5.3

Solving Trigonometric Equations

What You Should Learn

- Use standard algebraic techniques to solve trigonometric equations.
- Solve trigonometric equations of quadratic type.
- Solve trigonometric equations involving multiple angles.
- Use inverse trigonometric functions to solve trigonometric equations.

Introduction

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To solve a trigonometric equation, use standard algebraic techniques such as collecting like terms and factoring.

Your preliminary goal is to isolate the trigonometric function involved in the equation.

Example 1 – *Solving a Trigonometric Equation*

Solve $2 \sin x - 1 = 0$. Copy this slide, but not the next.

Solution:

$2 \sin x - 1 = 0$ Write original equation.

$2 \sin x = 1$ Add 1 to each side.

$\sin x = \frac{1}{2}$ Divide each side by 2.

Example 1 – *Solution*

cont'd

To solve for *x*, note in Figure 5.4 that the equation $\sin x = \frac{1}{2}$ has solutions $x = \pi/6$ and $x = 5\pi/6$ in the interval $[0, 2\pi)$.

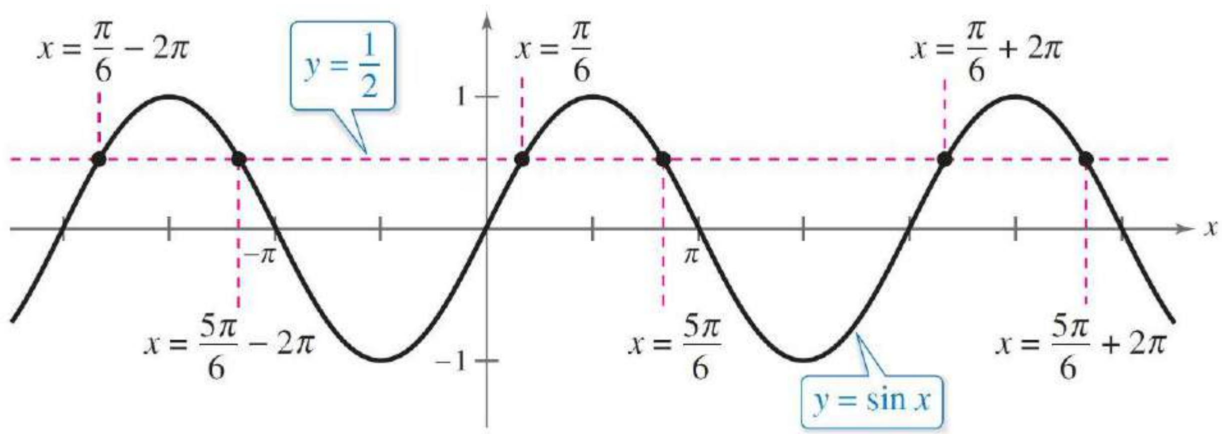

Figure 5.4

Example 1 – *Solution*

cont'd

Moreover, because sin x has a period of 2π, there are infinitely many other solutions, which can be written as

$$x = \frac{\pi}{6} + 2n\pi \quad \text{and} \quad x = \frac{5\pi}{6} + 2n\pi$$

General solution

where n is an integer, as shown in Figure 5.4.

Equations of Quadratic Type

Equations of Quadratic Type

Many trigonometric equations are of quadratic type $ax^2 + bx + c = 0$, as shown below. To solve equations of this type, factor the quadratic or, when factoring is not possible, use the Quadratic Formula.

Quadratic in sin x	*Quadratic in sec x*
$2\sin^2 x - \sin x - 1 = 0$	$\sec^2 x - 3\sec x - 2 = 0$
$(\sin x)^2 - \sin x - 1 = 0$	$(\sec x)^2 - 3\sec x - 2 = 0$

Example 5 – *Factoring an Equation of Quadratic Type*

Find all solutions of $2 \sin^2 x - \sin x - 1 = 0$ in the interval $[0, 2\pi)$.

Solution:

Treating the equation as a quadratic in $\sin x$ and factoring produces the following.

$$2 \sin^2 x - \sin x - 1 = 0 \qquad \text{Write original equation.}$$

$$(2 \sin x + 1)(\sin x - 1) = 0 \qquad \text{Factor.}$$

Example 5 – *Solution*

cont'd

Setting each factor equal to zero, you obtain the following solutions in the interval $[0, 2\pi)$.

$2 \sin x + 1 = 0$ and $\sin x - 1 = 0$

$$\sin x = -\frac{1}{2} \qquad \sin x = 1$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6} \qquad x = \frac{\pi}{2}$$

Functions Involving Multiple Angles

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The next example involves trigonometric functions of multiple angles of the forms sin *ku* and cos *ku*.

To solve equations of these forms, first solve the equation for *ku,* then divide your result by *k*.

Example 8 – *Functions Involving Multiple Angles*

Solve $2 \cos 3t - 1 = 0$

Solution:

$2 \cos 3t - 1 = 0$ Write original equation.

$2 \cos 3t = 1$ Add 1 to each side.

$\cos 3t = \frac{1}{2}$ Divide each side by 2.

Example 8 – *Solution*

cont'd

In the interval $[0, 2\pi)$, you know that $3t = \pi/3$ and $3t = 5\pi/3$ are the only solutions. So in general, you have
$3t = \pi/3 + 2n\pi$ and $3t = 5\pi/3 + 2n\pi$.

Dividing this result by 3, you obtain the general solution

$$t = \frac{\pi}{9} + \frac{2n\pi}{3} \quad \text{and} \quad t = \frac{5\pi}{9} + \frac{2n\pi}{3}$$ General solution

where n is an integer. This solution is confirmed graphically in Figure 5.12.

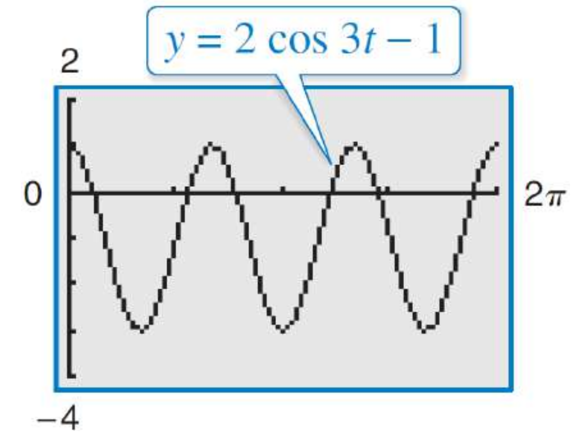

Figure 5.12