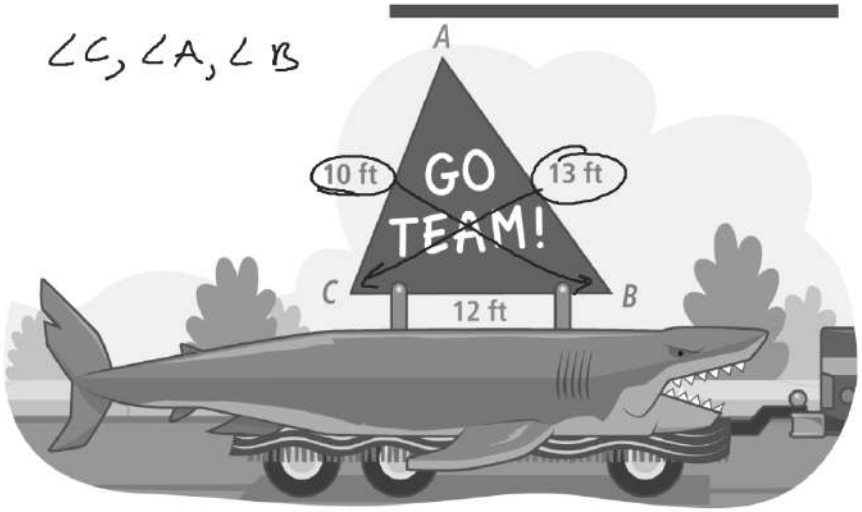


Then... $m\angle B > m\angle A$

To support a triangular piece of a float, a brace is placed at the largest angle and a guide wire is placed at the smallest angle.
Which angle is the largest? ∠C
Which angle is the smallest? ∠B

SOLUTION

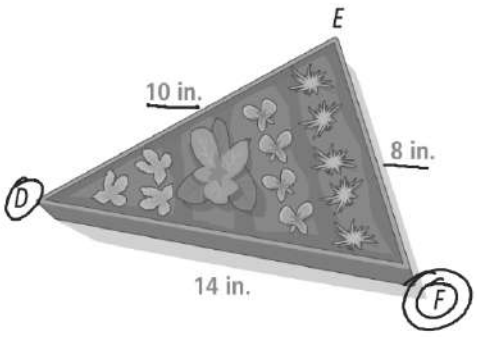
∠C, ∠A, ∠B



Lucas sketched a diagram for a garden box.

List the angles from least to greatest.

$\angle D, \angle F, \angle E$



$8+14>10$

$8+10>14$

$14+10>8$

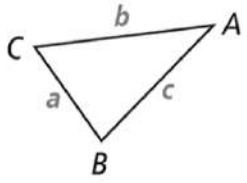
Converse of Theorem 5-9

If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.

PROOF: SEE EXAMPLE 3.

If... $m\angle B > m\angle A$

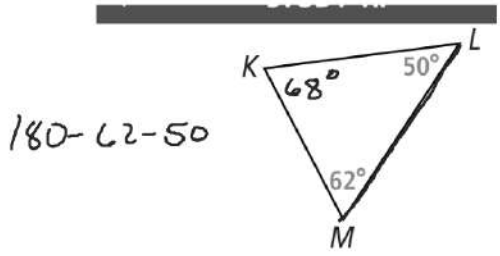
Then... $b > a$



Which side of $\triangle KLM$ is the longest?

$\overline{LM}$

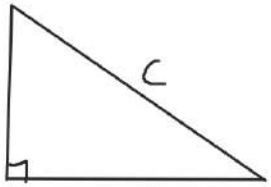
SOLUTION



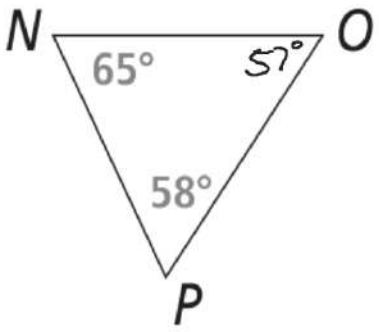
180-62-50

Shortest?

$\overline{KM}$



List the sides of triangles NOP from least to greatest.



$m\angle O = 180 - 65 - 58$

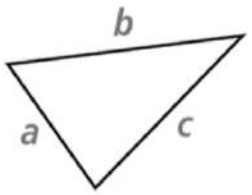
$\overline{NP}, \overline{NO}, \overline{OP}$

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

PROOF: SEE EXERCISE 14.

If...

