1.5 Combinations of Functions

What You Should Learn

- Add, subtract, multiply, and divide functions.
- Find compositions of one function with another function.
- Use combinations of functions to model and solve real-life problems.

Arithmetic Combinations of Functions

Arithmetic Combinations of Functions

Just as two real numbers can be combined by the operations of addition, subtraction, multiplication, and division to form other real numbers, two *functions* can be combined to create new functions. When

$f(x) = 2x - 3$ and $g(x) = x^2 - 1$

you can form the sum, difference, product, and quotient of *f* and *g* as follows.

$$f(x) + g(x) = (2x - 3) + (x^2 - 1)$$

$$= x^2 + 2x - 4$$

Sum

Arithmetic Combinations of Functions

$f(x) - g(x) = (2x - 3) - (x^2 - 1)$

$= -x^2 + 2x - 2$ Difference

$f(x) \cdot g(x) = (2x - 3)(x^2 - 1)$

$= 2x^3 - 3x^2 - 2x + 3$ Product

$$\frac{f(x)}{g(x)} = \frac{2x - 3}{x^2 - 1}, \quad x \neq \pm 1$$ Quotient

Arithmetic Combinations of Functions

The domain of an **arithmetic combination** of functions *f* and *g* consists of all real numbers that are common to the domains of *f* and *g*. In the case of the quotient

$$\frac{f(x)}{g(x)}$$

there is the further restriction that $g(x) \neq 0$.

Example 1 – *Finding the Sum of Two Functions*

Given $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$, find $(f + g)(x)$. Then evaluate the sum when $x = 2$.

Solution:

$$(f + g)(x) = f(x) + g(x)$$

$$= (2x + 1) + (x^2 + 2x - 1)$$

$$= x^2 + 4x$$

When $x = 2$, the value of this sum is

$$(f + g)(2) = 2^2 + 4(2)$$

$$= 12.$$

Compositions of Functions

Compositions of Functions

Another way of combining two functions is to form the **composition** of one with the other.

For instance, when $f(x) = x^2$ and $g(x) = x + 1$, the composition of *f* with *g* is

$$\begin{aligned} f(g(x)) &= f(x + 1) \\ &= (x + 1)^2. \end{aligned}$$

This composition is denoted as $f \circ g$ and is read as "*f* composed with *g*".

Compositions of Functions

Definition of Composition of Two Functions

The **composition** of the function f with the function g is

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f. (See Figure 1.50.)

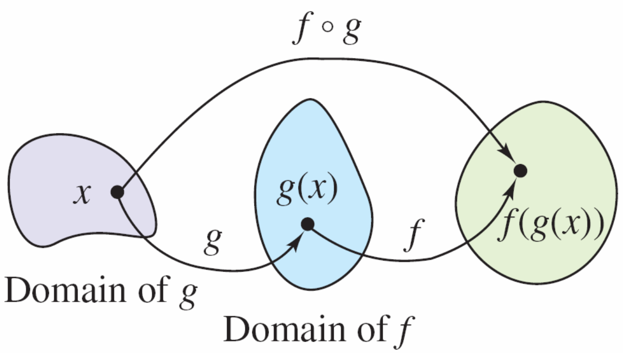

Figure 1.50

Example 5 – *Forming the Composition of f with g*

Find $(f \circ g)(x)$ for $f(x) = \sqrt{x},\ x \geq 0,$ and $g(x) = x - 1,\ x \geq 1.$
If possible, find $(f \circ g)(2)$ and $(f \circ g)(0)$.

Solution:

The composition of *f* with *g* is

$(f \circ g)(x) = f(g(x))$ — Definition of $f \circ g$

$= f(x - 1)$ — Definition of $g(x)$

$= \sqrt{x - 1}, \quad x \geq 1.$ — Definition of $f(x)$

Example 5 – *Solution*

cont'd

The domain of $f \circ g$ is $[1, \infty)$ (See Figure 1.51).

