1.1 Lines in the Plane

What You Should Learn

- Find the slopes of lines.
- Write linear equations given points on lines and their slopes.
- Use slope-intercept forms of linear equations to sketch lines.
- Use slope to identify parallel and perpendicular lines.

The Slope of a Line

In this section, you will study lines and their equations. The **slope** of a nonvertical line represents the number of units the line rises or falls vertically for each unit of horizontal change from left to right.

For instance, consider the two points

(x_1, y_1) and (x_2, y_2)

on the line shown in Figure 1.1.

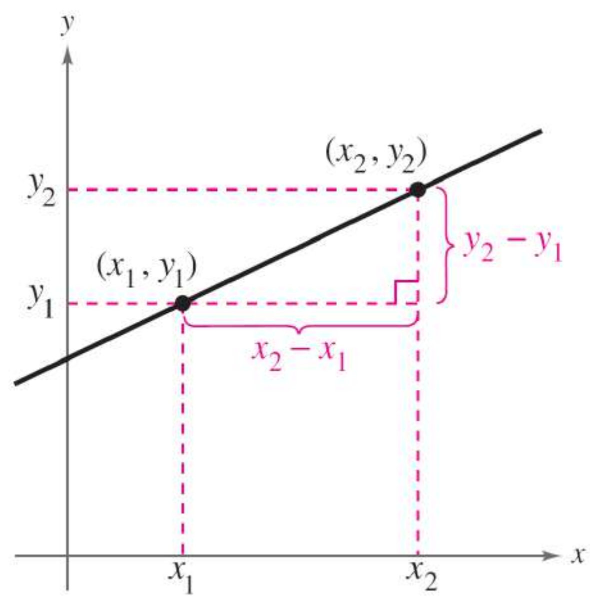


Figure 1.1

The Slope of a Line

Definition of the Slope of a Line

The **slope** m of the nonvertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

where $x_1 \neq x_2$.

Example 1 – *Finding the Slope of a Line*

Find the slope of the line passing through each pair of points.

a. (–2, 0) and (3, 1)

b. (–1, 2) and (2, 2)

c. (0, 4) and (1, –1)

Example 1 – *Solution*

a. $$m = \overbrace{y_2 - y_1}^{\text{Difference in } y\text{-values}} \Big/ \underbrace{x_2 - x_1}_{\text{Difference in } x\text{-values}} = \frac{1 - 0}{3 - (-2)} = \frac{1}{3 + 2} = \frac{1}{5}$$

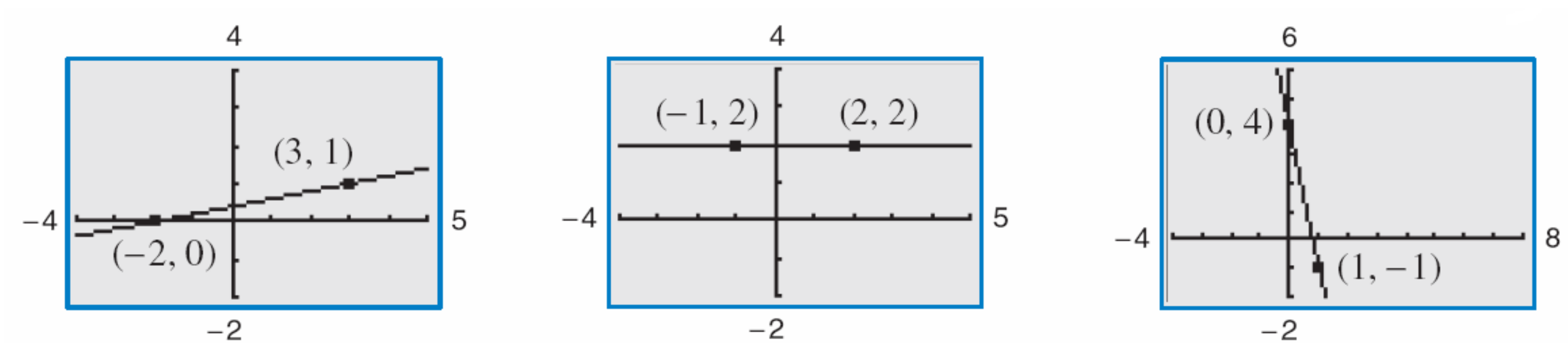
b. $$m = \frac{2 - 2}{2 - (-1)} = \frac{0}{3} = 0$$

