4

Trigonometric Functions

4.3

Right Triangle Trigonometry

What You Should Learn

- Evaluate trigonometric functions of acute angles and use a calculator to evaluate trigonometric functions.
- Use the fundamental trigonometric identities.
- Use trigonometric functions to model and solve real-life problems.

The Six Trigonometric Functions

The Six Trigonometric Functions

Consider a right triangle, with one acute angle labeled θ, as shown in Figure 4.24. Relative to the angle θ, the three sides of the triangle are the **hypotenuse,** the **opposite side** (the side opposite the angle θ), and the **adjacent side** (the side adjacent to the angle θ).

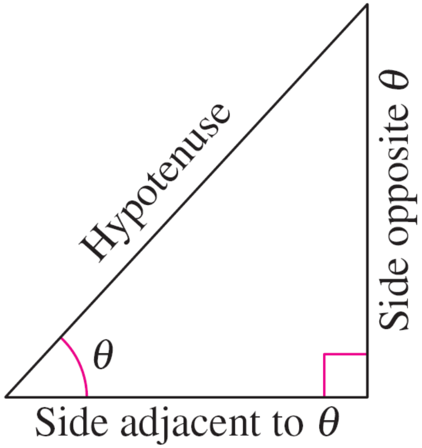

Figure 4.24

The Six Trigonometric Functions

Using the lengths of these three sides, you can form six ratios that define the six trigonometric functions of the acute angle θ.

sine	**cosecant**
cosine	**secant**
tangent	**cotangent**

In the following definitions it is important to see that

$$0° < \theta < 90°$$

θ lies in the first quadrant

and that for such angles the value of each trigonometric function is *positive*.

The Six Trigonometric Functions

Right Triangle Definitions of Trigonometric Functions

Let θ be an *acute* angle of a right triangle. Then the six trigonometric functions *of the angle* θ are defined as follows. (Note that the functions in the second column are the *reciprocals* of the corresponding functions in the first column.)

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

The abbreviations

"opp," "adj," and "hyp"

represent the lengths of the three sides of a right triangle.

opp = the length of the side *opposite* θ

adj = the length of the side *adjacent* to θ

hyp = the length of the *hypotenuse*

Example 1 – *Evaluating Trigonometric Functions*

Use the triangle in Figure 4.25 to find the exact values of the six trigonometric functions of θ.

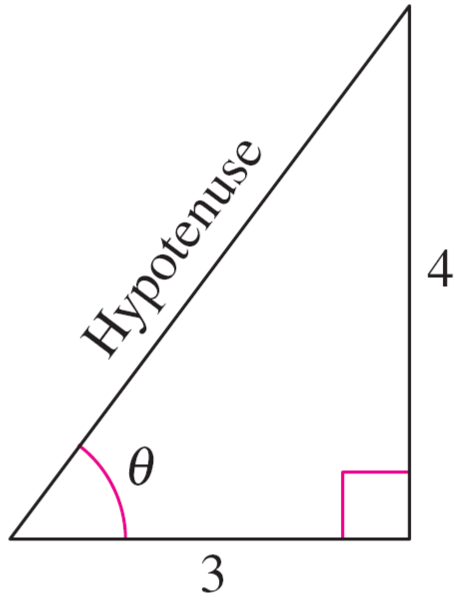

Figure 4.25

Example 1 – *Solution*

cont'd

By the Pythagorean Theorem, $(\text{hyp})^2 = (\text{opp})^2 + (\text{adj})^2$, it follows that

$$\text{hyp} = \sqrt{4^2 + 3^2}$$

$$= \sqrt{25}$$

$$= 5.$$

Example 1 – *Solution*

cont'd

So, the six trigonometric functions of θ are

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5} \qquad \csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5} \qquad \sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3} \qquad \cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$$

The Six Trigonometric Functions

Sines, Cosines, and Tangents of Special Angles

$\sin 30° = \sin \frac{\pi}{6} = \frac{1}{2}$	$\cos 30° = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$	$\tan 30° = \tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$
$\sin 45° = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$\cos 45° = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$\tan 45° = \tan \frac{\pi}{4} = 1$
$\sin 60° = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$	$\cos 60° = \cos \frac{\pi}{3} = \frac{1}{2}$	$\tan 60° = \tan \frac{\pi}{3} = \sqrt{3}$

In the box, note that $\sin 30° = \frac{1}{2} = \cos 60°$. This occurs because 30° and 60° are complementary angles, and, in general, it can be shown from the right triangle definitions that *cofunctions of complementary angles are equal*.

The Six Trigonometric Functions

That is, if θ is an acute angle, then the following relationships are true.

$\sin(90° - \theta) = \cos\theta$ $\qquad$ $\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$ $\qquad$ $\cot(90° - \theta) = \tan\theta$

$\sec(90° - \theta) = \csc\theta$ $\qquad$ $\csc(90° - \theta) = \sec\theta$

Example 4 – *Using a Calculator*

Use a calculator to evaluate cos(5° 40′ 12″).

