AP® CALCULUS AB
2010 SCORING GUIDELINES

Question 6

Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

(a) $f'(1) = \left.\frac{dy}{dx}\right|_{(1,\,2)} = 8$

An equation of the tangent line is $y = 2 + 8(x - 1)$.

$2 : \begin{cases} 1 : f'(1) \\ 1 : \text{answer} \end{cases}$

(b) $f(1.1) \approx 2.8$

Since $y = f(x) > 0$ on the interval $1 \le x < 1.1$,

$\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2) > 0$ on this interval.

Therefore on the interval $1 < x < 1.1$, the line tangent to the graph of $y = f(x)$ at $x = 1$ lies below the curve and the approximation 2.8 is less than $f(1.1)$.

$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{conclusion with explanation} \end{cases}$

(c) $\frac{dy}{dx} = xy^3$

$\int \frac{1}{y^3}\,dy = \int x\,dx$

$-\frac{1}{2y^2} = \frac{x^2}{2} + C$

$-\frac{1}{2 \cdot 2^2} = \frac{1^2}{2} + C \Rightarrow C = -\frac{5}{8}$

$y^2 = \frac{1}{\frac{5}{4} - x^2}$

$f(x) = \frac{2}{\sqrt{5 - 4x^2}}, \quad \frac{-\sqrt{5}}{2} < x < \frac{\sqrt{5}}{2}$

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

AP® CALCULUS AB
2010 SCORING GUIDELINES (Form B)

Question 5

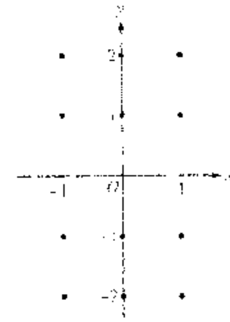
Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

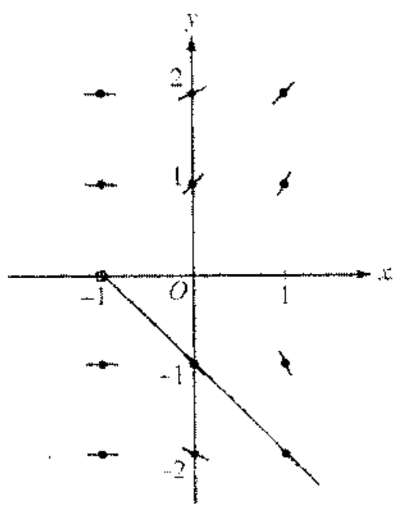
(Note: Use the axes provided in the exam booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.



(a)



3 : { 1 : zero slopes
1 : nonzero slopes
1 : solution curve through $(0, -1)$ }

(b) $-1 = \frac{x+1}{y} \Rightarrow y = -x - 1$

$\frac{dy}{dx} = -1$ for all (x, y) with $y = -x - 1$ and $y \neq 0$

1 : description

(c) $\int y\,dy = \int (x+1)\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + x + C$

$\frac{(-2)^2}{2} = \frac{0^2}{2} + 0 + C \Rightarrow C = 2$

$y^2 = x^2 + 2x + 4$

Since the solution goes through $(0, -2)$, y must be negative. Therefore $y = -\sqrt{x^2 + 2x + 4}$.

5 : { 1 : separates variables
1 : antiderivatives
1 : constant of integration
1 : uses initial condition
1 : solves for y }

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

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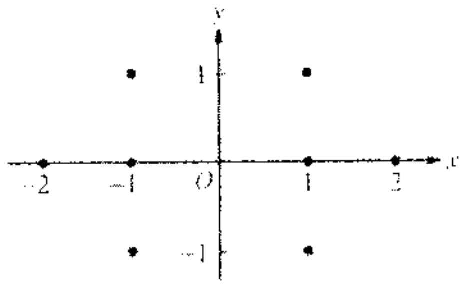
Question 5

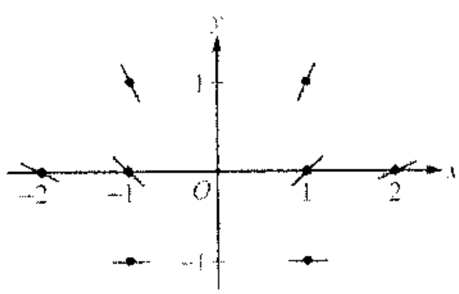
Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.
(Note: Use the axes provided in the pink exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

(a)



2 : sign of slope at each point and relative steepness of slope lines in rows and columns

(b) $\frac{1}{1+y}\,dy = \frac{1}{x}\,dx$

$\ln|1+y| = \ln|x| + K$

$|1+y| = e^{\ln|x| + K}$

$1 + y = C|x|$

$2 = C$

$1 + y = 2|x|$

$y = 2|x| - 1$ and $x < 0$

or

$y = -2x - 1$ and $x < 0$

7 :
- 6 :
  - 1 : separates variables
  - 2 : antiderivatives
  - 1 : constant of integration
  - 1 : uses initial condition
  - 1 : solves for y

  Note: max 3/6 [1-2-0-0-0] if no constant of integration

  Note: 0/6 if no separation of variables
- 1 : domain