c. $$m = \frac{-1 - 4}{1 - 0} = \frac{-5}{1} = -5$$

Example 1 – *Solution*

cont'd

The graphs of the three lines are shown in Figure 1.2. Note that the square setting gives the correct "steepness" of the lines.



(a) **(b)** **(c)**

Figure 1.2

The Slope of a Line

The Slope of a Line

1. A line with positive slope $(m > 0)$ *rises* from left to right.

2. A line with negative slope $(m < 0)$ *falls* from left to right.

3. A line with zero slope $(m = 0)$ is *horizontal*.

4. A line with undefined slope is *vertical*.

The Point-Slope Form of the Equation of a Line

The Point-Slope Form of the Equation of a Line

This equation in the variables and can be rewritten in the **point-slope form** of the equation of a line.

Point-Slope Form of the Equation of a Line

The **point-slope form** of the equation of the line that passes through the point (x_1, y_1) and has a slope of m is

$$y - y_1 = m(x - x_1).$$

Example 2 – *The Point-Slope Form of the Equation of a Line*

Find an equation of the line that passes through the point (1, –2) and has a slope of 3.

Solution:

$$y - y_1 = m(x - x_1)$$ **Point-slope form.**

$$y - (-2) = 3(x - 1)$$ **Substitute for y_1, m and x_1.**

$$y + 2 = 3x - 3$$ **Simplify.**

$$y = 3x - 5$$ **Solve for y.**

Example 2 – *Solution*

cont'd

The line is shown in Figure 1.5.

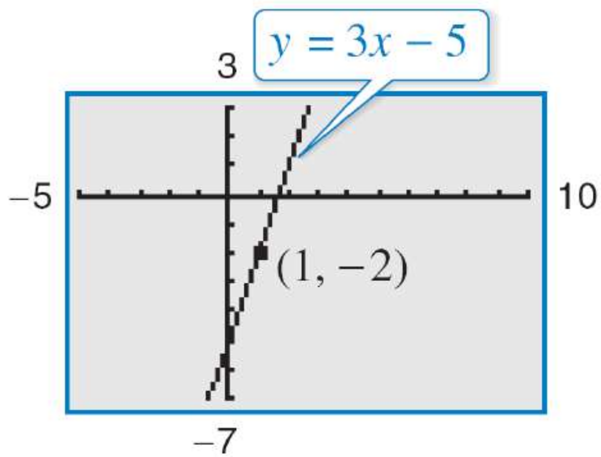


Figure 1.5

The Point-Slope Form of the Equation of a Line

The point-slope form can be used to find an equation of a nonvertical line passing through two points

(x_1, y_1) and (x_2, y_2).

First, find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

Then use the point-slope form to obtain the equation

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

This is sometimes called the **two-point form** of the equation of a line.

Sketching Graphs of Lines

Sketching Graphs of Lines

Many problems in coordinate geometry are as follows.

1. Given a graph (or parts of it), find its equation.

2. Given an equation, sketch its graph.

The form that is better suited to graphing linear equations is the slope-int. form of the equation of a line, $y = mx + b$.

Slope-Intercept Form of the Equation of a Line

The graph of the equation

$$y = mx + b$$

is a line whose slope is m and whose y-intercept is $(0, b)$.

Example 4 – *Using the Slope-Intercept Form*

Determine the slope and *y*-intercept of each linear equation. Then describe its graph.

a. $x + y = 2$ **b.** $y = 2$

Solution:

a. Begin by writing the equation in slope-intercept form.

