4

Trigonometric Functions

4.8

Applications and Models

What You Should Learn

- Solve real-life problems involving right triangles.
- Solve real-life problems involving directional bearings.
- Solve real-life problems involving harmonic motion.

Applications Involving Right Triangles

Applications Involving Right Triangles

Instructions for setting up your triangle:

The three angles of a right triangle are denoted by the letters *A*, *B* and *C* (where *C* is the right angle), and the lengths of the sides opposite these angles by the letters *a*, *b* and *c* (where *c* is the hypotenuse).

Example 1 – *Solving a Right Triangle*

Solve the right triangle shown in Figure 4.77 for all unknown sides and angles.

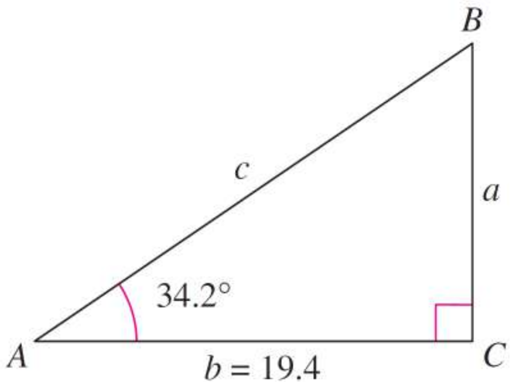

Figure 4.77

Solution:

Because $C = 90°$, it follows that

$$A + B = 90°$$

and

$$B = 90° - 34.2°$$

$$= 55.8°.$$

Example 1 – *Solution*

cont'd

To solve for *a*, use the fact that

$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{a}{b} \quad \Rightarrow \quad a = b \tan A$$

So, $a = 19.4 \tan 34.2° \approx 13.18$. Similarly, to solve for *c*, use the fact that

$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{b}{c} \quad \Rightarrow \quad c = \frac{b}{\cos A}.$$

$$\text{So, } c = \frac{19.4}{\cos 34.2°} \approx 23.46$$

Trigonometry and Bearings

Trigonometry and Bearings

In surveying and navigation, directions are generally given in terms of **bearings.** A bearing measures the acute angle a path or line of sight makes with a fixed north-south line, as shown in Figure 4.81. For instance, the bearing of S 35° E in Figure 4.81(a) means 35° degrees east of south.

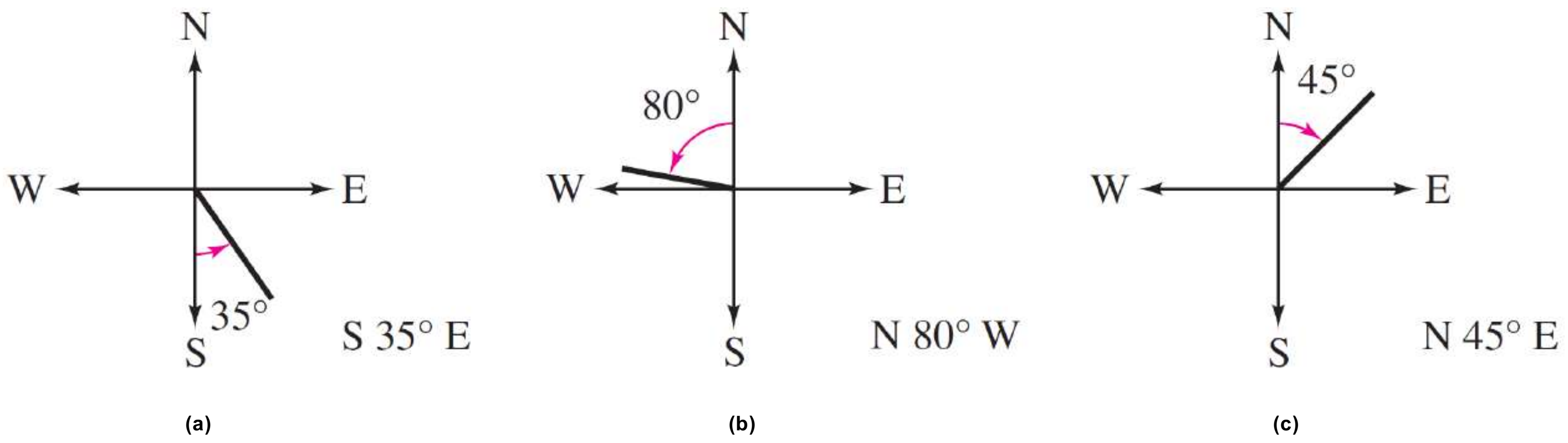

Figure 4.81

Example 5 – *Finding Directions in Terms of Bearings*

A ship leaves port at noon and heads due west at 20 knots, or 20 nautical miles (nm) per hour. At 2 P.M. the ship changes course to N 54° W, as shown in Figure 4.82. Find the ship's bearing and distance from the port of departure at 3 P.M.

