Notes 4 B The Power of Compounding Day 1

On July 18, 1461, King Edward IV of England borrowed the modern equivalent of $384 from New College of Oxford. The King soon paid back $160, but never repaid the remaining $224. The debt was forgotten for 535 years. Upon its rediscovery in 1996, a New College administrator wrote to the Queen of England asking for repayment with interest. Assuming an interest rate of 4% per year, he calculated that the college was owed $290 billion. ….Is this possible????? How were interest calculated????

Principal	In financial formulas is the _____upon which interest is paid.
Simple Interest	Interest is paid on your _____or_____ and **NOT** on any _____
Compound Interest	Interest paid on _____ on the _____ and on _____

SIMPLE Interest	**COMPOUND Interest**
Imagine you deposit $1000 in Honest John's Money Holding Service, which promises to pay 5% interest each year.	Suppose you place $1000 in a bank account that pays the same 5% interest per year. But instead of paying you the interest directly, the bank <u>adds</u> the interest to your account.
After one year:	After one year:
After the second year:	After the second year:
After the third year:	After the third year:

Example 1) Savings Bond

While banks almost always pay compound interest, bonds usually pay simple interest. Suppose you invest $1000 in a savings bond that pays simple interest of 10% per year. How much total interest will you receive in 5 years? If the bond paid compound interest, would you receive more or less total interest? Explain.

End of Year	**Simple Interest**		**Compound Interest**	
	Interest	New Balance	Interest	New Balance

THE COMPOUND INTEREST FORMULA
(For Interest Paid Once a Year)

$$A = P \times (1 + APR)^{Y}$$

Where:

A = Accumulated balance after *Y* years. Also called Future Value (FV)

P = Starting Principal. Also called Present Value (PV)

APR = Annual Percentage Rate. Express it as a decimal!!!

Y = Number of Years

Example: P=$100, APR= 10%, and Y=5 years, Find Accumulated balance A = ?

Procedure to use with calculator:

$$A = 100 \times (1 + 0.1)^{5}$$

	Calculator Steps	Ouput
Step 1. Parentheses	**1 + 0.1**	**=**
Step 2. Exponent	**^ 5**	**=**
Step 3. Multiply	**x 100**	**=**

Note: It is very important that you not round any of the answers intermediate steps, even though you will round the final answer to the nearest cent.

Example 2) Simple and Compound Interest

You invest $100 in two accounts that each pay an interest rate of 10% per year. However, one account pays simple interest and one account pays compound interest. Make a table that shows the growth of each account over a 5-year period. Use the compound interest formula to verify the result in the table for the compound interest case.

End of Year	**Simple Interest**		**Compound Interest**	
	Interest	New Balance	Interest	New Balance

Use the compound formula now:

Example 3) King Edward IV of England New College Debt

Calculate the amount due to New College if the interest rate is 2% and 4%, using

a. simple interest

At 2%= _____

At 4%= _____

b. compound interest

At 2%= _____

At 4%= _____

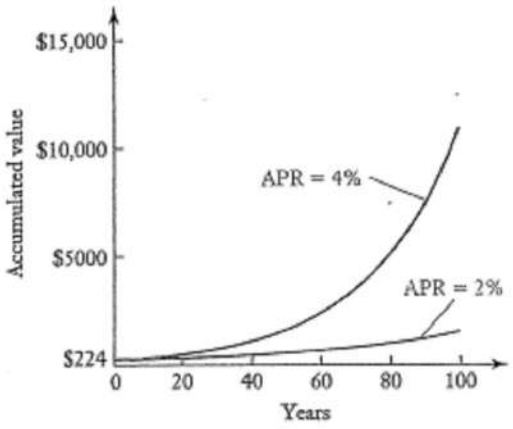

FIGURE 4.3 This figure contrasts the debt in the New College case during the first 100 years at interest rates of 2% and 4%.
Note that the rate change does not make much difference for the first few years, but over time the higher rate becomes far more valuable.

Compound Interest paid More Than Once a Year

THE COMPOUND INTEREST FORMULA
(For Interest Paid "*n*" times per Year)

$$A = P \times \left(1 + \frac{APR}{n}\right)^{(nY)}$$

Where:

A = Accumulated balance after *Y* years. Also called Future Value (FV)

P = Starting Principal. Also called Present Value (PV)

APR = Annual Percentage Rate. Express it as a decimal!!!

***n* = Number of compounding periods per year**

Y = Number of Years

Note that Y is not necessarily an integer; for example, a calculation for three and a half years would have Y = 3.5

Example: P=$5000, APR= 3%, n = 12, and Y=5 years, Find Accumulated balance A = ?

Procedure to use with calculator:

$$A = 5000 \times \left(1 + \frac{0.03}{12}\right)^{(12 \times 5)}$$

	Calculator Steps	Ouput
Step 1. Parentheses	**1 + 0.03 ÷ 12**	**=**
Step 2. Exponent	**^ (12 × 5)**	**=**
Step 3. Multiply	**x 5000**	**=**

Note: It is very important that you not round any of the answers intermediate steps, even though you will round the final answer to the nearest cent.

Example 4) Monthly Compounding at 3%

You deposit $5000 in a bank account that pays an APR of 3% and compounds interest monthly. How much money will you have after 5 years? Compare this amount to the amount you'd have if interest were paid only once each year.

Example 5) Mattress Investments

Your grandfather put $100 under his mattress 50 years ago. If he had instead invested it in a bank account paying 3.5% interest compounded yearly (roughly the average U.S. rate of inflation during that period), how much would it be worth now?

Annual Percentage Yield (APY):

$$APY = relative \cdot increase \cdot in \cdot 1 \cdot year = \frac{absolute \cdot increase}{starting \cdot principal}$$

- Is the actual percentage by which a balance increases **in one year**.
- It is equal to the APR if interest is **compounded annually**.
- It is greater than the APR if interest is compounded **more than once a year**.
- The APY **does not** depend on the starting principal.
- The APY is sometimes also called the **effective yield** or simply the **yield**.

Example 6) More Compounding Means a Higher Yield

You deposit $1000 into an account with APR = 8%. Find the annual percentage yield with *monthly* compounding and with *daily* compounding.

Continuous Compounding

TABLE 4.4 Annual Yield (APY) for APR = 8% with Various Numbers of Compounding Periods (n)

n	APY	n	APY
1	8.0000000%	1000	8.3283601%
4	8.2432160%	10,000	8.3286721%
12	8.2999507%	1,000,000	8.3287064%
365	8.3277572%	10,000,000	8.3287067%
500	8.3280135%	1,000,000,000	8.3287068%

THE CONTINUOUS COMPOUND INTEREST FORMULA
(For Interest Paid "*n*" times per Year)

$$A = P \times e^{(APR \bullet Y)}$$

Where:

A = Accumulated balance after *Y* years. Also called Future Value (FV)

P = Starting Principal. Also called Present Value (PV)

APR = Annual Percentage Rate. Express it as a decimal!!!

Y = Number of Years

Example 7) Continuous Compounding

You deposit $100 in an account with an APR of 8% and continuous compounding. How much will you have after 10 years?

Planning Ahead with Compound Interest:

Example 8) College Fund at 3%

Suppose you put money in an investment with an interest rate of APR = 3%, compounded annually, and leave it there for the next 18 years. How much would you have to deposit now to realize $100,000 after 18 years?

Example 9) College Fund at 5%, Compounded Monthly

Repeat Example 8, but with an interest rate of 5% and monthly compounding. Compare the results.