$x + y = 2$ **Write original equation.**

$y = 2 - x$ **Subtract from each side.**

$y = -x + 2$ **Write in slope-intercept form.**

Example 4 – *Solution*

cont'd

From the slope-intercept form of the equation, the slope is –1 and the y–intercept is (0, 2).

Because the slope is negative, you know that the graph of the equation is a line that falls one unit for every unit it moves to the right.

b. By writing the equation $y = 2$ in slope-intercept form

$$y = (0)x + 2$$

you can see that the slope is 0 and the -intercept is (0, 2).

A zero slope implies that the line is horizontal.

Sketching Graphs of Lines

Summary of Equations of Lines

1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Slope-intercept form: $y = mx + b$
5. Point-slope form: $y - y_1 = m(x - x_1)$

Sketching Graphs of Lines

From the slope-intercept form of the equation of a line, you can see that a horizontal line ($m = 0$) has an equation of the form

$y = b$ **Horizontal line**

This is consistent with the fact that each point on a horizontal line through $(0,b)$ has a y-coordinate of b. Similarly, each point on a vertical line through $(a,0)$ has an x-coordinate of a So, a vertical line has an equation of the form

$x = a$ **Vertical line**

Sketching Graphs of Lines

This equation cannot be written in slope-intercept form because the slope of a vertical line is undefined. However, every line has an equation that can be written in the general form

$Ax + By + C = 0$ **General form of the equation of a line**

where A and B are not *both* zero.

Parallel and Perpendicular Lines

Parallel and Perpendicular Lines

The slope of a line is a convenient tool for determining whether two lines are parallel or perpendicular.

Parallel Lines

Two distinct nonvertical lines are **parallel** if and only if their slopes are equal. That is,

$$m_1 = m_2.$$

Example 6 – *Equations of Parallel Lines*

Find the slope-intercept form of the equation of the line that passes through the point (2, –1) and is parallel to the line $2x - 3y = 5$.

Solution:

Begin by writing the equation of the given line in slope – intercept form.

$2x - 3y = 5$ **Write original equation.**

$-2x + 3y = -5$ **Multiply by –1**

$3y = 2x - 5$ **Add 2*x* to each side.**

Example 6 – *Solution*

cont'd

$y = \frac{2}{3}x - \frac{5}{3}$ 	**Write in slope-intercept form.**

Therefore, the given line has a slope of

$m = \frac{2}{3}.$

Any line parallel to the given line must also have a slope of $\frac{2}{3}.$ So, the line through (2, –1) has the following equation.

$y - y_1 = m(x - x_1)$ 	**Point-slope form.**

$$y - (-1) = \frac{2}{3}(x - 2)$$

Substitute for y_1, m, and x_1.

$$y + 1 = \frac{2}{3}x - \frac{4}{3}$$

Simplify.

Example 6 – *Solution*

cont'd

$$y = \frac{2}{3}x - \frac{7}{3}$$

Write in slope-intercept form.

Notice the similarity between the slope–intercept form of the original equation and the slope–intercept form of the parallel equation. The graphs of both equations are shown in Figure 1.8.

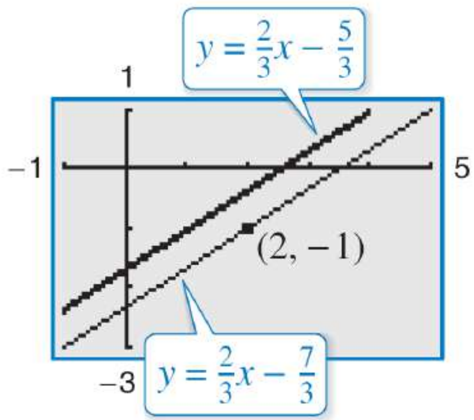


Figure 1.8

Parallel and Perpendicular Lines

Perpendicular Lines

Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other. That is,

$$m_1 = -\frac{1}{m_2}.$$