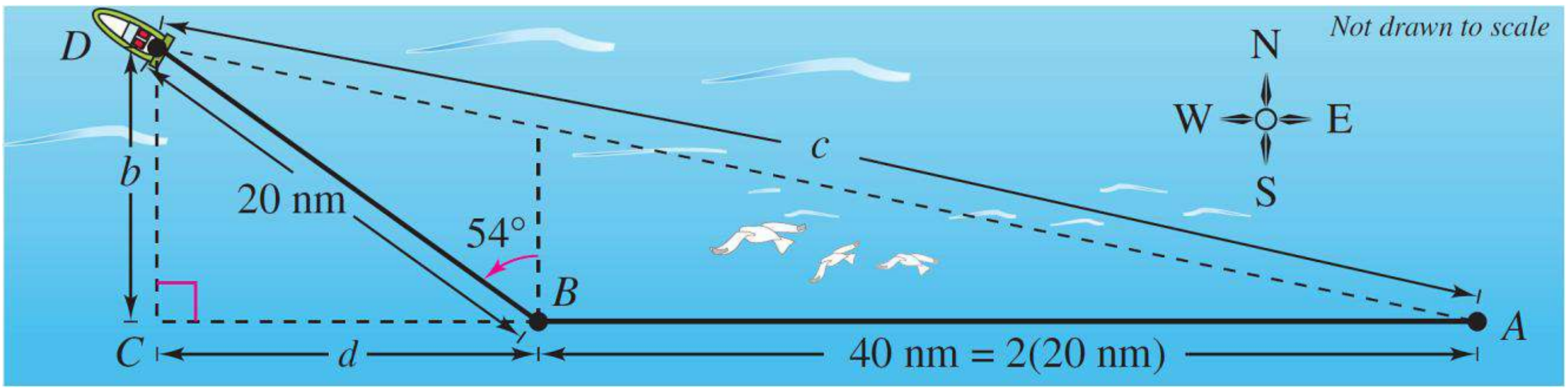

Figure 4.82

Example 5 – *Solution*

For triangle *BCD* you have

$B = 90° - 54°$

$= 36°.$

The two sides of this triangle can be determined to be

$b = 20 \sin 36°$ and $d = 20 \cos 36°$.

In triangle *ACD,* you can find angle *A* as follows.

$$\tan A = \frac{b}{d + 40} = \frac{20 \sin 36°}{20 \cos 36° + 40}$$

Example 5 – *Solution*

cont'd

$$A \approx \arctan 0.2092494$$

$$\approx 0.2062732 \text{ radian}$$

$$\approx 11.82$$

The angle with the north-south line is

$90° - 11.82° = 78.18°$.

Example 5 – *Solution*

cont'd

So, the bearing of the ship is N 78.18° Finally, from triangle *ACD* you have

$$\sin A = \frac{b}{c}$$

which yields

$$c = \frac{b}{\sin A}$$

$$= \frac{20 \sin 36^\circ}{\sin 11.82^\circ}$$

$$\approx 57.39 \text{ nautical miles}$$

Distance from port

Harmonic Motion

Harmonic Motion

The periodic nature of the trigonometric functions is useful for describing the motion of a point on an object that vibrates, oscillates, rotates, or is moved by wave motion.

For example, consider a ball that is bobbing up and down on the end of a spring, as shown in Figure 4.83.

