$_nP_r = \frac{n!}{(n-r)!}$

STATS 4: COMBINATIONS

$\cancel{_7P_{10}}$

Warm-Up: Evaluate each expression.

1. $_5P_3$ **2.** $_6P_3$ **3.** $_8P_2$ **4.** $_7P_4$

$\frac{8!}{(8-2)!} = \frac{8!}{6!} = \frac{8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!}} = 56$

A and *B* are independent events. Find *P*(*A* and *B*) for the given probabilities.

5. $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}$

6. $P(A) = \frac{1}{8}, P(B) = \frac{5}{9}$

$P(A \text{ and } B) = \frac{1}{8} \cdot \frac{5}{9} = \frac{5}{12}$

Objectives

- To find combinations
- To decide if a scenario is a combination or a permutation

VOCABULARY

Combination

A **combination** is an arrangement of objects without regard to order.

Mike, Bob, Sue

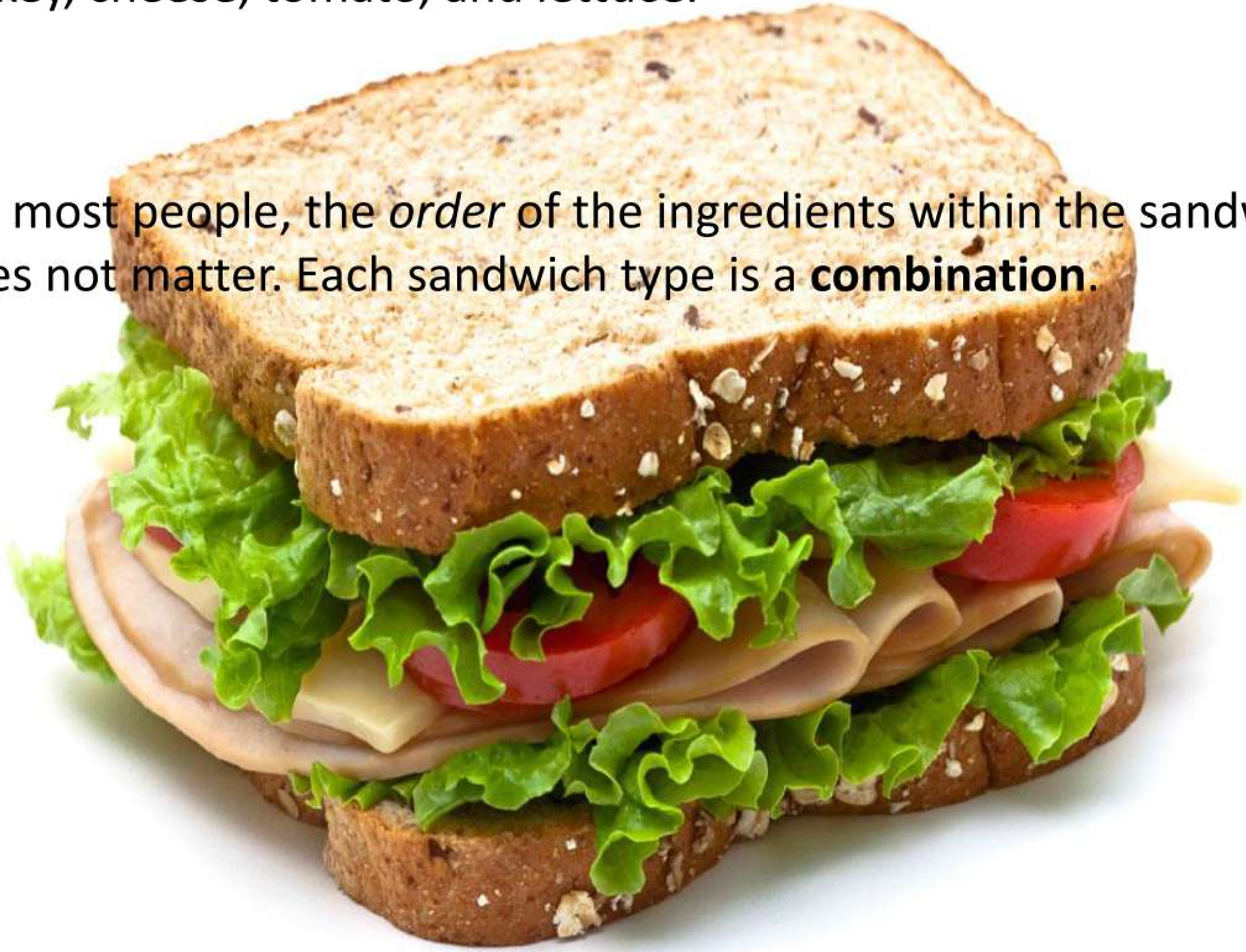
P = 6

M, B, S S, B, M ; B M, S
M, S, B S, M, B ; B, S, M

C

1

Suppose you are making a sandwich with three of these ingredients: turkey, cheese, tomato, and lettuce.



For most people, the *order* of the ingredients within the sandwich does not matter. Each sandwich type is a **combination**.

So how do we find the number of different sandwiches?

VOCABULARY

Combination Notation

The expression ${}_nC_r$ represents the number of combinations of *n* objects arranged *r* at a time.