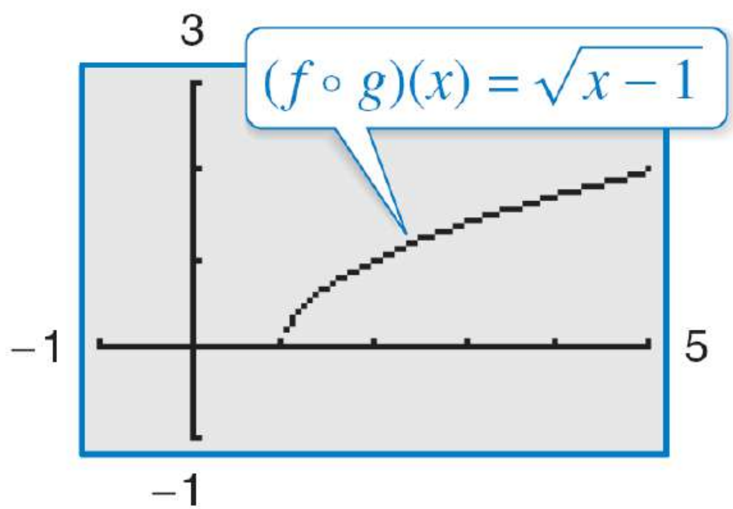

Figure 1.51

So,

$(f \circ g)(2) =$ $= \sqrt{2 - 1}$

is defined, but $(f \circ g)(0)$ is not defined because 0 is not in the domain of $f \circ g$.

Application

Example 10 – *Bacteria Count*

The number N of bacteria in a refrigerated petri dish is given by

$$N(T) = 20T^2 - 80T + 500, \quad 2 \le T \le 14$$

where T is the temperature of the petri dish (in degrees Celsius). When the petri dish is removed from refrigeration, the temperature of the petri dish is given by

$$T(t) = 4t + 2, \quad 0 \le t \le 3$$

where t is the time (in hours).

a. Find the composition $N(T(t))$ and interpret its meaning in context.

Example 10 – *Bacteria Count*

cont'd

b. Find the number of bacteria in the petri dish when $t = 2$ hours.

c. Find the time when the bacteria count reaches 2000.

Solution:

a. $N(T(t)) = 20(4t + 2)^2 - 80(4t + 2) + 500$

$$= 20(16t^2 + 16t + 4) - 320t - 160 + 500$$

$$= 320t^2 + 320t + 80 - 320t - 160 + 500$$

$$= 320t^2 + 500$$

The composite function $N(T(t))$ represents the number of bacteria as a function of the amount of time the petri dish has been out of refrigeration.

Example 10 – *Solution*

cont'd

b. When $t = 2$ the number of bacteria is

$N = 320(2)^2 + 420$

$= 1280 + 420$

$= 1700.$

c. The bacteria count will reach $N = 2000$ when $320t^2 + 420 = 2000$.

You can solve this equation for t algebraically as follows.

$320t^2 + 420 = 2000$

$320t^2 = 1580$

Example 10 – *Solution*

cont'd

$$t^2 = \frac{79}{16}$$

$$t = \frac{\sqrt{79}}{4} \quad \Rightarrow \quad t \approx 2.22 \text{ hours}$$

So, the count will reach 2000 when $t \approx 2.22$ hours. Note that the negative value is rejected because it is not in the domain of the composite function.

Example 10 – *Solution*

cont'd

To confirm your solution, graph the equation $N = 320t^2 + 420$ as shown in Figure 1.54. Then use the *zoom* and *trace* features to approximate $N = 2000$ when $t \approx 2.22$ as shown in Figure 1.55.

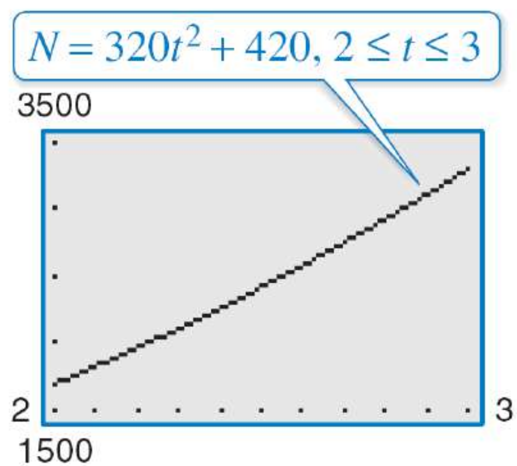

Figure 1.54

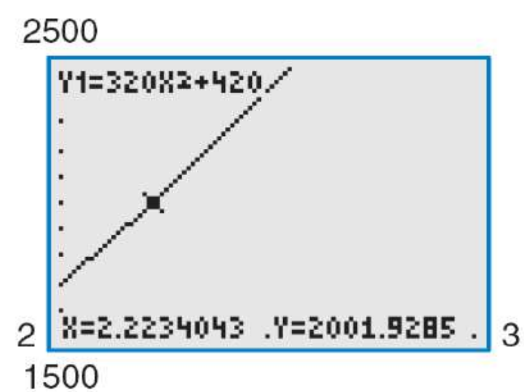

Figure 1.55