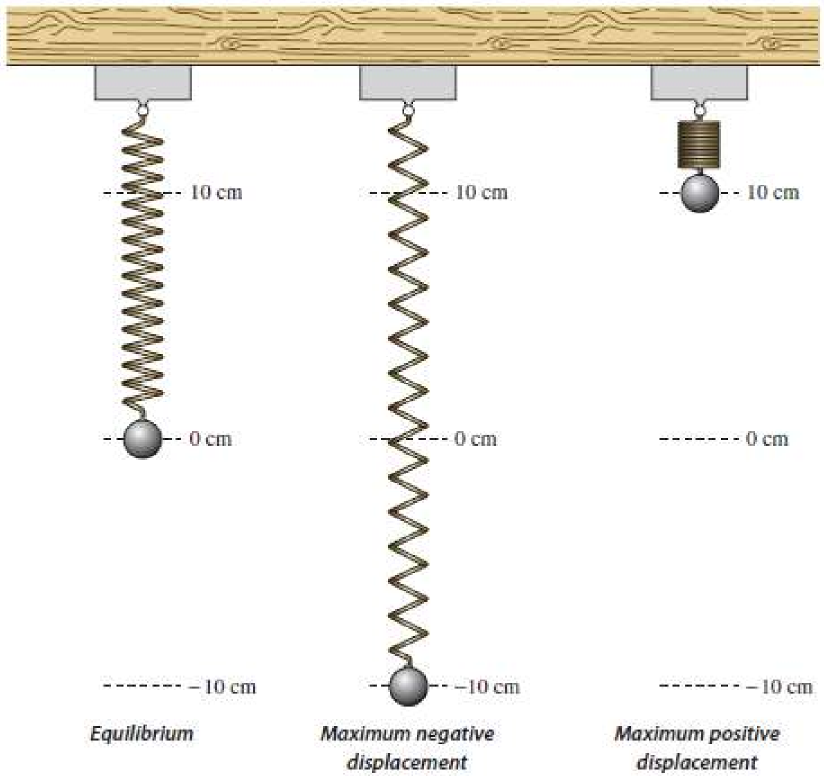

Figure 4.83

Harmonic Motion

Suppose that 10 centimeters is the maximum distance the ball moves vertically upward or downward from its equilibrium (at-rest) position. Suppose further that the time it takes for the ball to move from its maximum displacement above zero to its maximum displacement below zero and back again is

t = 4 seconds.

Assuming the ideal conditions of perfect elasticity and no friction or air resistance, the ball would continue to move up and down in a uniform and regular manner.

Harmonic Motion

From this spring you can conclude that the period (time for one complete cycle) of the motion is

Period = 4 seconds

its amplitude (maximum displacement from equilibrium) is

Amplitude = 10 centimeters

and its **frequency** (number of cycles per second) is

Frequency = $\frac{1}{4}$ cycle per second.

Motion of this nature can be described by a sine or cosine function, and is called **simple harmonic motion.**

Harmonic Motion

Definition of Simple Harmonic Motion

A point that moves on a coordinate line is said to be in **simple harmonic motion** when its distance d from the origin at time t is given by either

$$d = a \sin \omega t \qquad \text{or} \qquad d = a \cos \omega t$$

where a and ω are real numbers such that $\omega > 0$. The motion has amplitude $|a|$, period $2\pi/\omega$, and frequency $\omega/(2\pi)$.

Example 6 – *Simple Harmonic Motion*

Write the equation for the simple harmonic motion of the ball illustrated in Figure 4.83, where the period is 4 seconds. What is the frequency of this motion?

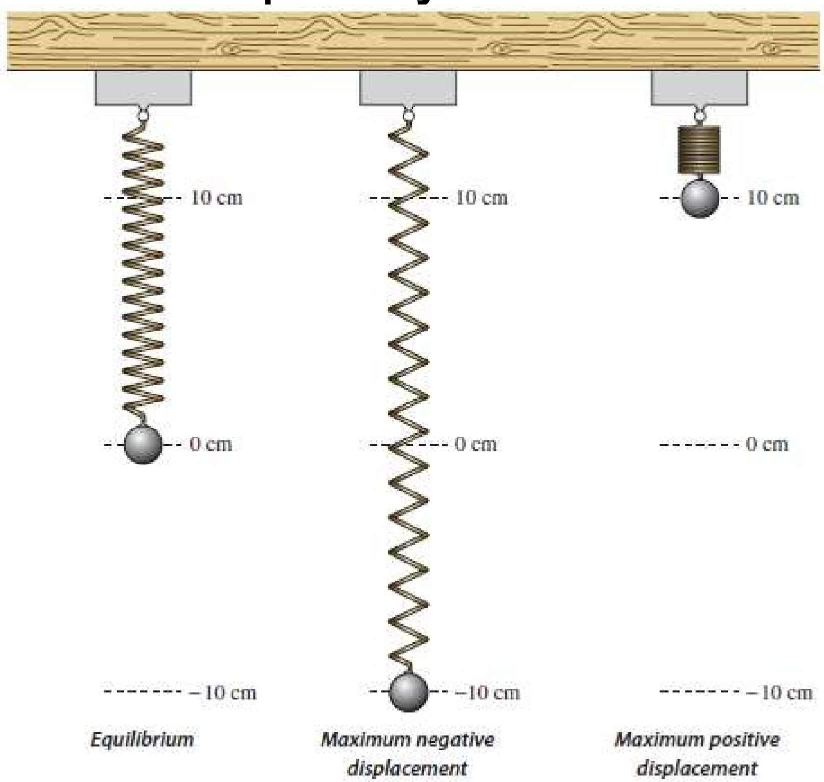

Figure 4.83

Example 6 – *Solution*

Because the spring is at equilibrium ($d = 0$) when $t = 0$, you use the equation

$$d = a \sin \omega t.$$

Moreover, because the maximum displacement from zero is 10 and the period is 4, you have the following.

Amplitude = $|a| = 10$

$$= \text{Period} = \frac{2\pi}{\omega} \quad \Rightarrow \quad \omega = \frac{\pi}{2}$$

Consequently, the equation of motion is

$$d = 10 \sin \frac{\pi}{2} t.$$

Example 6 – *Solution*

cont'd

Note that the choice of a = 10 or a = –10 depends on whether the ball initially moves up or down. The frequency is

$$\text{Frequency} = \frac{\omega}{2\pi}$$

$$= \frac{\pi/2}{2\pi}$$

$$= \frac{1}{4}$$

cycle per secor