Exponent Rules

Parts

- When a number, variable, or expression is raised to a power, the number, variable, or expression is called the base and the power is called the exponent.

$$b^{n}$$

What is an Exponent?

- An exponent means that you multiply the base by itself that many times.
- For example

$$x^4 = x \bullet x \bullet x \bullet x$$

$$2^6 = 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 = 64$$

The Invisible Exponent

- When an expression does not have a visible exponent its exponent is understood to be 1.

$$x = x^1$$

Exponent Rule #1

- When **multiplying** two expressions with the same base you **add** their exponents.

$$b^n \cdot b^m = b^{n+m}$$

- For example

$$x^2 \cdot x^4 = x^{2+4} = x^6$$

$$2 \cdot 2^2 = 2^1 \cdot 2^2 = 2^{1+2} = 2^3 = 8$$

Exponent Rule #1

$$b^n \cdot b^m = b^{n+m}$$

- Try it on your own:

$$1.\ h^3 \cdot h^7 = h^{3+7} = h^{10}$$

$$2.\ 3^2 \cdot 3 = 3^{2+1} = 3^3$$

$$= 3 \cdot 3 \cdot 3 = 27$$

Exponent Rule #2

- When **dividing** two expressions with the same base you **subtract** their exponents.

$$\frac{b^n}{b^m} = b^{n-m}$$

- For example

$$\frac{x^4}{x^2} = x^{4-2} = x^2$$

Exponent Rule #2

$$\frac{b^n}{b^m} = b^{n-m}$$

- Try it on your own:

3. $\dfrac{h^6}{h^2} = h^{6-2} = \boxed{h^4}$

4. $\dfrac{3^3}{3} = 3^{3-1} = 3^2 = \boxed{9}$

Exponent Rule #3

- When raising a **power to a power** you **multiply** the exponents

$$(b^n)^m = b^{n \cdot m}$$

- For example

$$(x^2)^4 = x^{2 \cdot 4} = x^8$$

$$(2^2)^2 = 2^{2 \cdot 2} = 2^4 = 16$$

Exponent Rule #3

$$(b^n)^m = b^{n \cdot m}$$

- Try it on your own

$$5.\ (h^3)^2 = h^{3 \cdot 2} = \boxed{h^6}$$

$$6.\ (3^2)^2 = 3^{2 \cdot 2} = 3^4 = \boxed{81}$$

Note

- When using this rule the exponent can not be brought in the parenthesis **if there is addition or subtraction**

$$(x^2 + 2^2)^2 \neq x^4 + 2^4$$

You would have to use FOIL in these cases

Exponent Rule #4

- When a product is raised to a power, each piece is raised to the power

$$(ab)^m = a^m b^m$$

- For example

$$(xy)^2 = x^2 y^2$$

$$(2 \cdot 5)^2 = 2^2 \cdot 5^2 = 4 \cdot 25 = 100$$

Exponent Rule #4

$$(ab)^m = a^m b^m$$

- Try it on your own

$$7.\ (hk)^3 = h^3 k^3$$

$$8.\ (2 \cdot 3)^2 = 2^2 \cdot 3^2 = 4 \cdot 9 = 36$$

Note

- This rule is for products only. When using this rule the exponent can not be brought in the parenthesis **if there is addition or subtraction**

$$(x+2)^2 \neq x^2 + 2^2$$

You would have to use FOIL in these cases

Exponent Rule #5

- When a quotient is raised to a power, both the numerator and denominator are raised to the power

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

- For example

$$\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$$

Exponent Rule #5

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

- Try it on your own

$$9.\ \left(\frac{h}{k}\right)^2 = \frac{h^2}{k^2}$$

$$10.\ \left(\frac{4}{2}\right)^2 = \frac{4^2}{2^2} = \frac{16}{4} = 4$$

Zero Exponent

- When anything, except 0, is raised to the zero power it is 1.

$$a^0 = 1 \quad (\text{ if a} \neq 0)$$

- For example

$$x^0 = 1 \quad (\text{ if x} \neq 0)$$

$$25^0 = 1$$

Zero Exponent

$$a^0 = 1 \quad (\text{ if } a \neq 0)$$

- Try it on your own

11. $h^0 = \boxed{1} \quad (\text{ if } h \neq 0)$

12. $1000^0 = \boxed{1}$

13. $0^0 = \boxed{0}$

Negative Exponents

- If b ≠ 0, then $$b^{-n} = \frac{1}{b^n}$$
- For example

$$x^{-2} = \frac{1}{x^2}$$

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

Negative Exponents

- If b ≠ 0, then $b^{-n} = \dfrac{1}{b^n}$
- Try it on your own:

14. $h^{-3} = \dfrac{1}{h^3}$

15. $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

Negative Exponents

- The negative exponent basically flips the part with the negative exponent to the other half of the fraction.

$$\left(\frac{1}{b^{-2}}\right) = \left(\frac{b^2}{1}\right) = b^2$$

$$\left(\frac{2}{x^{-2}}\right) = \left(\frac{2x^2}{1}\right) = 2x^2$$

Math Manners

- For a problem to be completely simplified there should not be any negative exponents

Mixed Practice

1. $$\frac{6d^5}{3d^9} = 2d^{5-9} = 2d^{-4} = \frac{2}{d^4}$$

2. $$2e^4 4e^5 = 8e^{4+5} = 8e^9$$

Mixed Practice

3. $\left(q^{4}\right)^{5} = q^{4\cdot 5} = q^{20}$

4. $\left(2lp\right)^{5} = 2^{5} l^{5} p^{5} = 32 l^{5} p^{5}$

Mixed Practice

5. $$\frac{(x^2y)^4}{(xy)^2} = \frac{x^8y^4}{x^2y^2} = x^{8-2}y^{4-2} = \boxed{x^6y^2}$$

6. $$\frac{(x^3x^5)^2}{x^9} = \frac{(x^8)^2}{x^9} = \frac{x^{16}}{x^9} = x^{16-9} = \boxed{x^7}$$

Mixed Practice

7. $(m^6n^4)^2(m^3n^2p^5)^6$

$$= m^{12}n^8 \cdot m^{18}n^{12}p^{30}$$

$$= m^{12+18}n^{8+12}p^{30}$$

$$= \boxed{m^{30}n^{20}p^{30}}$$

Mixed Practice

8. $$\frac{(x-2y)^6}{(x-2y)^4} = (x-2y)^{6-4} = (x-2y)^2$$

$$= (x-2y)(x-2y)$$

F O I L

$$= x^2 - 2xy - 2xy + 4y^2$$

$$= x^2 - 4xy + 4y^2$$

Mixed Practice

9. $$\frac{a^6 d^5}{a^4 d^9} = a^{6-4} d^{5-9} = a^2 d^{-4}$$

$$= \frac{a^2}{d^4}$$