Solution:

Begin by converting to decimal degree form.

$$5° \, 40' \, 12'' = 5° + \left(\frac{40}{60}\right)^{\circ} + \left(\frac{12}{3600}\right)^{\circ} = 5.67°$$

Then use a calculator in *degree* mode to evaluate cos 5.67°

Function	*Graphing Calculator Keystrokes*	*Display*
cos(5° 40′ 12″) = cos 5.67°	COS (5.67) ENTER	0.9951074

Trigonometric Identities

Trigonometric Identities

In trigonometry, a great deal of time is spent studying relationships between trigonometric functions (identities).

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Example 5 – *Applying Trigonometric Identities*

Let θ be an acute angle such that cos $\theta = 0.8$. Find the values of (a) sin θ and (b) tan θ using trigonometric identities.

Solution:

a. To find the value of sin θ, use the Pythagorean identity

$$\sin^2\theta + \cos^2\theta = 1.$$

So, you have

$$\sin^2\theta + (0.8)^2 = 1.$$

Substitute 0.8 for cos θ.

Example 5 – *Solution*

cont'd

$\sin^2 \theta = 1 - (0.8)^2$ Subtract $(0.8)^2$ from each side.

$\sin^2 \theta = 0.36$ Simplify.

$\sin \theta = \sqrt{0.36}$ Extract positive square root.

$\sin \theta = 0.6$ Simplify.

Example 5 – *Solution*

cont'd

b. Now, knowing the sine and cosine of θ, you can find the tangent of θ to be

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{0.6}{0.8}$$

$$= 0.75.$$

Use the definitions of sin θ and tan θ and the triangle shown in Figure 4.28 to check these results.

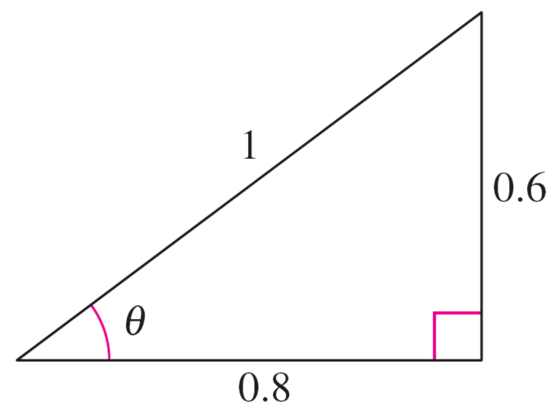

Figure 4.28

Applications

Applications

Many applications of trigonometry involve a process called **solving right triangles.**

In this type of application, you are usually given one side of a right triangle and one of the acute angles and are asked to find one of the other sides, *or* you are given two sides and are asked to find one of the acute angles.

Applications

In Example 8, the angle you are given is the **angle of elevation,** which represents the angle from the horizontal upward to the object.

In other applications you may be given the **angle of depression,** which represents the angle from the horizontal downward to the object. (See Figure 4.30.)

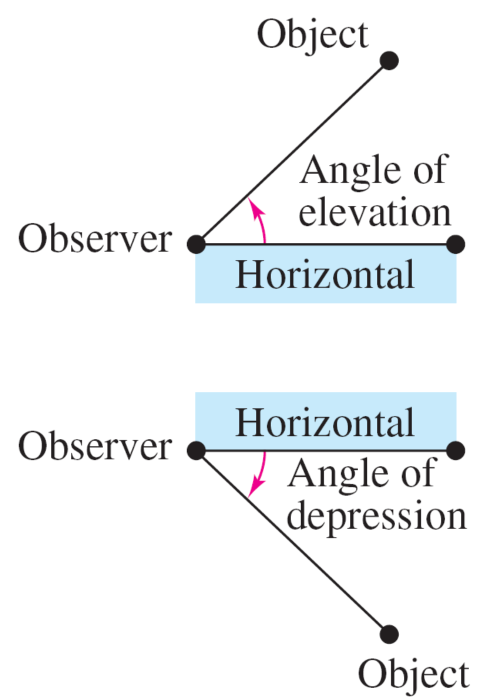

Figure 4.30

Example 8 – *Using Trigonometry to Solve a Right Triangle*

A surveyor is standing 50 feet from the base of a large tree, as shown in Figure 4.31. The surveyor measures the angle of elevation to the top of the tree as 71.5°. How tall is the tree?

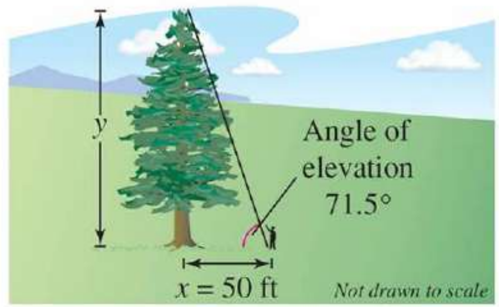

Figure 4.31

Solution:

From Figure 4.31, you can see that

$$\tan 71.5° = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

Example 8 – *Solution*

cont'd

Where x = 50 and y is the height of the tree. So, the height of the tree is

$$y = x \tan 71.5°$$

$$= 50 \tan 71.5°$$

$$\approx 149.43 \text{ feet.}$$