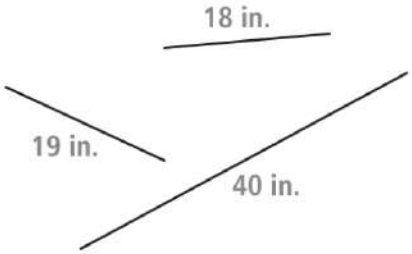


Then... $a + b > c$

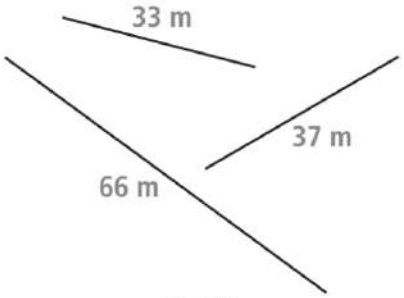
$a + c > b$

$b + c > a$

A. Which of the following sets of segments could be the sides of a triangle?



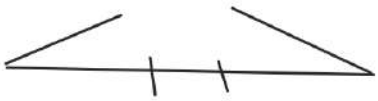
Set 1



Set 2

SOLUTION

$18+19>40$

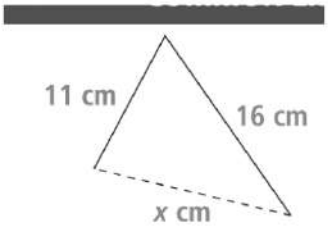


$33+37>66$

$66+33>37$

$66+37>33$

B. A triangle has sides that measure 11 cm and 16 cm. What are the possible lengths of the third side?



SOLUTION

13 14 6

III IIII

$11+6>16$ $11+14>16$ $11+13>16$

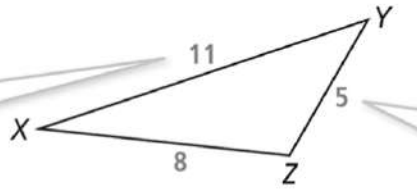
$16+6>11$ $11+16>14$ $16+13>11$

$16+11>6$ $14+16>11$ $11+16>13$

Inequalities in One Triangle

THEOREMS 5-9 AND 5-10

The longest side is opposite the largest angle.



The shortest side is opposite the smallest angle.

$ZY < XZ < XY$

$m\angle X < m\angle Y < m\angle Z$

THEOREM 5-11 Triangle Inequality Theorem

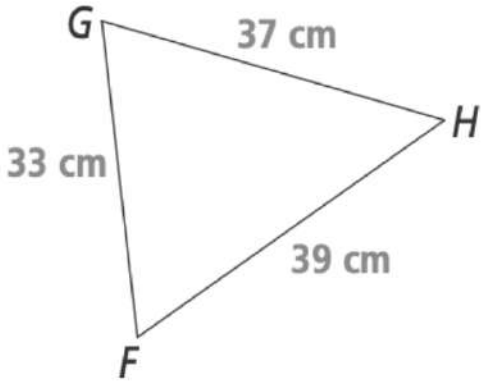
The sum of the lengths of any two sides is greater than the length of the third side.

$5 + 8 > 11$

$5 + 11 > 8$

$8 + 11 > 5$

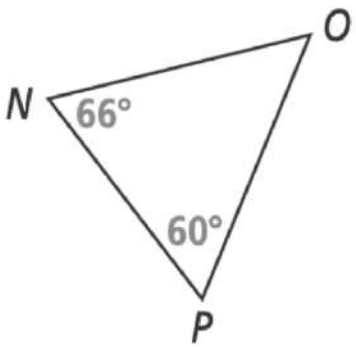
Identify the angles of $\triangle FGH$. SEE EXAMPLE 2.



18. Which angle is the smallest?

19. Which angle is the largest?

Identify the sides of $\triangle NOP$. SEE EXAMPLES 3 AND 4.



20. Which side is the longest?

21. Which side is the shortest?

Determine whether the side lengths could form a triangle. SEE EXAMPLE 5.

22. 13, 15, 9 Yes

22) $13+15>9$
$13+9>15$
$15+9>13$

23. 8, 15, 7 No

23) $8+15>7$
$7+15>8$
$8+7>15$
$15>15 \rightarrow$ False

24. 35, 20, 11 No

24) $35+20>11$
$20+11>35$ ← False
$35+11>20$

25. 65, 32, 40 Yes

25) $65+32>40$
$32+40>65$
$65+40>32$

Given two sides of a triangle, determine the range of possible lengths of the third side. SEE EXAMPLE 5.

26. 10 in. and 12 in.

27. 5 ft and 10 ft

28. 200 m and 300 m

29. 90 km and 150 km