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

Example

$${}_4C_3 = \frac{4!}{3!(4-3)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 1} = \frac{24}{6} = 4$$

EXAMPLE 1: COUNTING COMBINATIONS

Simplify $_8C_5$.

$$\frac{8!}{r!(8-5)!} = \frac{8!}{5!\,3!} = \frac{8\cdot7\cdot\cancel{6}\cdot\cancel{5\cdot4\cdot3\cdot2\cdot1}}{\cancel{5\cdot4\cdot3\cdot2\cdot1}\cdot\cancel{3}\cdot2\cdot1} = 56$$

EXAMPLE 1: QUICK CHECK

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

Simplify each expression.

a. ${}_4C_2$ $\frac{4!}{2!(4-2)!}$

$$\frac{4!}{2!2!} = \frac{\cancel{4}^{2}\cdot 3\cdot \cancel{2!}}{\cancel{2!}\cdot\cancel{2}\cdot 1}$$

6

b. ${}_7C_3$ $\frac{7!}{3!(7-3)!}$

$$\frac{7!}{3!4!} = \frac{7\cdot\cancel{6}\cdot 5\cdot\cancel{4!}}{\cancel{3}\cdot\cancel{2}\cdot 1\cdot\cancel{4!}}$$

35

c. ${}_{10}C_4$ $\frac{10!}{4!(10-4)!}$

$$\frac{10!}{4!6!} = \frac{\cancel{10}^{5}\cdot\cancel{9}^{3}\cdot\cancel{8}^{2}\cdot 7\cdot\cancel{6!}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1\cdot\cancel{6!}}$$

$\boxed{210}$

EXAMPLE 2: REAL-WORLD PROBLEM SOLVING

Twenty people report for jury duty. How many different twelve-person juries can be chosen?

$${}_{20}C_{12} \qquad \frac{20!}{12!(20-12)!} \qquad \frac{20!}{12!\,8!} =$$

$$\frac{\cancel{20}\cdot 19\cdot \cancel{18}\cdot 17\cdot \cancel{16}\cdot 15\cdot \overset{2}{\cancel{14}}\cdot 13\cdot \cancel{12!}}{\cancel{12!}\,\cancel{8}\cdot\cancel{7}\cdot\cancel{6}\cdot\cancel{5\cdot 4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1}$$

$\boxed{125970 \text{ juries}}$

EXAMPLE 2: QUICK CHECK

For your history report, you can choose to write about two of a list of five presidents of the United States. Calculate the number of combinations possible for your report.

$$_5C_2 = \frac{5!}{2!(5-2)!} = \frac{5!}{2!\,3!} = \frac{5 \cdot \overset{2}{\cancel{4}} \cdot \cancel{3!}}{\cancel{2} \cdot 1 \cdot \cancel{3!}}$$

10 combinations

EXAMPLE 2: QUICK CHECK

A reading list for a literature course has 20 books on it. In how many ways can you choose four books to read?

$$_{20}C_4 = \frac{20!}{4!(20-4)!} = \frac{20!}{4!\,16!} = \frac{\overset{5}{\cancel{20}}\cdot 19\cdot \overset{\overset{3}{\cancel{6}}}{\cancel{18}}\cdot 17\cdot \cancel{16!}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1\cdot \cancel{16!}}$$

$\boxed{4845 \text{ ways}}$

EXAMPLE 2: QUICK CHECK

There are 20 people in a talent contest. In how many ways can the top 10 be chosen?

$$_{20}C_{10} = \frac{20!}{10!(20-10)!}$$

$$= \frac{20!}{10!\,10!} = \frac{\cancel{20}\cdot 19\cdot \cancel{18}\cdot 17\cdot \cancel{16}\cdot \cancel{15}\cdot \cancel{14}\cdot 13\cdot \cancel{12}\cdot 11\cdot \cancel{10!}}{\cancel{10}\cdot \cancel{9}\cdot \cancel{8}\cdot \cancel{7}\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1\cdot \cancel{10!}}$$

2 5 2 2

$$\boxed{184756 \text{ ways}}$$

EXAMPLE 2: QUICK CHECK

At a local pizzeria, you can order a pizza with nine different toppings. How many different pizzas can be made using three of the toppings?

$${}_9C_3 = \frac{9!}{3!\,6!} = \frac{\overset{3}{\cancel{9}} \cdot \overset{4}{\cancel{8}} \cdot 7 \cdot \cancel{6!}}{\cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{6!}} = \boxed{84 \text{ pizzas}}$$

EXAMPLE 3: COMBINATION OR PERMUTATION?

A locker contains eight books. You select three books at random. How many different sets of books could you select?

$${}_8C_3 = \frac{8!}{3!5!} = \frac{8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5!}}{\cancel{3 \cdot 2} \cdot 1 \cdot \cancel{5!}}$$

56 sets

EXAMPLE 3: QUICK CHECK

You take four books out of the library to read during spring break. In how many different orders can you read the four books?

$$_4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4\cdot3\cdot2\cdot1}{1} = \boxed{\text{24 orders}}$$

EXAMPLE 3: QUICK CHECK

You want to have three servings of dairy products without having the same food more than once. Milk, yogurt, cottage cheese, and cheddar cheese are in the refrigerator. How many different ways can you get your dairy?

$_4P_3 \quad \frac{4!}{(4-3)!} \quad \frac{4\cdot 3\cdot 2\cdot 1}{1}$

24 ways

$\underset{S1}{4} \cdot \underset{S2}{3} \cdot \underset{S3}{2}$

24 ways

EXAMPLE 3: QUICK CHECK

Suppose you have nine different shirts. How many ways can you select five shirts to wear in order?

$$ {}_{9}P_{5} = \frac{9!}{4!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = 15120 $$

ways